Lecture 3: Detection of Molecules

Salyk, Pontoppidan et al. 2008

Hot water in the planet-forming zones of disks

Figure 1. This figure presents several CO velocity channel maps of disks orbiting around MWC48v and Herbig Ae stars. The CO J=1 data have been obtained with the IRAM array. The angular resolution is about 1 arcminute. The last box in the bottom shows the observed velocity gradient which is in agreement with rotation since the velocity gradient appears to be along the major disk axis.
How do we detect particular species in space?

- Molecules emit & absorb radiation at characteristic frequencies
- Spectroscopy from the ground and space
- Laboratory spectra need to be known in advance
I. Molecular levels
Energy levels of molecules

Electronic Transitions:
$\Delta E = 1-15 \text{ eV}$
Visible-UV

Vibrational Transitions:
$\Delta E = 0.1-1 \text{ eV}$
Infrared

Rotational Transitions:
$\Delta E = 0.01-0.1 \text{ eV}$
(sub)-Millimeter
Electronic transitions (UV-optical)

- Example: H$_2$
- B-X: Lyman system (<1125 Å)
- C-X: Werner system (<1051 Å)
- Ozone, O$_3$: Hartley bands (<3000 Å)

courtesy of E. van Dishoeck (2011)

Abgrall et al. (1993)
Vibrational transitions (IR)

- Zero-point vibrational energy: $\frac{1}{2} \hbar \nu_0$
- Equidistant separation of energy levels only for harmonic approximation
- Many degrees of freedom => various vibrational modes: $3N-5$ (or -6)
Vibrational modes

<table>
<thead>
<tr>
<th>Stretching</th>
<th>$\nu_1$</th>
<th>$\nu_2$</th>
<th>$\nu_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{C}_2\text{H}_2$</td>
<td>3373 cm$^{-1}$</td>
<td>1974 cm$^{-1}$</td>
<td>3295 cm$^{-1}$</td>
</tr>
<tr>
<td>$\text{C}_2\text{D}_2$</td>
<td>2705 cm$^{-1}$</td>
<td>1765 cm$^{-1}$</td>
<td>2439 cm$^{-1}$</td>
</tr>
</tbody>
</table>

$\text{H}_2\text{O}$ vibration modes

<table>
<thead>
<tr>
<th>$\nu_1$</th>
<th>$\nu_2$</th>
<th>$\nu_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3657 cm$^{-1}$</td>
<td>3756 cm$^{-1}$</td>
<td>1595 cm$^{-1}$</td>
</tr>
</tbody>
</table>
# Vibrational modes: hydrocarbons

<table>
<thead>
<tr>
<th>Group</th>
<th>Mode</th>
<th>Frequency (cm(^{-1}))</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH stretch</td>
<td>≡C–H</td>
<td>3280–3340</td>
<td></td>
</tr>
<tr>
<td></td>
<td>=C–H</td>
<td>3000–3100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CO–CH(_3)</td>
<td>2900–3000</td>
<td>ketones</td>
</tr>
<tr>
<td></td>
<td>C–CH(_3)</td>
<td>2865–2885</td>
<td>symmetric</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2950–2975</td>
<td>asymmetric</td>
</tr>
<tr>
<td></td>
<td>O–CH(_3)</td>
<td>2815–2835</td>
<td>symmetric</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2955–2995</td>
<td>asymmetric</td>
</tr>
<tr>
<td></td>
<td>N–CH(_3)</td>
<td>2780–2805</td>
<td>aliphatic amines</td>
</tr>
<tr>
<td></td>
<td>N–CH(_3)</td>
<td>2810–2820</td>
<td>aromatic amines</td>
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<tr>
<td></td>
<td>CH(_2)</td>
<td>2840–2870</td>
<td>symmetric</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2915–2940</td>
<td>asymmetric</td>
</tr>
<tr>
<td></td>
<td>CH</td>
<td>2880–2900</td>
<td></td>
</tr>
<tr>
<td>C≡C stretch</td>
<td>C≡C</td>
<td>2100–2140</td>
<td>terminal group</td>
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<tr>
<td></td>
<td>C–C≡C–C–C</td>
<td>2190–2260</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C–C≡C–C≡C–</td>
<td>2040–2200</td>
<td></td>
</tr>
<tr>
<td>C=C stretch</td>
<td>–HC=C=CH(_2)</td>
<td>1945–1980</td>
<td></td>
</tr>
<tr>
<td></td>
<td>–HC=C=CH–</td>
<td>1915–1930</td>
<td></td>
</tr>
<tr>
<td>CH bend</td>
<td>CH(_3)</td>
<td>1370–1390</td>
<td>symmetric</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1440–1465</td>
<td>asymmetric</td>
</tr>
<tr>
<td></td>
<td>CH(_2)</td>
<td>1440–1480</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CH</td>
<td>1340</td>
<td></td>
</tr>
</tbody>
</table>

Tielens' book (2005)
Rotational transitions (FIR-millimeters)

• Rigid rotors: \( B \sim \hbar^2/2I \), where \( I \) - moment of inertia (depends on mass)

• Non-equidistant separation of energy levels

Energy levels

\[ E_r = B J(J + 1) \]

Selection rule:

\[ \Delta J = \pm 1 \]

Transitions:

\[ E_r(J + 1) - E_r(J) = B [(J + 1)(J + 2) - J(J + 1)] \]

\[ \Delta E = 2B[J + 1] \]


Rotation of polyatomic molecules

Any arbitraryily shaped solid body has three principal axes of rotation (A<B<C), the principle axes are orthogonal and they diagonalize the moment of inertia matrix

\[ I_A \leq I_B \leq I_C \]

Rotational constants:

\[ A = \frac{\hbar^2}{2I_C} \quad B = \frac{\hbar^2}{2I_B} \quad C = \frac{\hbar^2}{2I_C} \]

Class 1: linear molecules (\( I_A = 0, I_B = I_C \))
\( \text{H}_2, \text{C}_2, \text{C}_4, \ldots \)

Class 2: spherical molecules (\( I_A = I_B = I_C \))
\( \text{CH}_4, \text{C}_{60}, \ldots \)

Class 3: symmetric top molecules (\( I_B = I_C \) or \( I_A = I_B \))
\( \text{H}_3^+, \text{NH}_3, \ldots \)

Class 4: asymmetric top molecules (\( I_A \neq I_B \neq I_C \))
\( \text{Water, H}_2\text{CO, etc} \ldots \)

\[ E_r = B J(J + 1) \]

\[ E_r = B J(J + 1) \]

\[ E_r = B J(J + 1) - (A - B)K^2 \]

\[ E_r = B J(J + 1) - (C - B)K^2 \]

No simple general formula
Ro-vibrational transitions

- **P-branch**: $\Delta J = -1$, **R-branch**: $\Delta J = +1$, **Q-branch**: $\Delta J = 0$ (not always allowed)

- Statistical weight $g_J = 2J + 1$

- Quadrupole transitions: $\Delta J = \pm 2$
Gas vs Ice: CO (ro-)vibrational lines

Figure 2.5 The infrared absorption spectrum of CO in the gas phase, at different spectral resolutions (top three curves). To first order, the P and R branches consist of equidistant absorption lines with an absorption intensity that initially increases with $J$ owing to the statistical weight but, eventually, Boltzmann statistics take over. The bottom curve shows the absorption spectrum of solid CO. Absorption increases upwards in each curve. Figure reproduced with permission from L. J. Allamandola, 1984, in Galactic and Extragalactic Infrared Spectroscopy, ed. M. Kessler and P. Phillips, (Dordrecht: Reidel), p. 5.

- High resolution is needed: $R > 1000$
- Solid bands are shifted to lower frequencies

Each other, the peak positions are 3726 and 3709 cm$^{-1}$ (Ar matrix), 3715 and 3699 cm$^{-1}$ (N$_2$ matrix), and 3673 and 3493 cm$^{-1}$ (CO matrix). In crystalline H$_2$O ice, this effect is very strong, on account of H-bonding, and the $\nu_3$ mode is located at 3206 cm$^{-1}$. In contrast for CO, which forms only weak bonds, these shifts are always small; gas phase 2143 cm$^{-1}$, CO isolated in N$_2$ 2140 cm$^{-1}$, CO isolated in Ar 2138.4 cm$^{-1}$, and solid CO 2138.4 cm$^{-1}$. Different modes in a molecule are often affected differently. For water, the stretching mode shifts by some 300 cm$^{-1}$ while the bending mode shifts by only 65 cm$^{-1}$. Finally, besides the peak position, the width of a band is also severely affected by solid-state effects. In the gas phase, the width is set by the Doppler or turbulent broadening of the medium and is, for galactic objects, of the order of 3 km s$^{-1}$ (0.02 cm$^{-1}$).

A species trapped in the solid phase will experience variations in the interaction with the environment from one site to another. This will result in slight shifts in the peak frequency and hence broadening of the line. When the interaction with the environment is small (as for an inert species in a noble gas matrix) this broadening is 1–3 cm$^{-1}$, but this broadening is very large for H-bonding species in an H-bonding environment (e.g., 300 cm$^{-1}$ for solid H$_2$O).
II. Molecular emission and absorption
Transport of Radiation

\[ dI_\nu = -\kappa_\nu I_\nu ds + \varepsilon_\nu ds, \]

with the opacity
\[ d\tau_\nu = -\kappa_\nu ds \]

and the source function
\[ S_\nu = \frac{\varepsilon_\nu}{\kappa_\nu} \]

Then:
\[ \frac{dI_\nu}{d\tau_\nu} = I_\nu + S_\nu \]

Constant source function:
\[ I_\nu = S_\nu (1 - e^{-\tau_\nu}) + I_{\nu,0}e^{-\tau_\nu} \]

- Line shape and width:
  - Intrinsic (t of excited level)
  - Thermal (Tkin)
  - Turbulence, rotation, etc.
Einstein coefficients $B$ & $A$:

$$g_i B_{ij} = g_j B_{ji}, \quad A_{ji} = \frac{2h \nu_j^3}{c^2} B_{ji}, \quad g_i \text{ - statistical weight for level } j$$

Oscillator strength (effective number of classical oscillators for transition):

$$B_{ji} = \frac{4\pi \pi e}{h \nu_{ji} m_e c} f_{ji} = \frac{32 \pi^4 e^2}{3h^2 c} \mu_{ji}^2$$
### Table 2.1

<table>
<thead>
<tr>
<th>Type of transition</th>
<th>( f_{ul} )</th>
<th>( A_{ul} \text{ (s}^{-1} )</th>
<th>Example</th>
<th>( \lambda )</th>
<th>( A_{ul} \text{ (s}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electric dipole</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UV</td>
<td>1</td>
<td>( 10^9 )</td>
<td>Ly( \alpha )</td>
<td>1216 Å</td>
<td>( 2.40 \times 10^8 )</td>
</tr>
<tr>
<td>Optical</td>
<td>1</td>
<td>( 10^7 )</td>
<td>H( \alpha )</td>
<td>6563 Å</td>
<td>( 6.00 \times 10^6 )</td>
</tr>
<tr>
<td>Vibrational</td>
<td>( 10^{-5} )</td>
<td>( 10^2 )</td>
<td>CO</td>
<td>4.67 ( \mu )m</td>
<td>34.00</td>
</tr>
<tr>
<td>Rotational</td>
<td>( 10^{-6} )</td>
<td>( 3 \times 10^{-6} )</td>
<td>CS(^b)</td>
<td>6.1 mm</td>
<td>( 1.70 \times 10^{-6} )</td>
</tr>
<tr>
<td><strong>Forbidden</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optical (Electric quadrupole)</td>
<td>( 10^{-8} )</td>
<td>1</td>
<td>[OIII]</td>
<td>4363 Å</td>
<td>1.7</td>
</tr>
<tr>
<td>Optical (Magnetic dipole)</td>
<td>( 2 \times 10^{-5} )</td>
<td>( 2 \times 10^2 )</td>
<td>[OIII]</td>
<td>5007 Å</td>
<td>( 2.00 \times 10^{-2} )</td>
</tr>
<tr>
<td>Far-IR fine structure</td>
<td>( \frac{2 \times 10^{-7}}{\lambda (\mu \text{m})} )</td>
<td>( \frac{10}{\lambda^3 (\mu \text{m})} )</td>
<td>[OIII]</td>
<td>52 ( \mu )m</td>
<td>( 9.80 \times 10^{-5} )</td>
</tr>
<tr>
<td>Hyperfine</td>
<td></td>
<td></td>
<td>HI</td>
<td>21 cm</td>
<td>( 2.90 \times 10^{-15} )</td>
</tr>
</tbody>
</table>

\(^a\) See text for details.

\(^b\) The \( J = 1 \rightarrow 0 \) transition.

#### Lifetime of an excited level:

\[
\Sigma A_{ul} l^l, l < u
\]
Absorption spectroscopy

- Optical spectroscopy often lacks resolving power
- Measure "equivalent width of line":

\[ W_{\nu} = \int_{-\infty}^{\infty} (1 - e^{-\tau_{\nu}}) d\nu = \int_{-\infty}^{\infty} \left[ \frac{I_{\nu}(0) - I_{\nu}}{I_{\nu}(0)} \right] d\nu \quad \text{Hz} \]
Absorption spectroscopy: diagnostics

- Gas kinematics
- If line is optically thin:

Column density of absorbing molecules: \( N \propto Wv/f \)

Snow et al. (2000)
Formation of emission lines

Depends on physical conditions and distribution of molecules
Population of levels:

Statistical equilibrium for level population for level $i$:

\[
\begin{align*}
    n_i \sum_{j \neq i}^{k} R_{ij} &= \sum_{j \neq i}^{k} n_j R_{ji}, & R_{ij} &= A_{ij} \beta(\tau_{ij}) (1 + Q_{ij}) + C_{ij}, & i > j \\
    n_x &= \sum_{j=0}^{k} n_j, & R_{ij} &= \frac{g_j}{g_i} A_{ij} \beta(\tau_{ij}) Q_{ij} + C_{ij}, & i < j.
\end{align*}
\]

$Q_{ij}$ describes background radiation: dust emission + 2.7K cosmic microwave background,

$\beta(\tau_{ij})$ is escape probability for radiation at depth $\tau_{ij}$,

$C_{ij}$ is collisional (de-) excitation rate

- Escape probability varies through environment => non-linear coupling
- Usually no simple recipe to obtain column density
Useful asymptotic regimes

In optically thin regime:

\[ \frac{n_u}{n_1} = \frac{g_u/g_1 \exp[-E_{ul}/kT]}{1 + n_{cr}/n}, \]

where critical density

\[ n_{cr} = \frac{\sum A_{ul, l<u}}{\sum C_{ul, l \neq u}} \]

If \( n > n_{cr} \) collisions dominate and LTE occurs (Boltzmann eq., \( T = T_{\text{kin}} \)):

\[ \frac{n_u}{n_1} = \frac{g_u/g_1 \exp[-E_{ul}/kT]}{1 + n_{cr}/n} \]

In general case: no LTE, lets define excitation temperature \( T_x \):

\[ \frac{T}{T_x} - 1 = \frac{kT}{E_{ul}} \ln \left( 1 + \frac{n_{cr}}{n} \right) \]

High densities (\( n \ll n_{cr} \)): \( T_x \sim T_{\text{kin}} \)

Low densities (\( n \gg n_{cr} \)): \( T_x < T_{\text{kin}} \)
Molecular emission: diagnostics

Consider thermal equilibrium (LTE): $T_X = T_{\text{kin}}$

In a uniform cloud the source function $S_\nu = B_\nu$:
$S_\nu = B_\nu (T_X) = 2h\nu^3/c^2 \left( \exp(h\nu/kT) - 1 \right)^{-1}$

Radiation transfer equation:
$I_\nu = B_\nu (T_X) \left( 1 - e^{-T_\nu} \right) + I_{\nu,0} e^{-T_\nu}$

Radio-frequencies => Rayleigh-Jeans limits ($h\nu \ll kT$):
$B = 2k\nu^2/c^2 T$

Then radiation transfer equation:
$T = J_\nu (T_X) \left( 1 - e^{-T_\nu} \right) + J_{\nu,0} (T_{bg}) e^{-T_\nu}, J_\nu = h\nu/k \left( \exp(h\nu/kT) - 1 \right)^{-1}$

Emission line - background:
$T_r = (J_\nu(T_X) - J_{\nu,0}(T_{bg})) \left( 1 - e^{-T_\nu} \right)$
**Molecular emission: diagnostics**

Plane-parallel infinite, homogeneous slab of turbulent gas:

- LTE and low optical depth, intensity scales with column density $N_u$:

\[
I = \frac{A_{ul} N_u \hbar \nu_{ul}}{4\pi}
\]

- LTE and high optical depth, intensity scales with $T$ and line width $b$:

\[
I \approx B(T) \frac{\nu b}{c}
\]

From molecular emission lines:

1) **Line strength: column densities**
2) **Line ratios (ladder): temperatures, densities**
3) **Line profiles: kinematics**
Pause
Earth atmosphere

- Hard to observe at 20\(\mu m\) – 1 mm => H\(_2\)O, OH, CO\(_2\),... absorption
Parameters of telescopes

• Angular resolution: $\theta'' \sim 2.1 \times 10^5 \frac{\lambda}{D}$, where $D$ - mirror diameter
  - UV: $\lambda \sim 1000$ Å, resolution $\sim 0.02''$ for $D = 100$ cm
  - Radio: $\nu = 230$ GHz (1mm), resolution $\sim 20''$ for $D = 100$ cm

• Spectral resolution, $R$:
  - UV-optical: up to $R = 100,000$
  - IR: $R \sim 10^3-10^4$
  - Radio: $R = 10^6$

• Sensitivity: minimum detectable signal above noise:
  - CCD (optics): $<80-90\%$ of quantum limit (single photons),
  - Superheterodyne receivers (FIR-radio): $\sim$ the same
UV-optical detections

• Historically the first technique: CH, CN, CH\(^+\) (1937-41), CO and H\(_2\) (1970), N\(_2\) in 2004
• Selection rules are not as restrictive: e.g. H\(_2\) and N\(_2\) do not usually emit at IR and radio

• Strength:
  - Less difficult to obtain data than for IR
  - CCD => high quality spectra

• Weakness:
  - Laboratory data for complex molecules not easy to acquire
  - Some molecules do not have intense electronic transitions
  - Dissociative excited states lead to broad features
  - FUV is only accessible from space
UV-optical telescopes

- **Ground:** Very Large Telescope (VLT; 8.2 m), Keck (10 m), Gemini (8.2 m), Large Binocular Telescope (LBT; 10 m), Subaru (8 m), Palomar (5 m),...
- **Space:** Hubble (2.4 m), Copernicus (0.5 m), FUSE (0.7 m),...
- **Resolutions:** <0.1" (10 AU at 100 pc)
- **Diffuse Interstellar Bands (DIBs), CO, H₂, N₂, OH, ...** (check respective web sites!)
Infrared detections

• Non-polar polyatomic molecules and ices
• Near-IR electronic spectra of radicals?

• Intense vibrational transitions: H$_2$CO, HF (recent), OH, benzene, C$_3$, C$_5$

• Strength:
  - Numerous (ro-)vibrational lines => identification
  - High sensitivity

• Weakness:
  - Atmosphere is largely opaque
  - Limited space missions
  - A less precise identification than via radio (resolution)
IR telescopes

- IRAS (1983): 0.6 m, 12–100 μm, first sky survey, dust (β Pictoris disk)

- Infrared Space Observatory (1995–1998): 0.6 m, 2.5–240 μm, dust & molecules (H₂O, HF, OH, OI, C₆H₆, CH₃, CO₂, ...), infrared cirrus clouds

- Spitzer Space Telescope (2003–2009): 0.8 m, 3–180 μm, high-sensitivity imaging and mapping, dust & molecules (OH, H₂O, C₂H₂, ...)

- Herschel Space Observatory (2009–2012): 2.4 m, 60–670 μm, high-sensitivity imaging and mapping, dust & molecules (CH₃OH, H₂S, HCN, SO₂, H₂CO, H₂O, ...)

- Ground-based: Keck, VLT, Subaru,...
SOFIA (Stratospheric Observatory For Infrared Astronomy)

- NASA and DLR, German Aerospace Center joint mission
- Operation: 2010 (20 years lifetime)
- The largest airborne observatory in the world
- Telescope: 2.5 meter diameter, 240 K temperature
- Spectral coverage: 1–700 μm (3 THz–430 GHz)
IR vs (sub-)mm telescopes

Submillimeter:
- Very high spectral resolution (R > 10^6, < 0.1 km/s)
- Many gas-phase molecules: abundances > 10^{-11}
- Mapping of emission
- Ground-based (<1 THz), long lifetime

Infrared:
- Moderate spectral resolution (R ~ 10^3–10^4)
- Gases and ices: abundances > 10^{-8}
- Probe major reservoirs of C, N and O
- Molecules without permanent dipole moments (H_2, C_2H_2, CH_4, CO_2, CH_3, ...)
- Absorption & emission
- If in space, must be cryogenically cooled => short lifetime

Courtesy of E. van Dishoeck (2011)
Radio detections

• Unambiguous detection of molecular species
• Searches are based on rotational Hamiltonians based on laboratory data

• Sensitivity goes with square of the molecular dipole moment

• First radio detection: OH in 1963; NH$_3$, H$_2$O, H$_2$CO, HCO$^+$ in 1970 (first positive ion), C$_6$H$^-$ in 2006 (first negative ion)

• Strength:
  - Observation of several lines at expected frequencies => clear identification
  - High sensitivity (complex organics was found)

• Weakness:
  - Bias towards polar molecules
  - Symmetric species cannot be observed
(Sub-)millimeter telescopes

Submillimeter emission (0.4-1 mm; 200-800 GHz)

Single-dish telescopes:
IRAM 30m, JCMT 15m, CSO 10m, APEX 12m, HHT 10m

Typical beam sizes 12–30": 2000-6000 AU at 150 pc

Interferometers:
IRAM 6×15m, CARMA 610m+9×6m, SMA 8×6m, Nobeyama 6×10m, early ALMA: 16×12m

100 GHz: 2-5" ~ 300-500 AU at 150 pc
230 GHz: 0.5-1" ~100 AU
Future: Atacama Large Millimeter Array

- ALMA – largest (sub-)mm interferometer, Chile
- 3 billions $: USA, EU, Japan, Taiwan
- Lifetime: 30 years
ALMA specifications

- Antennas: 54 12-m и 12 7-m (at 5000 m)
- Antenna configurations: 150 m ~18 km
- Angular resolution: ~0,04" at 100 GHz
- Frequency resolution: <40 m/s
- 10 bands, 31–950 GHz (0.35–7 mm)

Comparison with Plateau de Bure interferometer:
- Frequencies: 31–950 GHz vs. 80–270 GHz
- Angular resolution: >0.005" vs. 0.3–5"
- Sensitivity: >20x higher
Homework

• Check web sites of key telescopes: FUSE, VLT, Subaru, Keck, Plateau de Bure interferometer, ALMA

• Calculate angular resolution (in ")
  a) for ALMA at frequency 950 GHz, assuming overall "diameter" of the interferometer of 15 km,
  b) for the IRAM 30-m antenna at 230 GHz,
  c) for FUSE satellite at 912 Å (D=0.7 m)

• Calculate critical densities, assuming radiative and collisional (de-)excitation within a 2-level system:
  a) HCO+, J=1–0, ν = 89 GHz, A_{21}=4.2 \times 10^{-5} \text{ s}^{-1}, C_{21}=2.6 \times 10^{-10} \text{ cm}^3 \text{ s}^{-1}
  b) CS, J=1–0, ν = 49 GHz, A_{21}=1.7 \times 10^{-6} \text{ s}^{-1}, C_{21}=3.7 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1}
  c) CS, J=5–4, ν = 245 GHz, A_{21}=3.0 \times 10^{-4} \text{ s}^{-1}, C_{21}=3.7 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1}
**Homework**

• How large spectral resolution $R$ should be in order to resolve electronic-vibronic-rotational ladder of molecules, assuming a typical electronic transition of 10 eV and typical rotational transitions of 0.1 meV?

• How large will be a width of molecular lines in molecular clouds, assuming $T_{\text{kin}} = 10$ K and turbulent motions of 100 m/s?

• Assuming total particle density of $n_H = 10$ cm$^{-3}$ and $10^6$ cm$^{-3}$, $T_{\text{kin}} = 10$ K, calculate roughly lifetimes of excited electronic, vibrational, and rotational excited levels and compare it with the 2 corresponding collisional lifetimes. The values of representative $A_{ul}$ could be found on page 18.
Suggested literature


• Master course in Astrochemistry, Ewine van Dischoek (2011): http://www.strw.leidenuniv.nl/~sanjose/astrochem

• T. Wilson et al., "Tools of Radioastronomy" (2009), Springer

• Frederick R. Chromey, "To Measure the Sky: An Introduction to Observational Astronomy" (2010), Cambridge Uni. Press