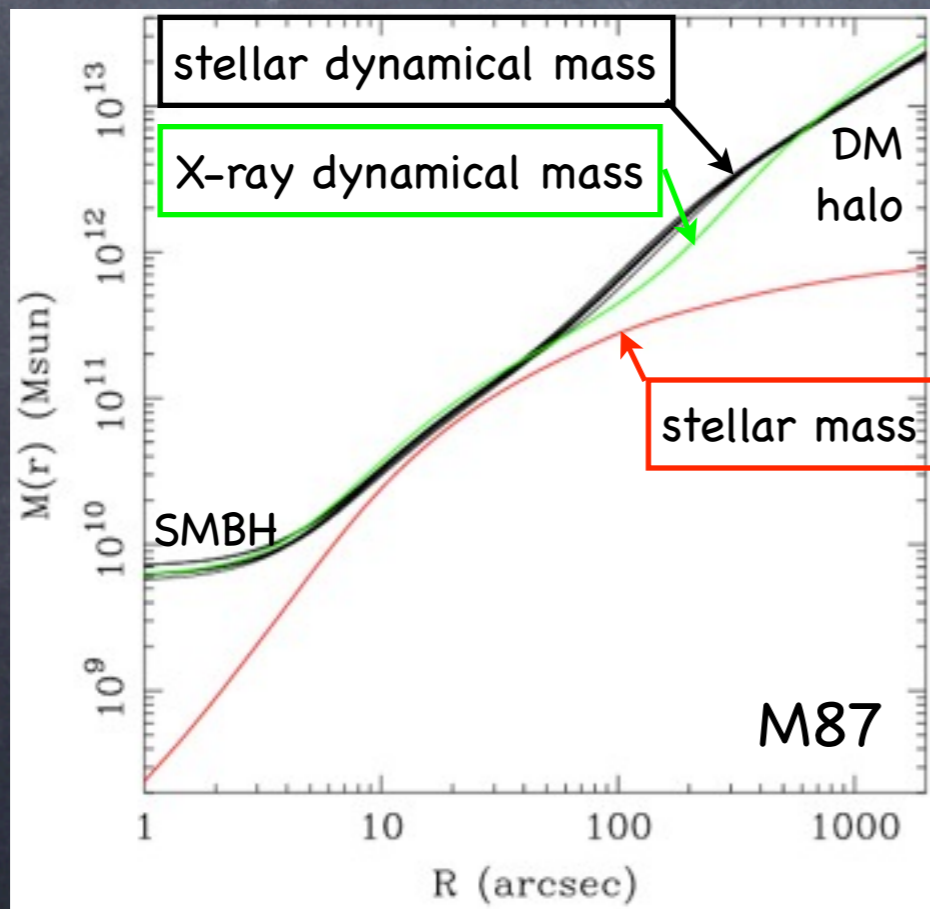


Overview of Dynamical Modeling

Glenn van de Ven
glenn@mpia.de

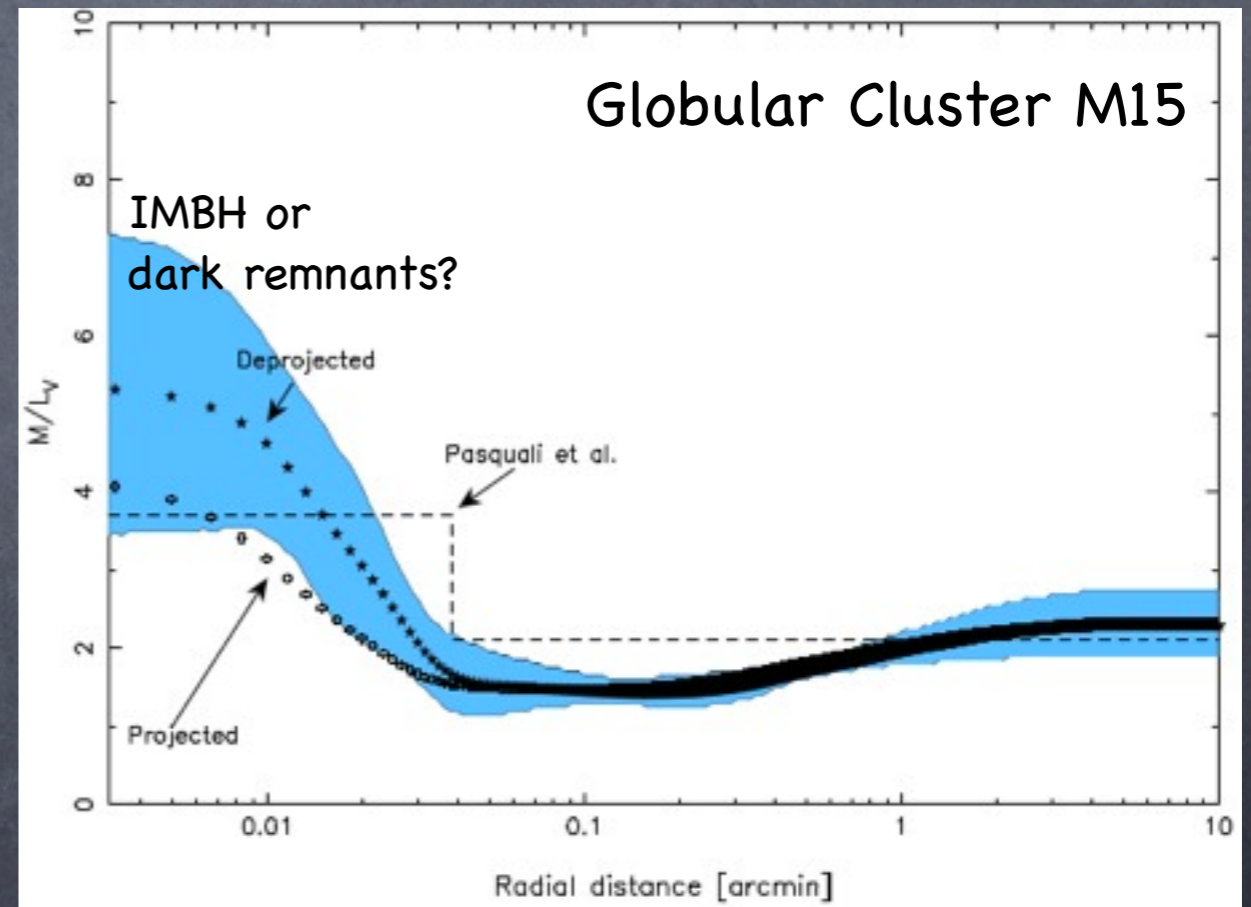
Why dynamical modeling? -- mass

- total mass stellar systems key is to their evolution
- compare luminous mass: constrain DM and/or IMF
- DM radial profile and shape agree with LCDM?
- central black hole mass



Gehardt & Thomas (2009)

Ringberg Dynamics, 10.04.2012



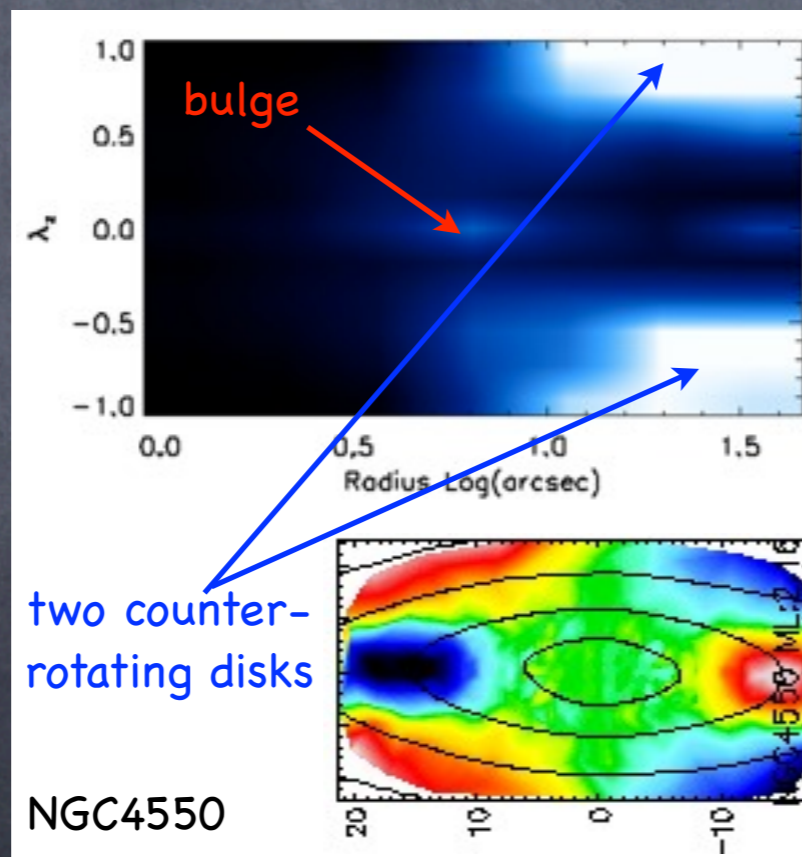
den Brok et al. (2012)

Glenn van de Ven, "Overview of Dynamical Modeling"

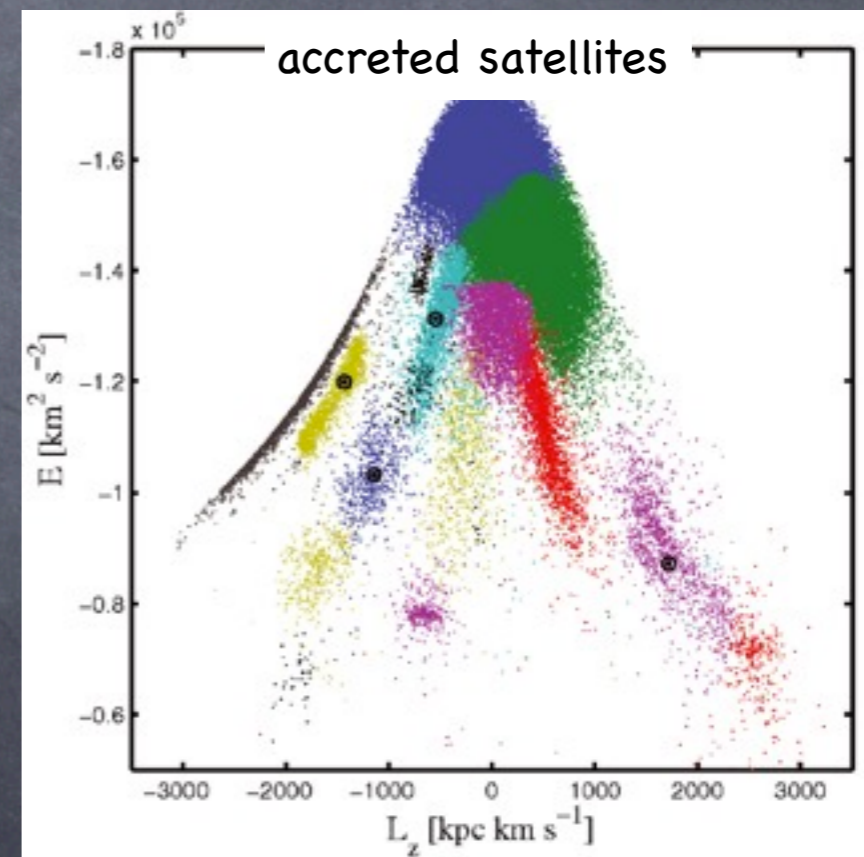
Why dynamical modeling? -- DF

- distribution function of (sub-)populations
- dynamical decomposition: bulge, disk, halo
- stellar merger/accretion conserved in phase-space
- chemical tagging: growth of disk and bulge

specific ang.
momentum
short axis



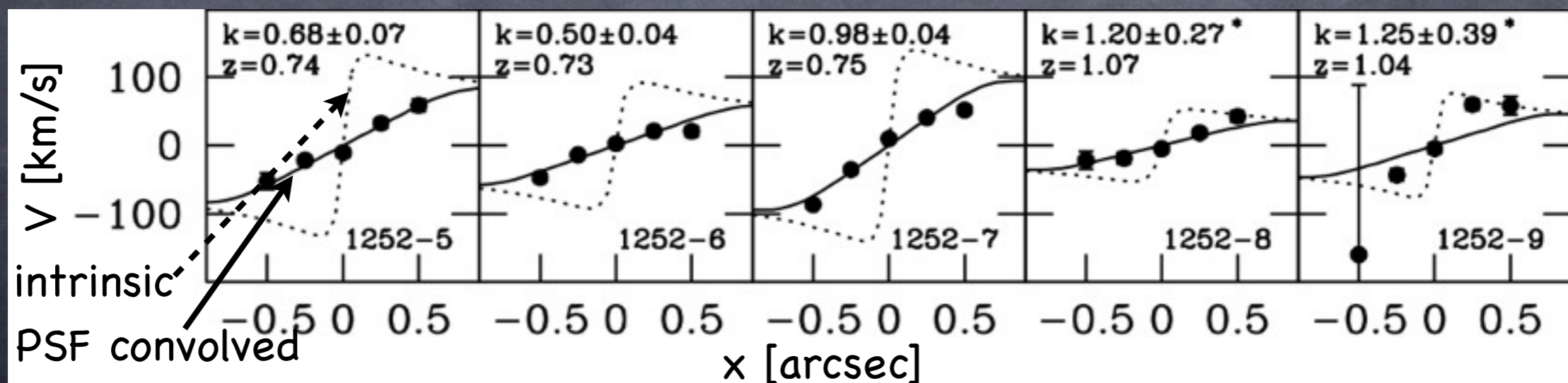
van den Bosch et al. (2012)



Gomez et al. (2010)

Kinematic tracers: integrated spectra

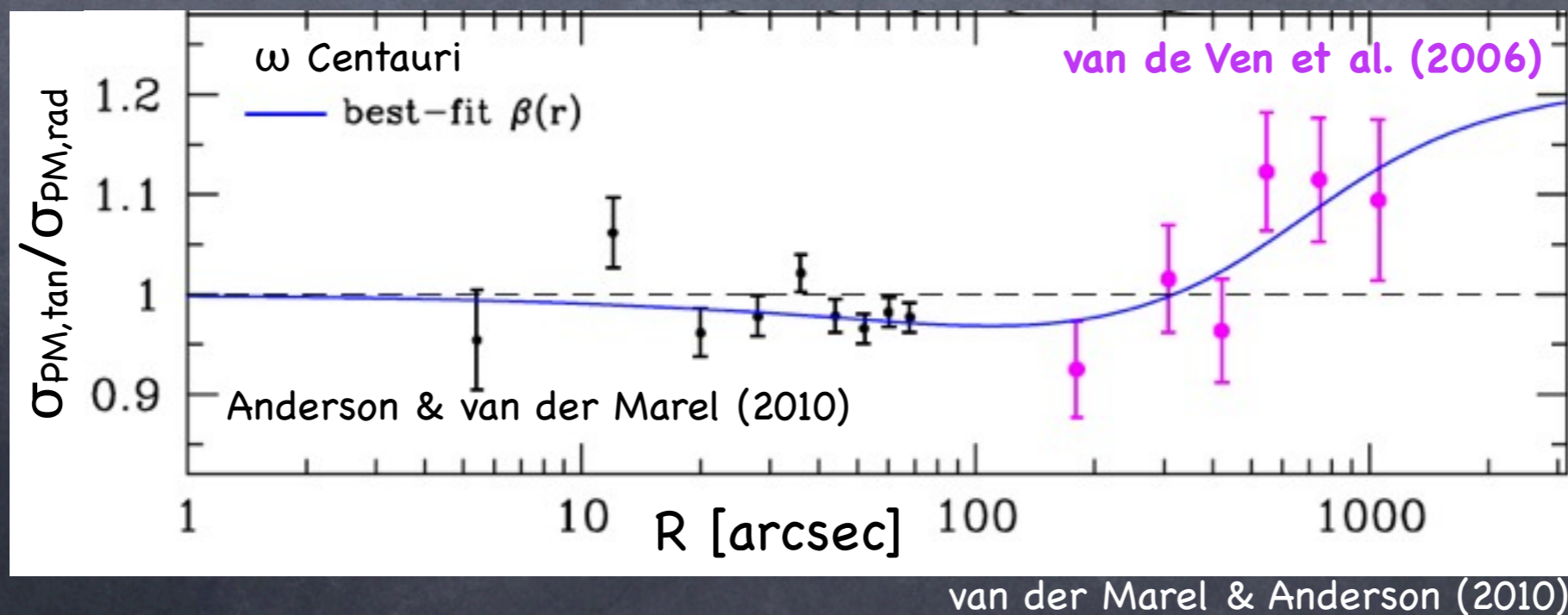
- inner parts external galaxies & distant universe; random errors mostly limited by exposure time
- observed spectrum = stellar spectra \otimes LOSVD
line-of-sight velocity distr. moments: V , σ , h_3 , h_4 , ...
- aperture: single value; physical size depends on z
- long-slit: radial profile; multiplexing at high(er) z
- integral-field spectroscopy: maps; uncover kinematical substructures, better constraints mass and DF



van der Wel & van der Marel (2008)

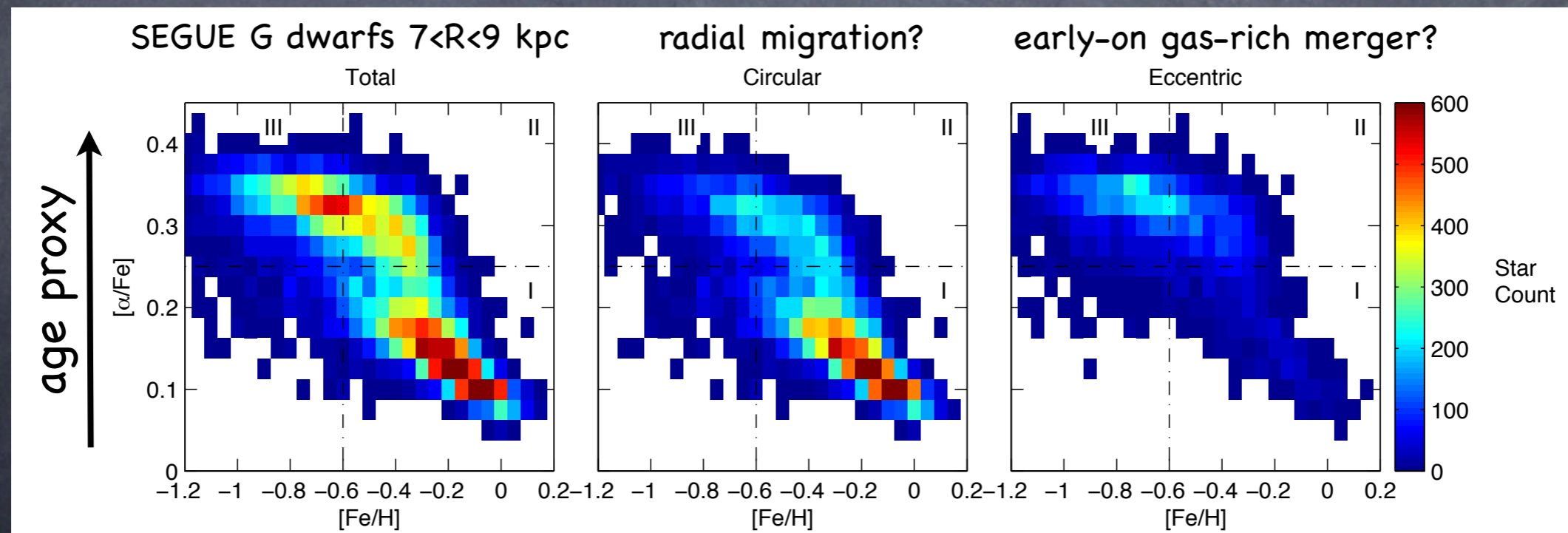
Kinematic tracers: discrete velocities

- outer parts external galaxies / groups & Local Group; random errors mostly limited by numbers [$d\sigma/\sigma=(2N)^{-1/2}$]
- observed spectrum of single, resolved object:
 - line-of-sight velocities (v_{los}): satellites, PNe, GCs, resolved stars
 - proper motions (μ_α & μ_δ): Galactic GCs, dSph galaxies? direct measure of velocity anisotropy $\beta = 1 - \sigma_{\text{tan}}^2/\sigma_{\text{rad}}^2$
 - plus distance ($\alpha, \delta, D, \mu_\alpha, \mu_\delta, v_{\text{los}}$): Milky Way stars



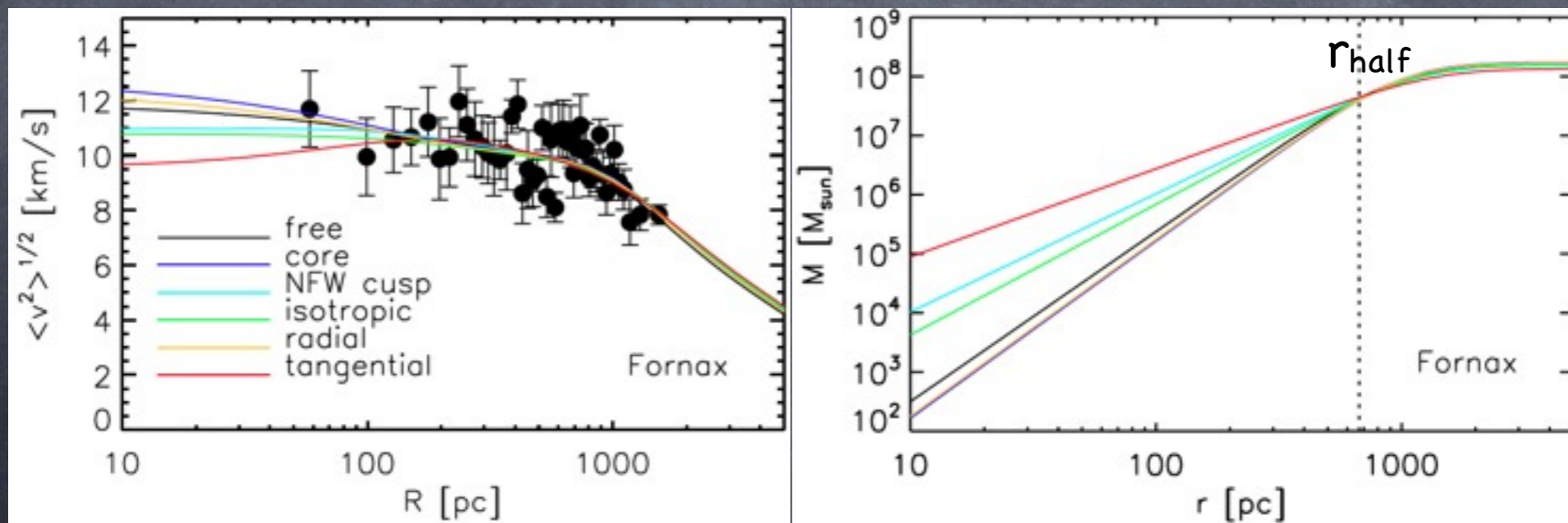
Kinematic tracers + chemical tags

- chemical tags are true integrals of motion
- integrated spectra: break degeneracy multiple stellar populations; additional info from UV & FIR?
- resolved stars photometry: (rough) estimate $[Fe/H]$
- resolved stars spectra: $[Fe/H]$, $[a/Fe]$ age proxy; beyond requires expensive high spectral resolution



Modeling methods: virial mass

- Scalar virial theorem: $M = v_{\text{RMS}}^2 r_g / G$
Virial mass estimate: $M = K \sigma_{\text{los}}^2 r_h / G$, $K \neq \text{constant}$:
 - r_g/r_h depends on surface brightness distribution, mass-to-light ratio profile (incl. dark halo), viewing direction
 - $v_{\text{RMS}}/\sigma_{\text{los}}$ depends velocity anisotropy, (model beyond) radial extend kinematic tracers, viewing direction
- still works due to insensitivity beyond $\sim r_h$, but dynamical modeling needed to go beyond single mass value

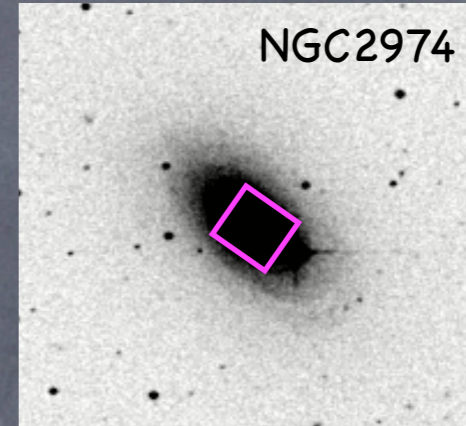


Walker (2011)

Modeling methods: Jeans equations

$$\frac{GM_{tot}(<R)}{R} = -R \frac{d\Phi}{dR} = V_c^2(R)$$

tracer density

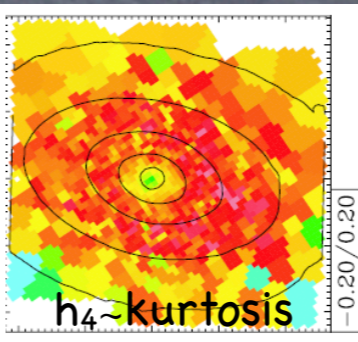
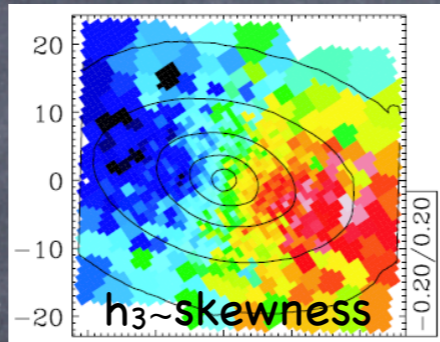
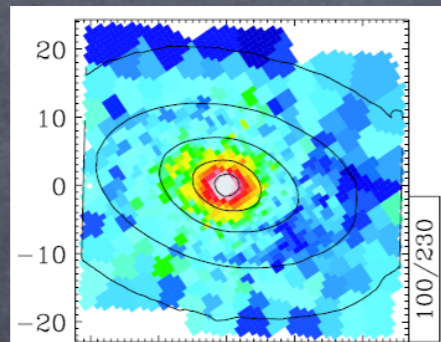
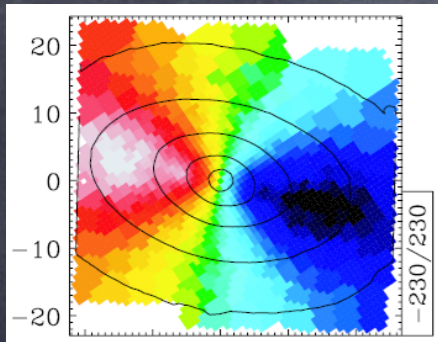


$$V_c^2(R) = V_\phi^2 + \sigma_R^2 \left[\frac{\partial \ln(v \sigma_R^2)^{-1}}{\partial \ln R} + \dots \right]$$

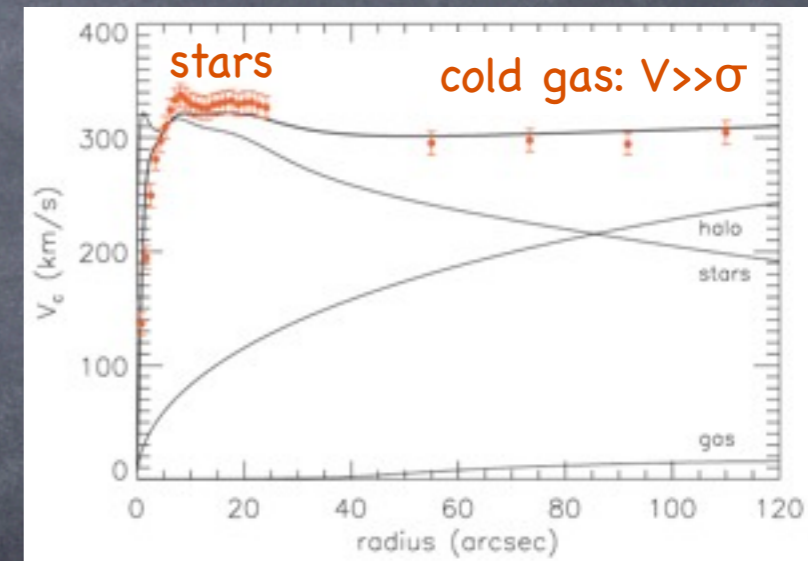
mean velocity

velocity dispersion

velocity anisotropy



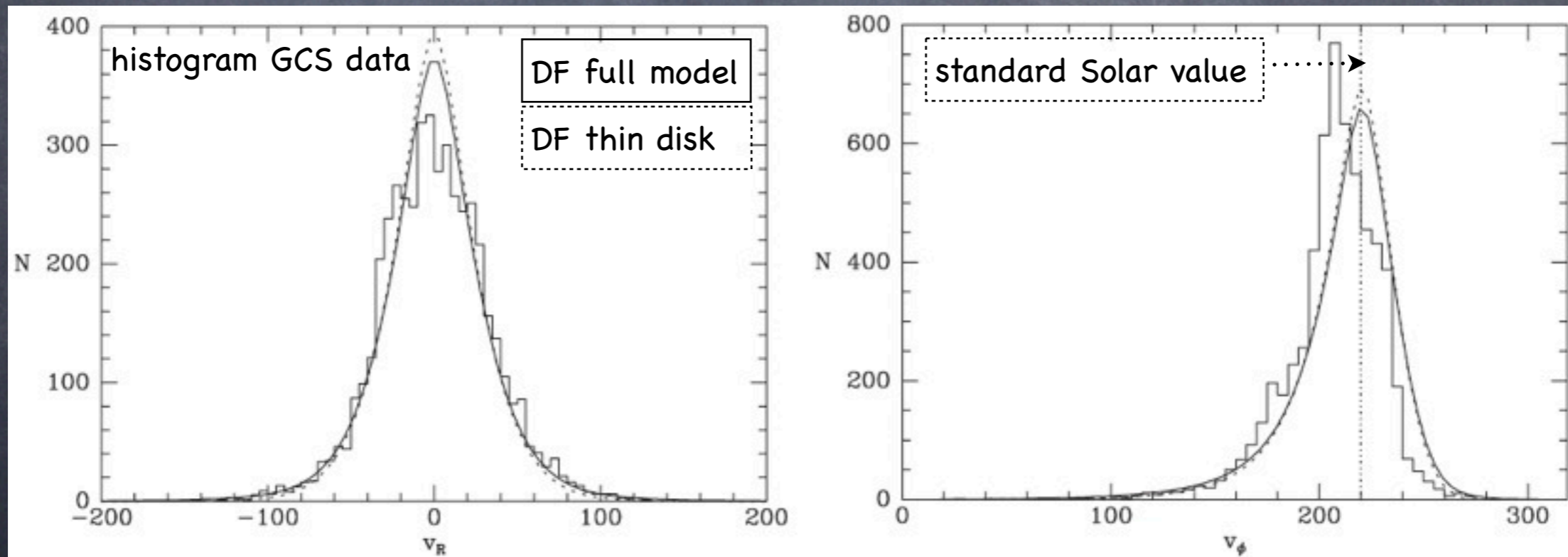
Weijmans et al. (2008)



Velocity anisotropy? Axial symmetry? DF non-negative?

Modeling methods: distribution function

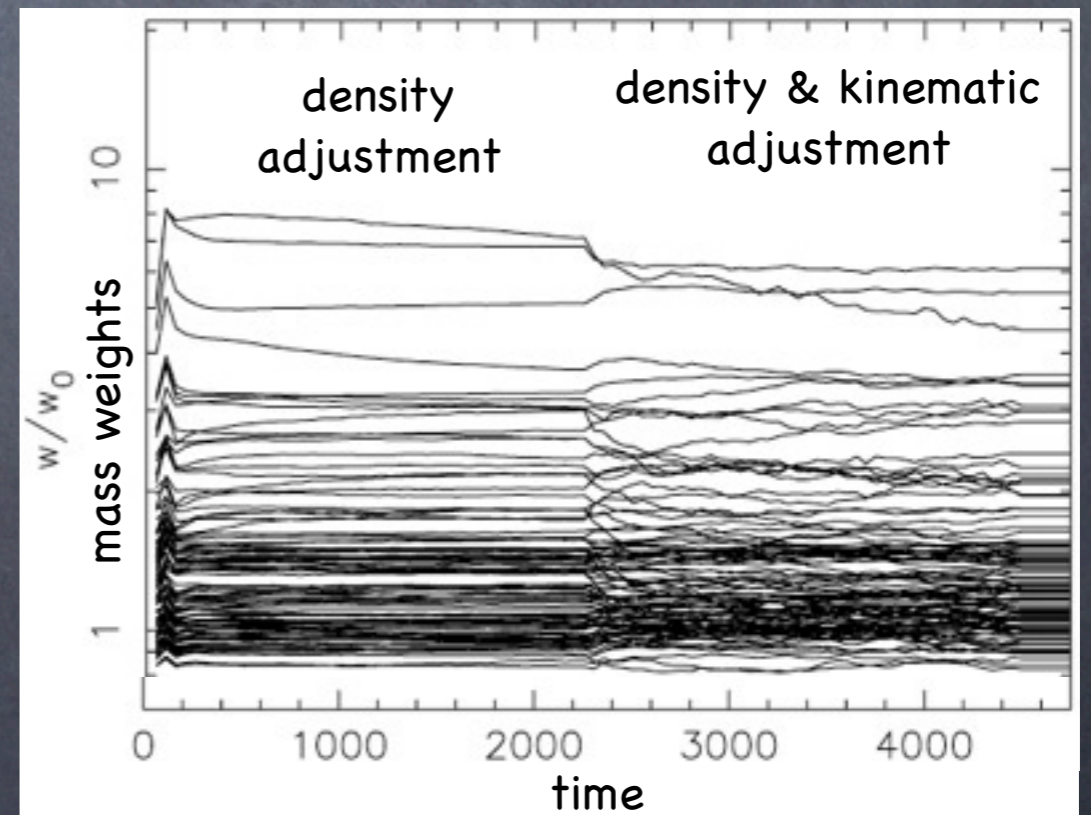
- Steady-state equilibrium: DF depends on 3 integrals of motion: energy E , second $I_2 = \{L, L_z, L_x, \dots\}$, third $I_3 = ?$
- Analytical DFs in literature, but specific forms assumed and I_3 unknown (exception separable Stäckel potentials)
- Numerical DFs via mass weighted superposition of orbits (Schwarzschild's method) or tori fitted to obs. kinematics



Binney (2010), McMillan & Binney (2010)

Modeling methods: particles

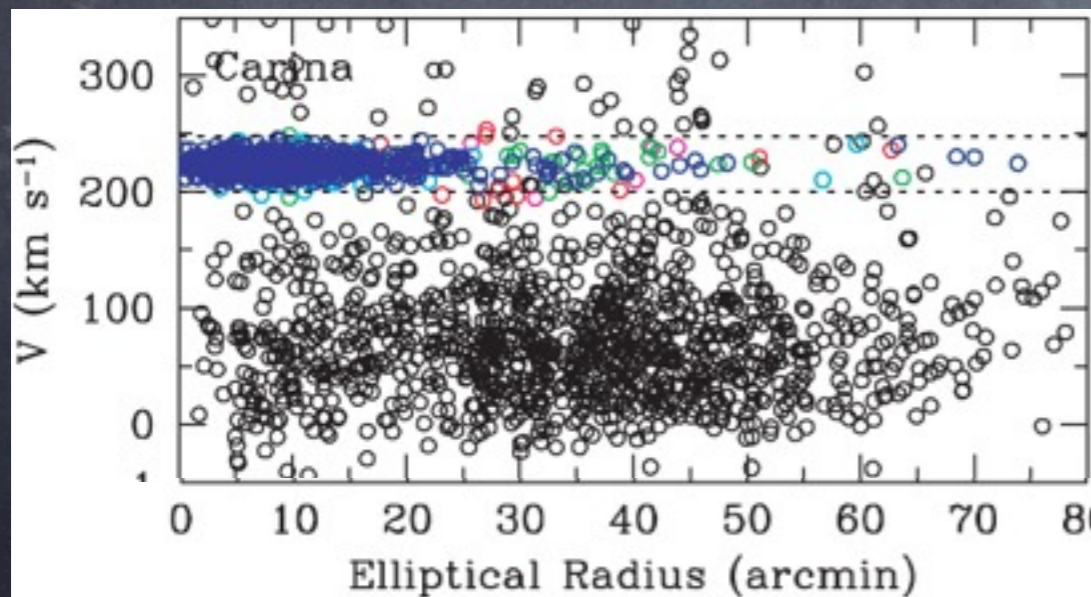
- evolving, non-equilibrium systems with particle models:
 - asymmetric mass distribution: warps, lopsided, mergers
 - rotating components: bars and spiral arms
 - evolution particles and gravitational potential & stability
- but computationally expensive and unknown initial conditions
- Made-to-Measure N-body: vary particle masses to converge toward observations (Syer & Tremaine 1996)



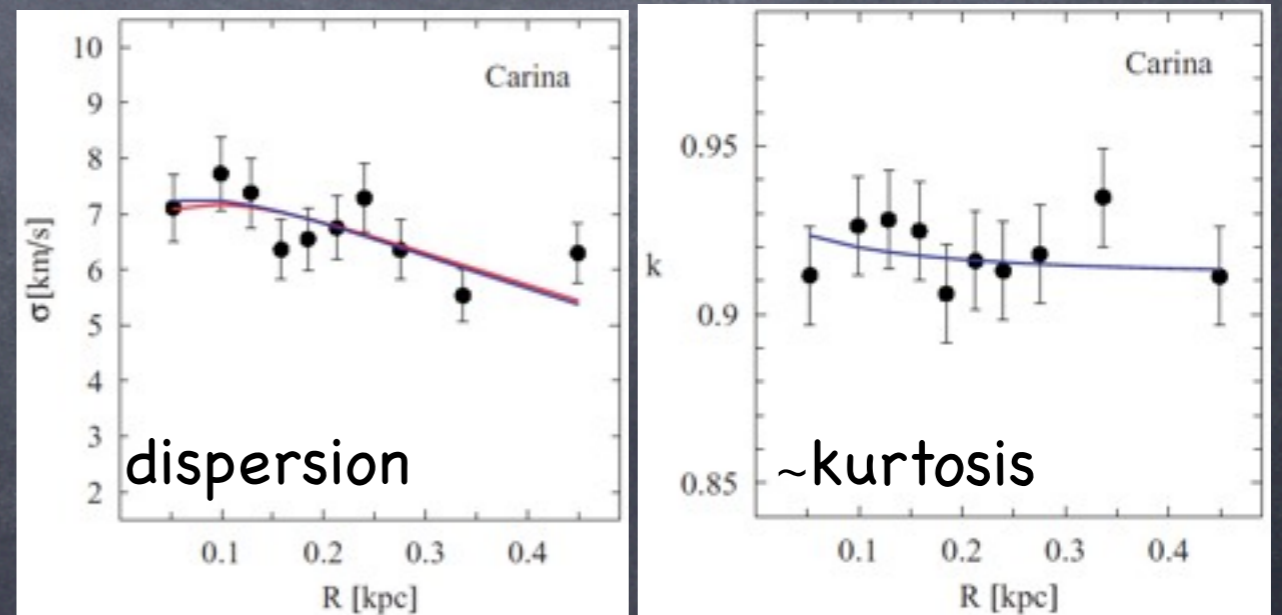
de Lorenzi et al. (2007)

Inference: velocity moments

- integrated spectra:
 - now fit extracted moments or histogram LOSVD
 - fit spectra directly using observed/model stellar templates?
- discrete velocities:
 - better cleaning for non-members also removes more members
 - more spatial binning to extract higher-order velocity moments
 - tracer density from photometry, but often challenging corrections for incompleteness and selection biases



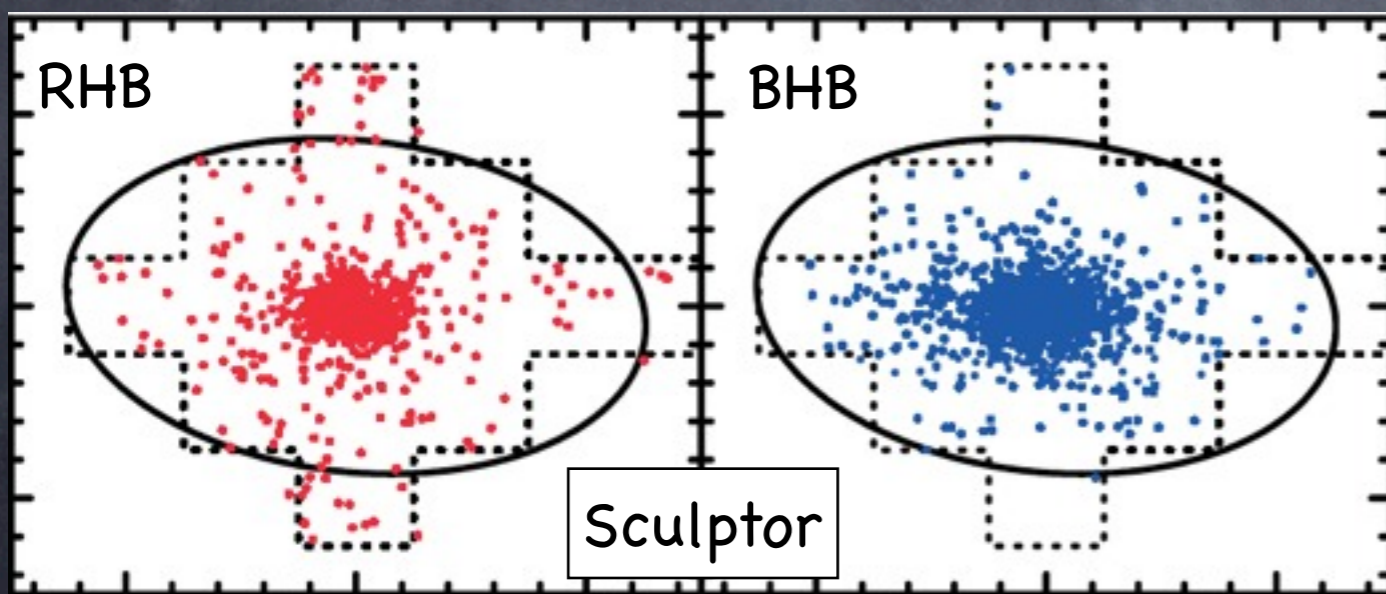
Walker et al. (2009)



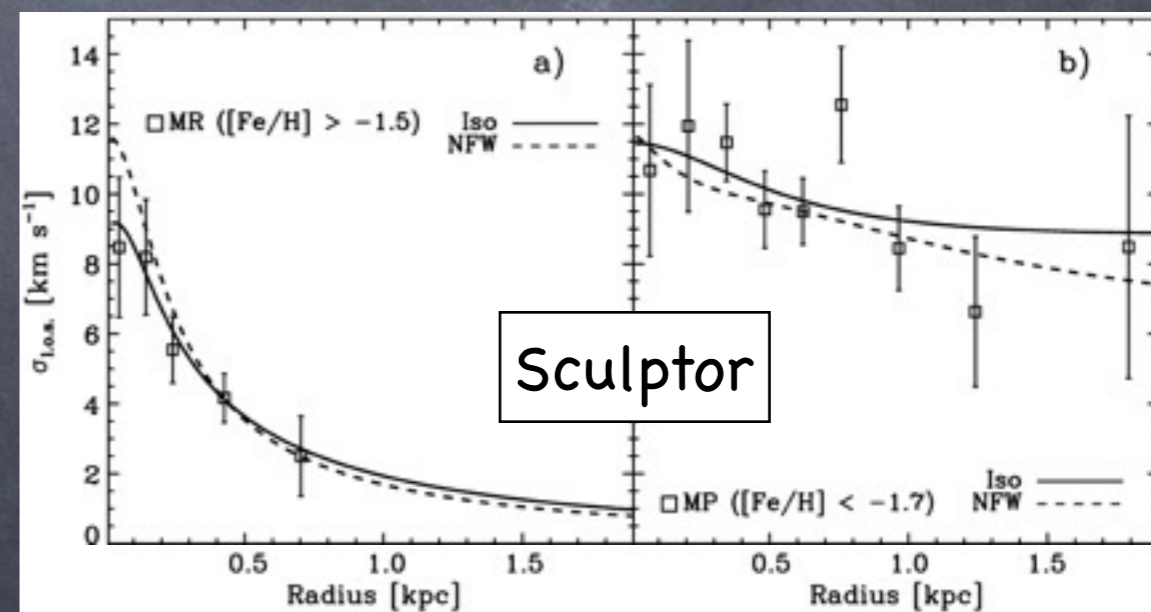
Lokas (2009)

Inference: discrete likelihood

- likelihood fitting discrete velocities
 - better spatial resolution; improve constraint (IM)BH?
 - non-members via background model in likelihood
 - computationally expensive; first constrain parameter range with simpler, faster (Jeans) models
- chemical tags
 - velocity moments: "hard cut" in e.g. $[\text{Fe}/\text{H}]$
 - discrete: additional chemical model in likelihood



Tolstoy et al. (2004)



Battaglia et al. (2008)

This workshop:

Dynamics meets kinematic tracers

- Kinematic tracers:
 - Session 1 – Hans-Walter Rix: What data can do for you
 - Session 2 – Discrete Data: incompleteness and selection effects
- Modeling methods:
 - Session 1 – James Binney: What models can do for you
 - Session 3 – Modeling approaches: speed versus accuracy
- Session 4 – Inference: the challenges of fitting models to data
- Session 5 – Results 1: versatile dynamics
- Session 6 – Results 2: cores or cusps?
- Session 7: Next steps