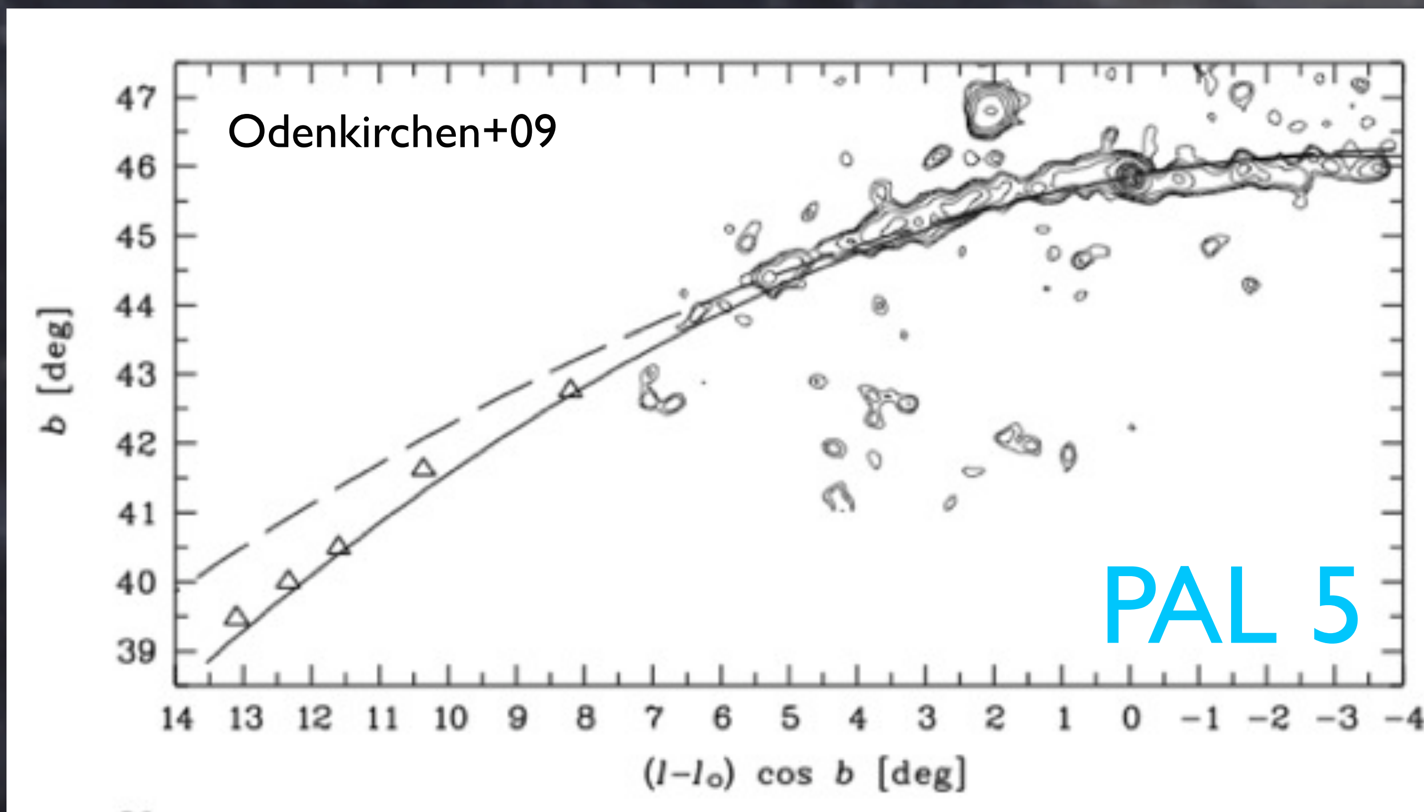
Measuring the Milky Way potential without dynamical models

Jorge Peñarrubia (IAA-CSIC)

in coll. w/ Sergey Koposov & Matt Walker

Ringberg 12th April 2012

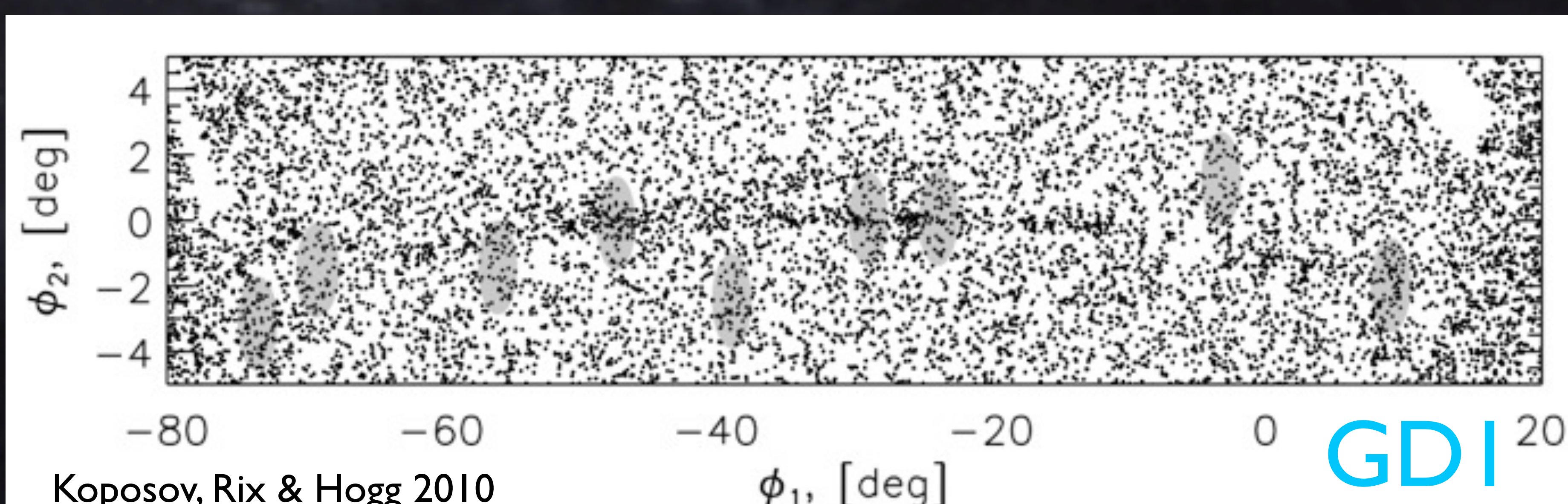
Tidal Streams as tracers of the potential



Stars moving on very similar orbits

↓

Strong constraints on MW potential



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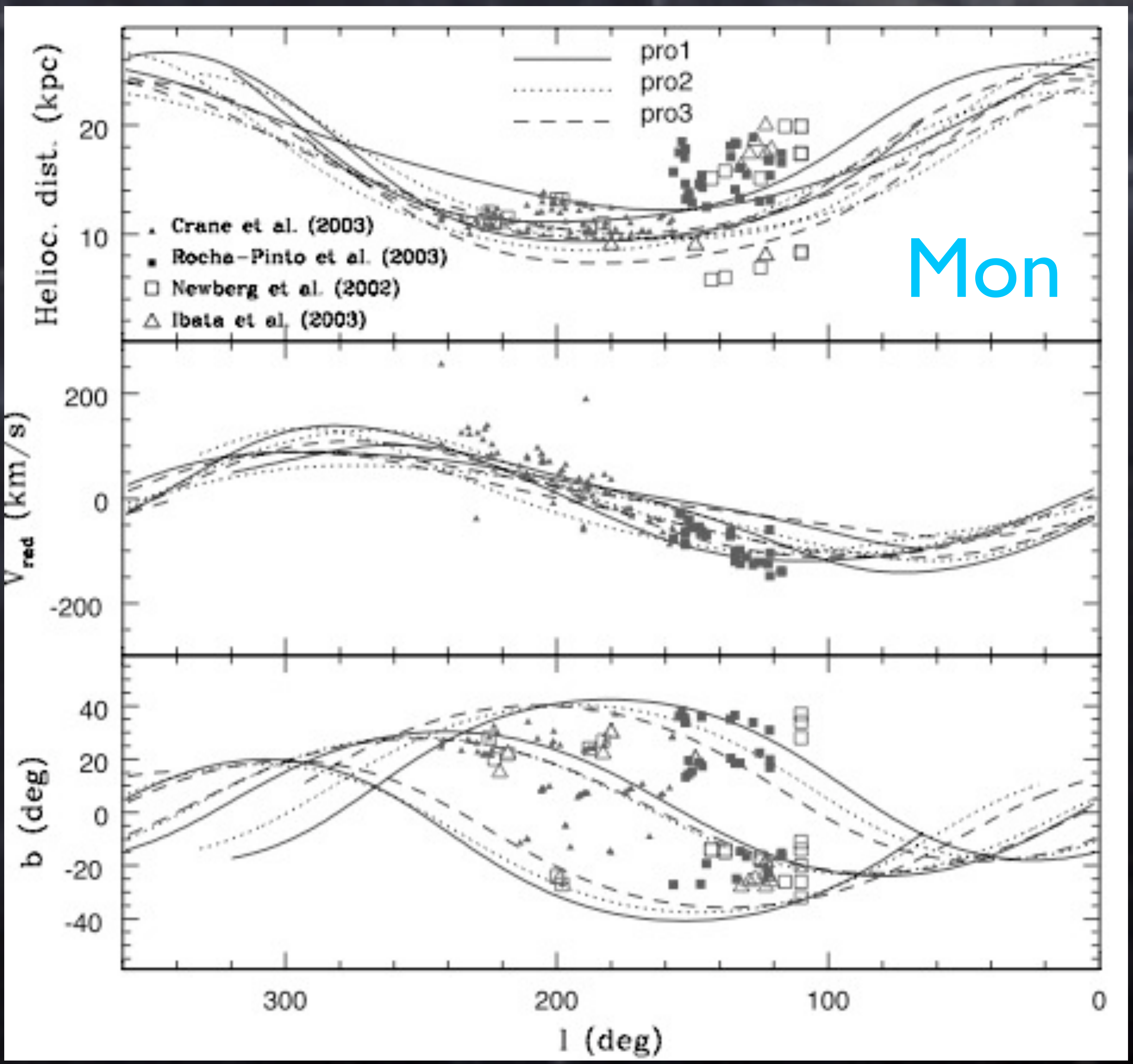

JP, Martinez-Delgado, Rix+05

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Strong constraints on MW potential

ISSUES:

- ★ **Complexity in the stream**
- ★ **Non-uniform maps**
- ★ **Membership probabilities**
- ★ **Phase-space mixing with age**

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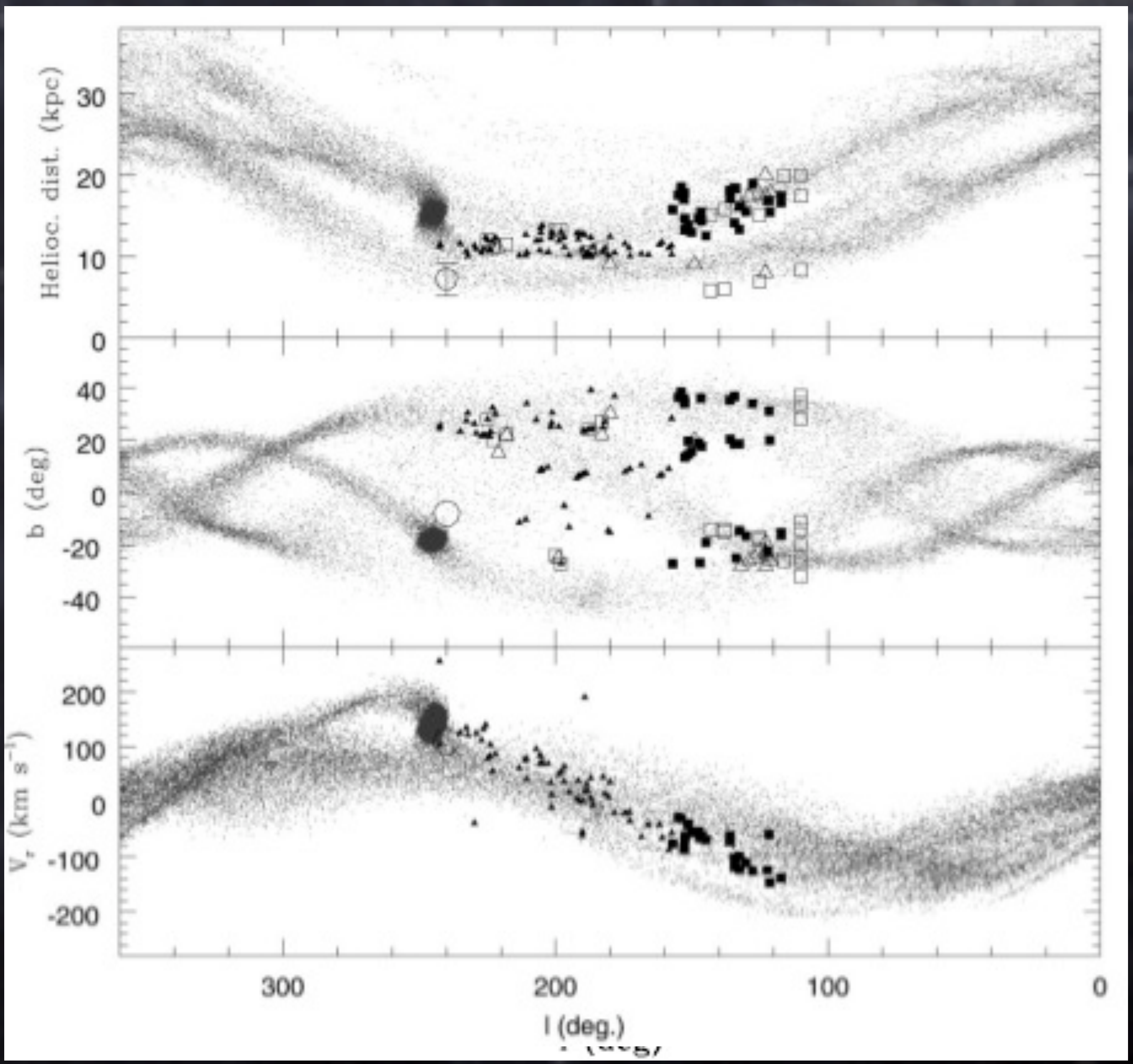

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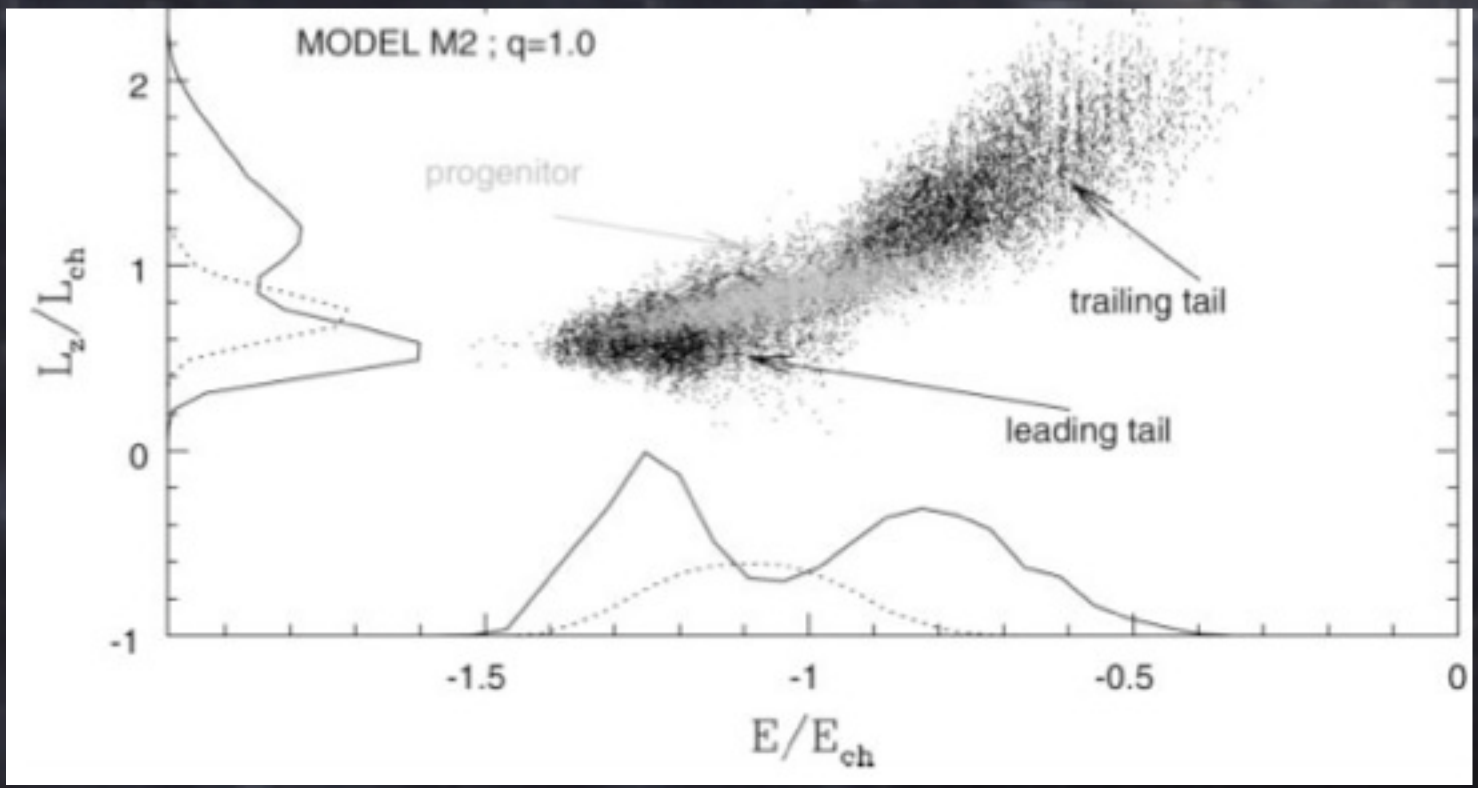

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Strong constraints on MW potential

Analysis of integrals of motion!

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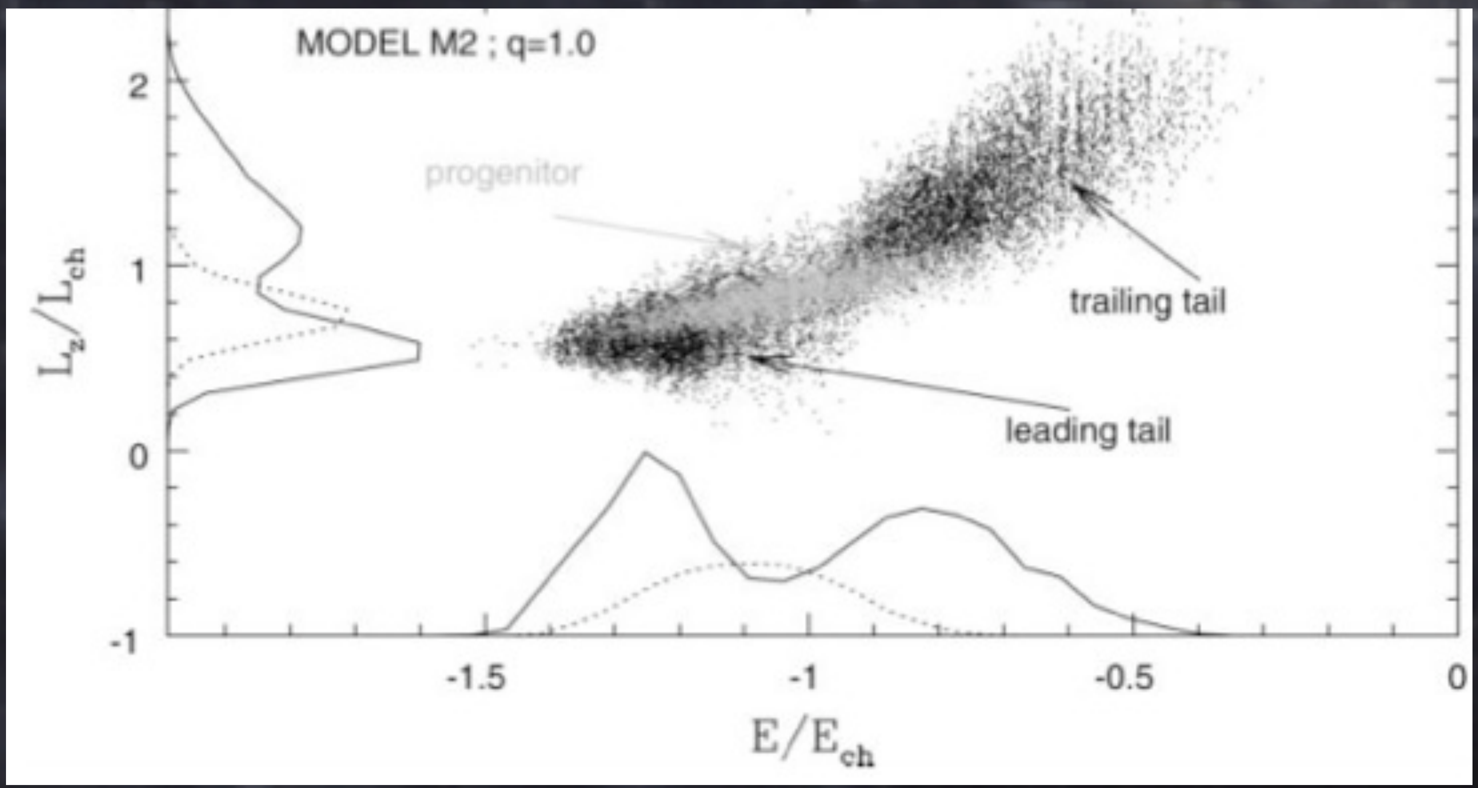

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Strong constraints on MW potential

Analysis of integrals of motion!

requires 6D phase-space information

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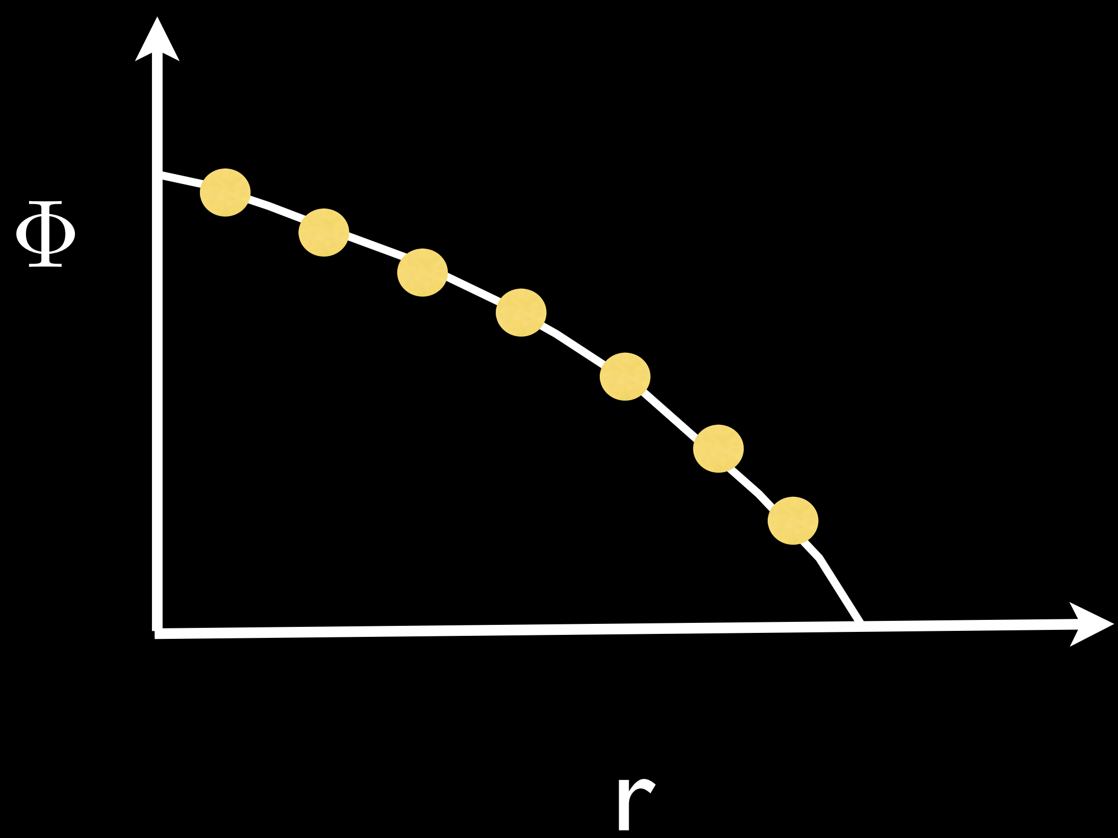
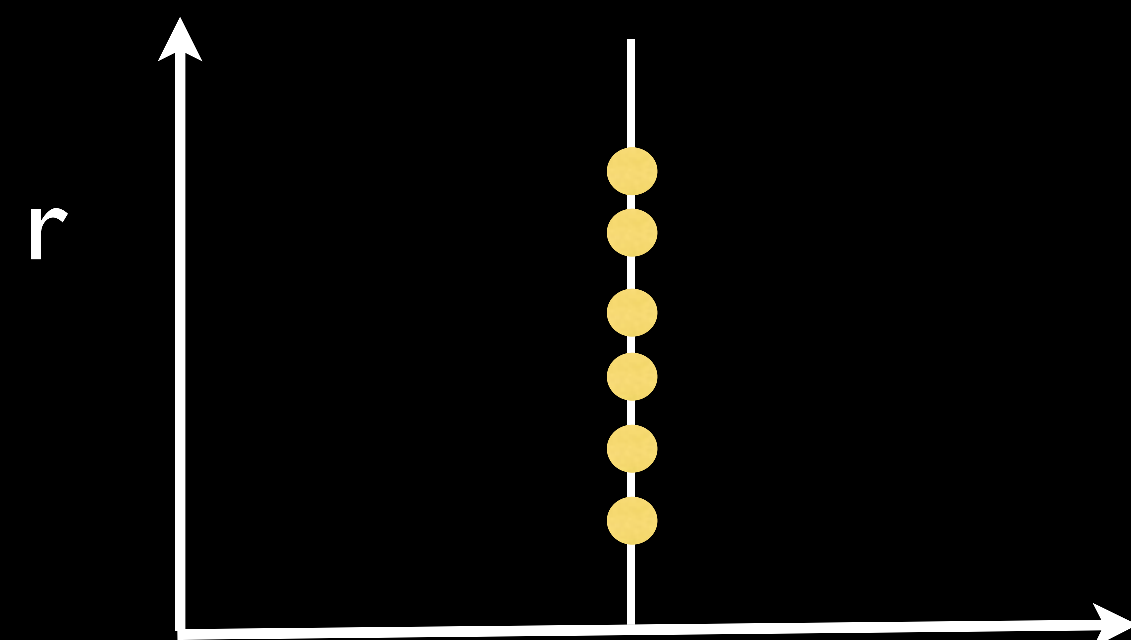
THE IDEA

Peñarrubia, Koposov & Walker (2012)

$$f(E) = \delta(E - E_0)$$

$$H \equiv -\int f(E) \ln f(E) dE = 0$$



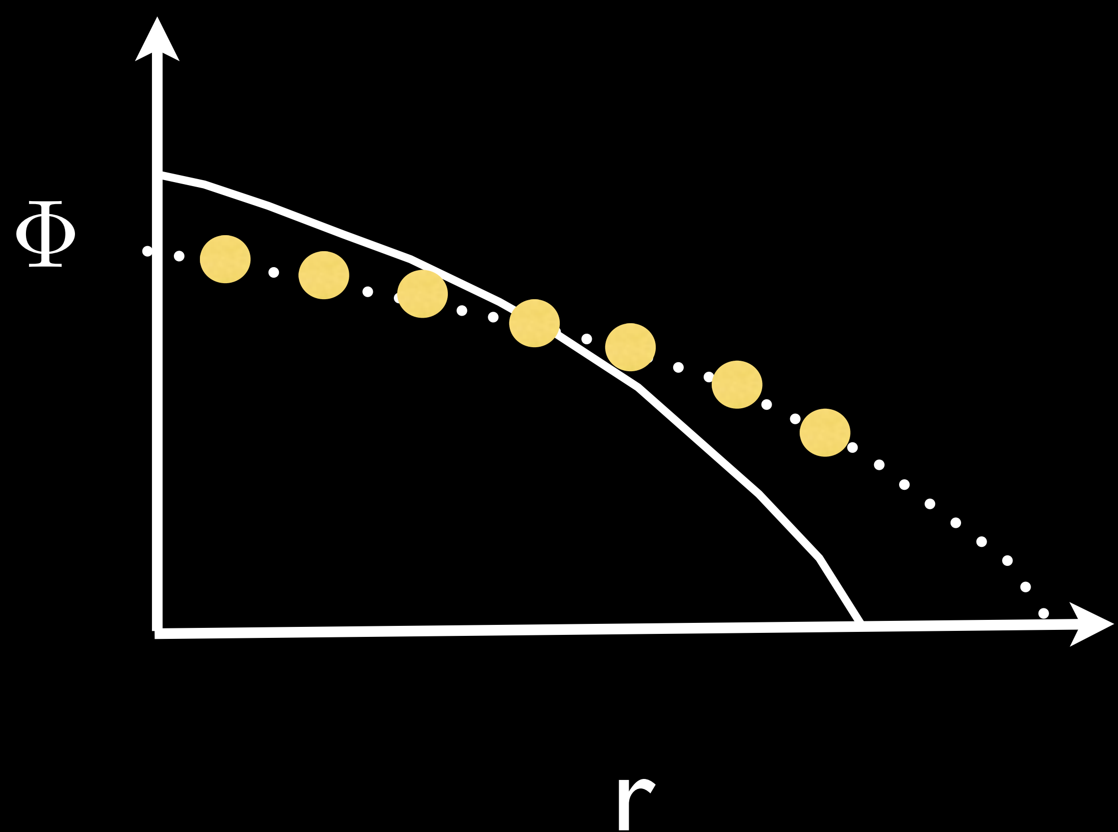


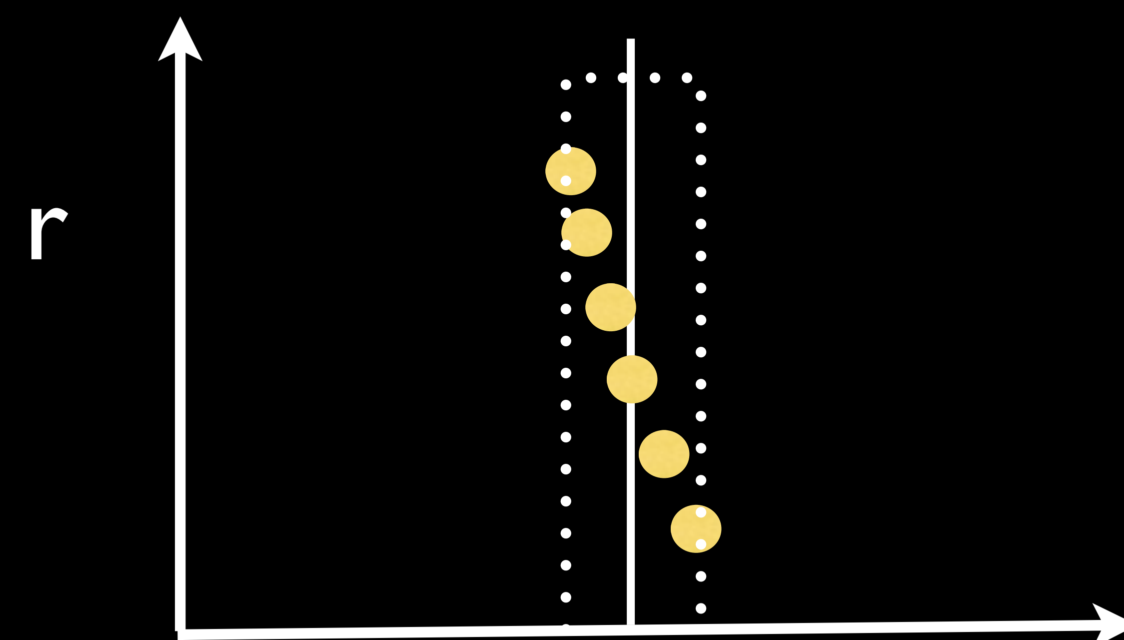
$$E = \frac{1}{2}v^2 + \Phi(r)$$

THE IDEA

Peñarrubia, Koposov & Walker (2012)

$$\tilde{f}(E) \longrightarrow \Delta H > 0$$





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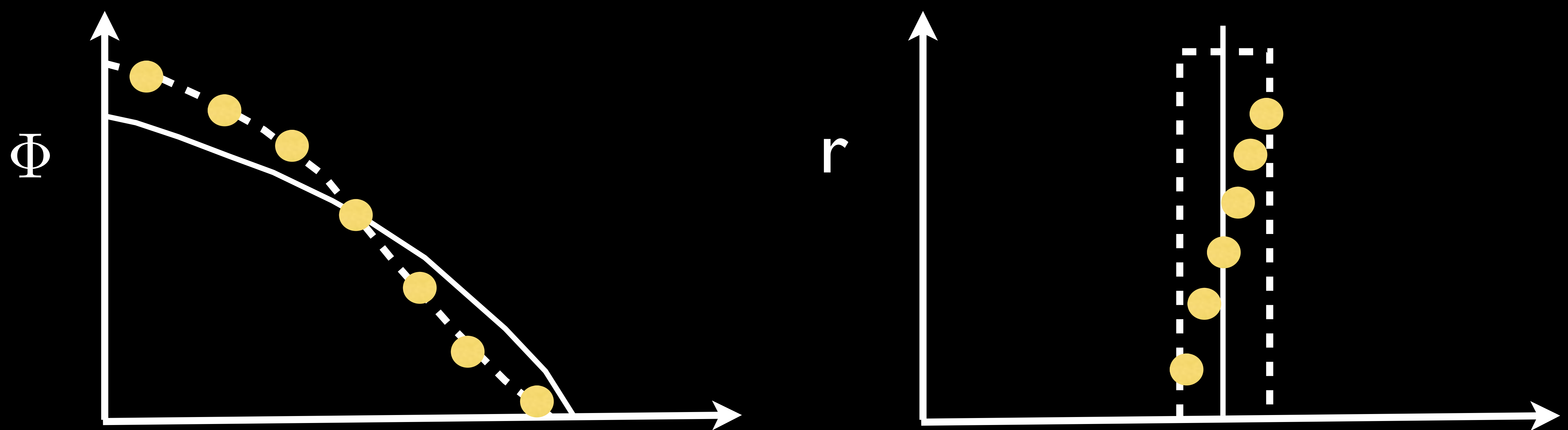

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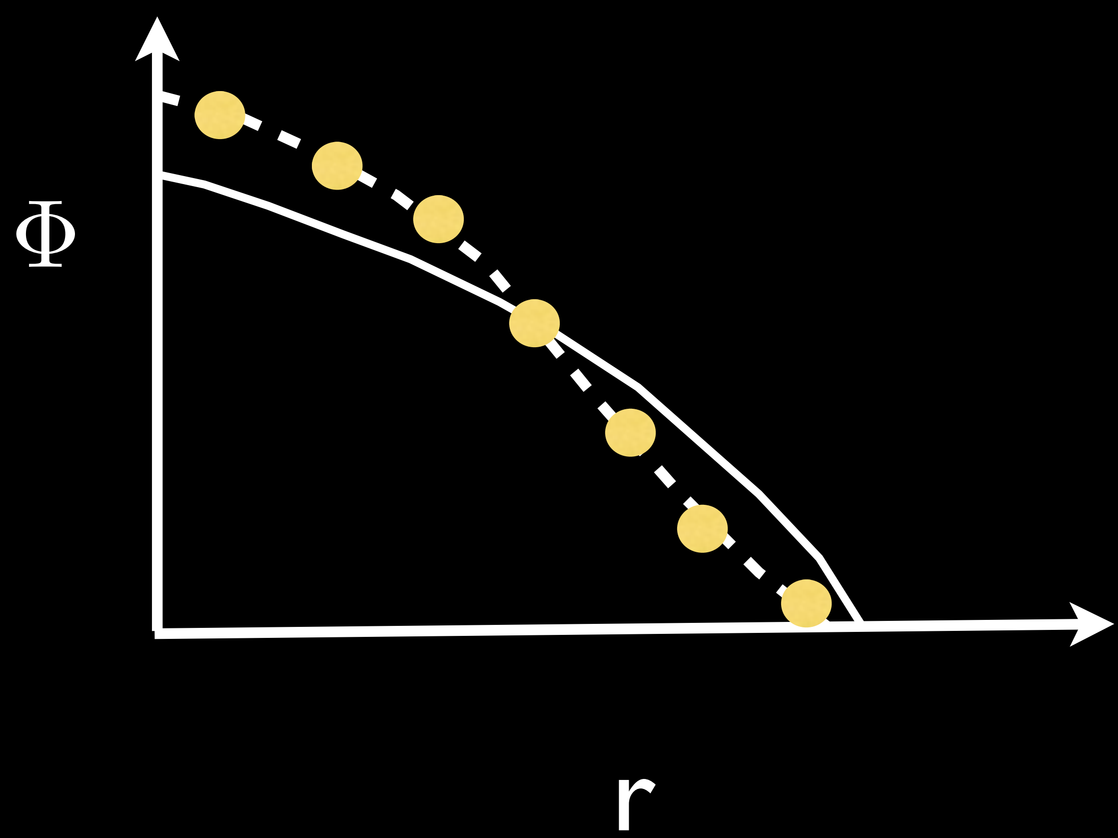

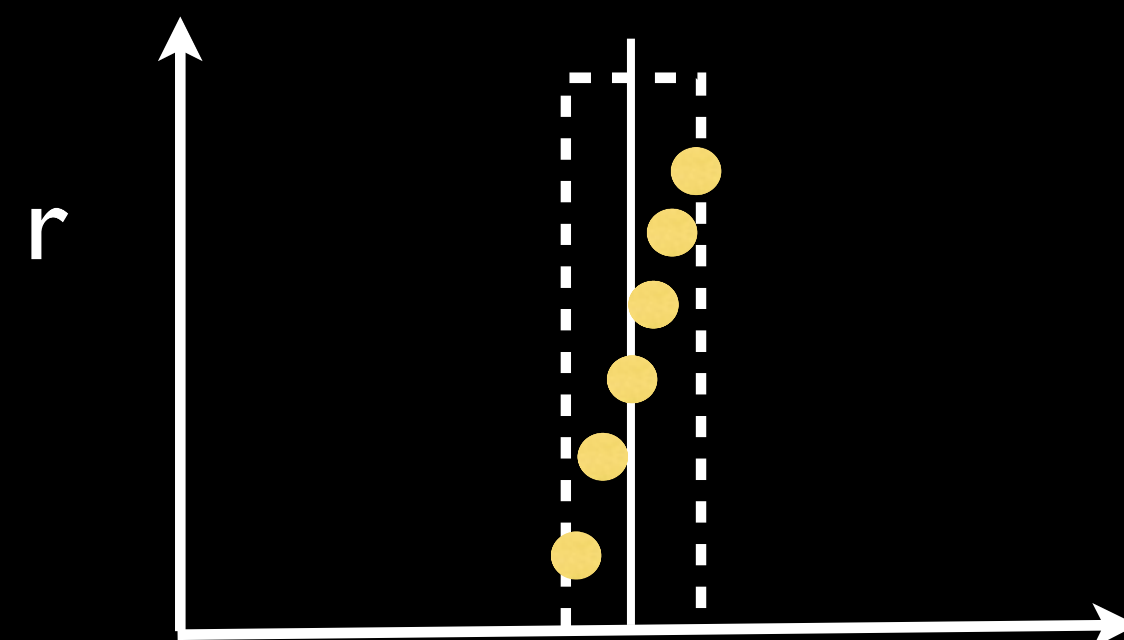

$$E = \frac{1}{2}v^2 + \Phi(r)$$

“Biases in the calculus of orbital energy yields and ***increase*** *in the entropy of the energy distribution”*

Entropy

Theorem:

"The entropy measured for stellar systems with separable energy distributions increases under the presence of biases in the theoretical modelling of the host's gravity"

$$\varepsilon = -E + \Phi_\infty$$ Relative energy

$$\tilde{\varepsilon}(\mathbf{r}) = \varepsilon(\mathbf{r}) + \delta\Phi(\mathbf{r})$$ **Energy Bias**

$$\tilde{f}(\varepsilon, \mathbf{r}) = f[\varepsilon - \delta\Phi(\mathbf{r}), \mathbf{r}] = f[\varepsilon - \delta\Phi(\mathbf{r})]g(\mathbf{r})$$ Separability condition

Measured energy distribution:

$$\tilde{f}(\varepsilon) = \int f(\varepsilon - \delta\Phi(\mathbf{r}))g(\mathbf{r})d^3\mathbf{r} \approx$$

$$f(\varepsilon)\int \left[1 - \delta\Phi(\mathbf{r})\frac{f'(\varepsilon)}{f(\varepsilon)} + \frac{\delta\Phi^2(\mathbf{r})}{2}\frac{f''(\varepsilon)}{f(\varepsilon)}\right]g(\mathbf{r})d^3\mathbf{r} =$$

$$f(\varepsilon)\left[1 - \langle\delta\Phi\rangle\frac{f'(\varepsilon)}{f(\varepsilon)} + \frac{\langle\delta\Phi^2\rangle}{2}\frac{f''(\varepsilon)}{f(\varepsilon)}\right].$$

Entropy

Theorem:

"The entropy measured for stellar systems with separable energy distributions increases under the presence of biases in the theoretical modelling of the host's gravity"

Measured Entropy

$$\tilde{H} = -\int d\varepsilon \tilde{f}(\varepsilon) \ln[\tilde{f}(\varepsilon)] =$$

$$H + \langle\delta\Phi\rangle \int d\varepsilon f'(\varepsilon)[1 + \ln f(\varepsilon)]$$

$$-\frac{\langle\delta\Phi\rangle^2}{2} \int d\varepsilon f(\varepsilon) \left[\frac{f'(\varepsilon)}{f(\varepsilon)}\right]^2 - \frac{\langle\delta\Phi^2\rangle}{2} \int d\varepsilon f''(\varepsilon)[1 + \ln f(\varepsilon)].$$

$$1) \int d\varepsilon f'(1 + \ln f) = \left(f \ln f\right)_0^{\Phi_\infty} = 0,$$

$$2) \int d\varepsilon f''(1 + \ln f) = -\int d\varepsilon f \left[\frac{f'}{f}\right]^2.$$

$$\tilde{H} = H + \frac{\langle\delta\Phi^2\rangle - \langle\delta\Phi\rangle^2}{2} \int d\varepsilon f(\varepsilon) \left[\frac{f'(\varepsilon)}{f(\varepsilon)}\right]^2 \equiv H + \frac{\sigma_\Phi^2}{2\sigma_\varepsilon^2} \quad \geq 0$$

Entropy

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Measured Entropy

$$\tilde{H} = H + \frac{\langle \delta\Phi^2 \rangle - \langle \delta\Phi \rangle^2}{2} \int d\varepsilon f(\varepsilon) \left[\frac{f'(\varepsilon)}{f(\varepsilon)} \right]^2 \equiv H + \frac{\sigma_\Phi^2}{2\sigma_\varepsilon^2}$$

- Entropy increases for $\delta\Phi = \delta\Phi(\mathbf{r}) \neq 0$
- Adding a constant value to the potential does not yield an increase in entropy
- Changes in entropy will be stronger for "cold" distributions

Tests

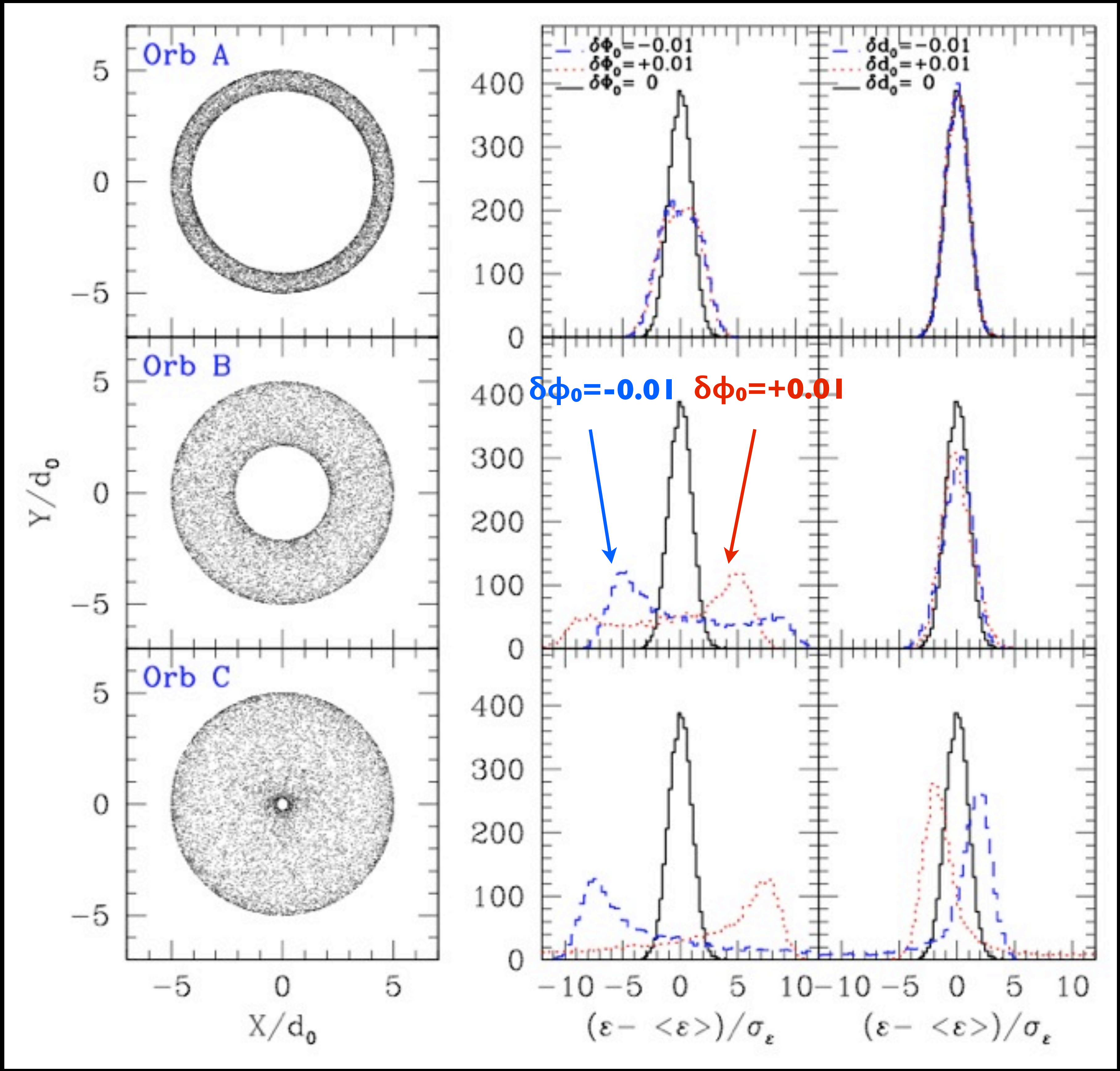
$$f(\varepsilon) = 1/\sqrt{2\pi\sigma_\varepsilon^2}\,\exp[-(\varepsilon - \varepsilon_{\rm orb})^2/(2\sigma_\varepsilon^2)$$

Unbiased (true) energy distribution

$$\Phi(r) = \Phi_0 \ln(d_0^2 + r^2)$$

Unbiased (true) Potential

Tests



Orb A
Orb B
Orb C
Y/d0
X/d0
$\delta\Phi_0 = -0.01$
$\delta\Phi_0 = +0.01$
$\delta\Phi_0 = 0$
$\delta d_0 = -0.01$
$\delta d_0 = +0.01$
$\delta d_0 = 0$
δϕ₀=-0.01
δϕ₀=+0.01
$(\varepsilon - \langle\varepsilon\rangle)/\sigma_\varepsilon$
$(\varepsilon - \langle\varepsilon\rangle)/\sigma_\varepsilon$

Tests

$$f(\varepsilon) = 1/\sqrt{2\pi\sigma_\varepsilon^2}\exp[-(\varepsilon-\varepsilon_{\rm orb})^2/(2\sigma_\varepsilon^2)$$ Unbiased (true) energy distribution

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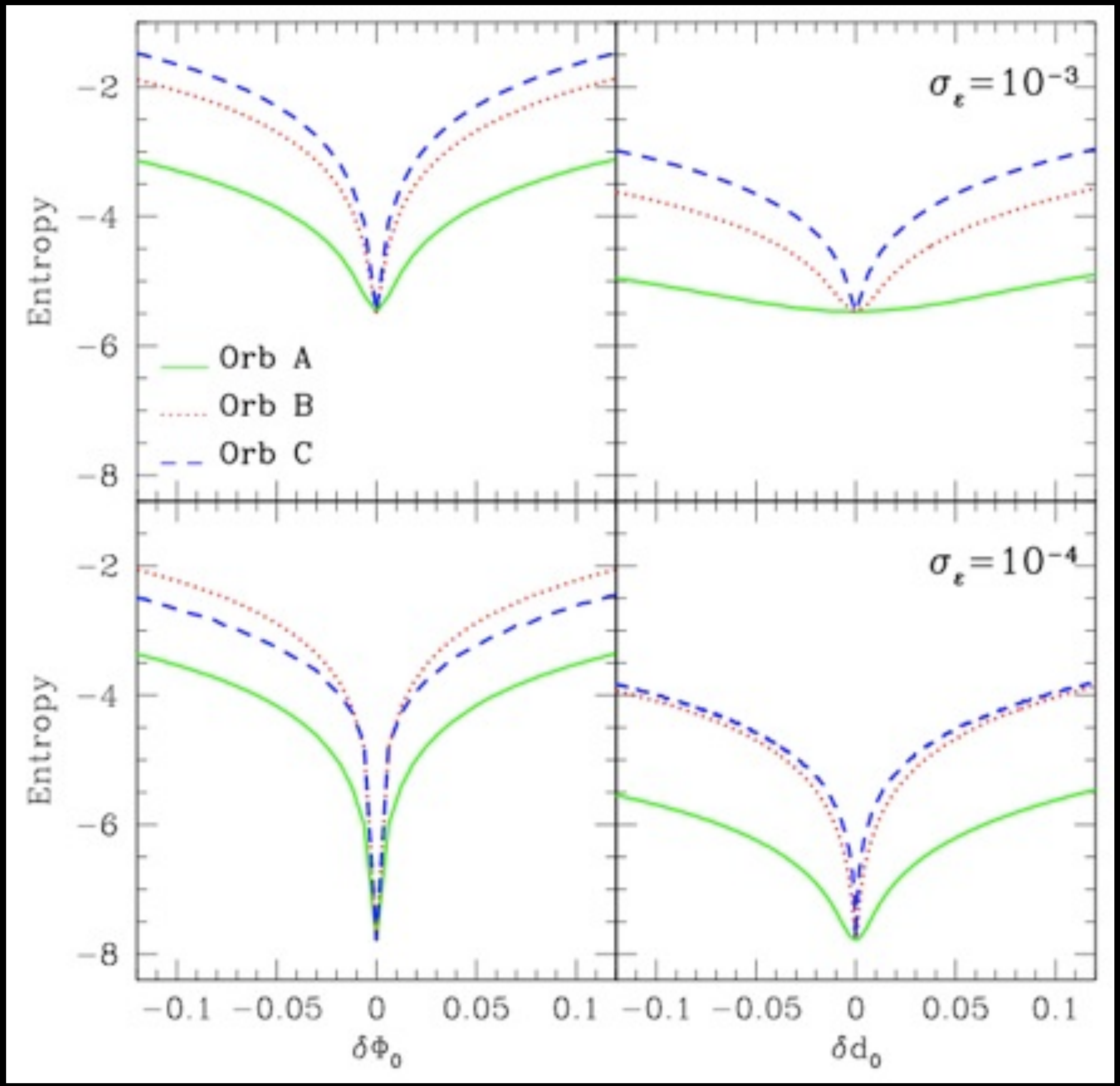

$$r_{\rm apo} = 5d_0$$

$$\sigma_\varepsilon = 10^{-3}\Phi_0$$

$$H_{\rm Gauss} = 1/2[\ln(2\pi\sigma_\varepsilon^2)+1]$$

Tests

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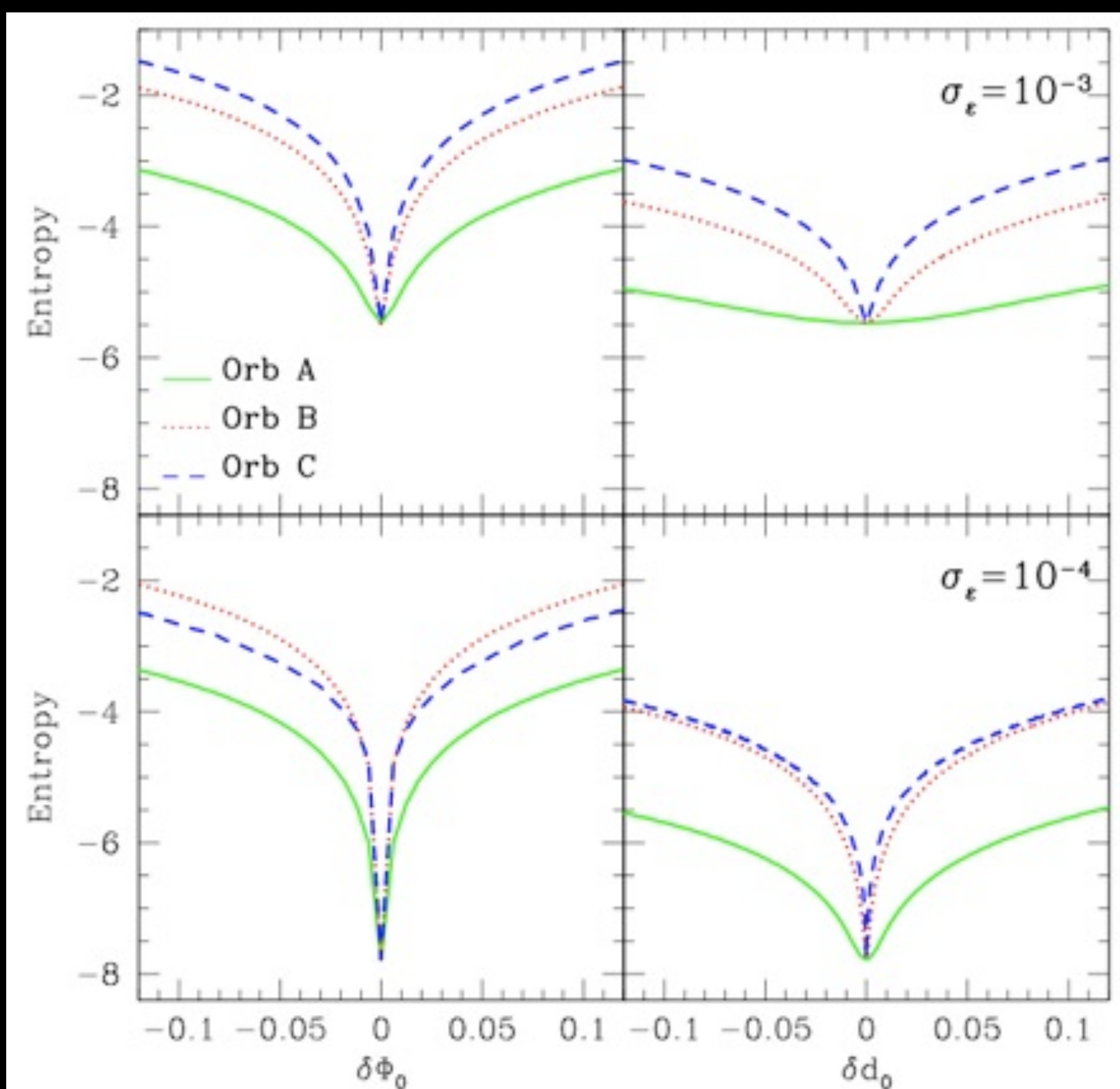

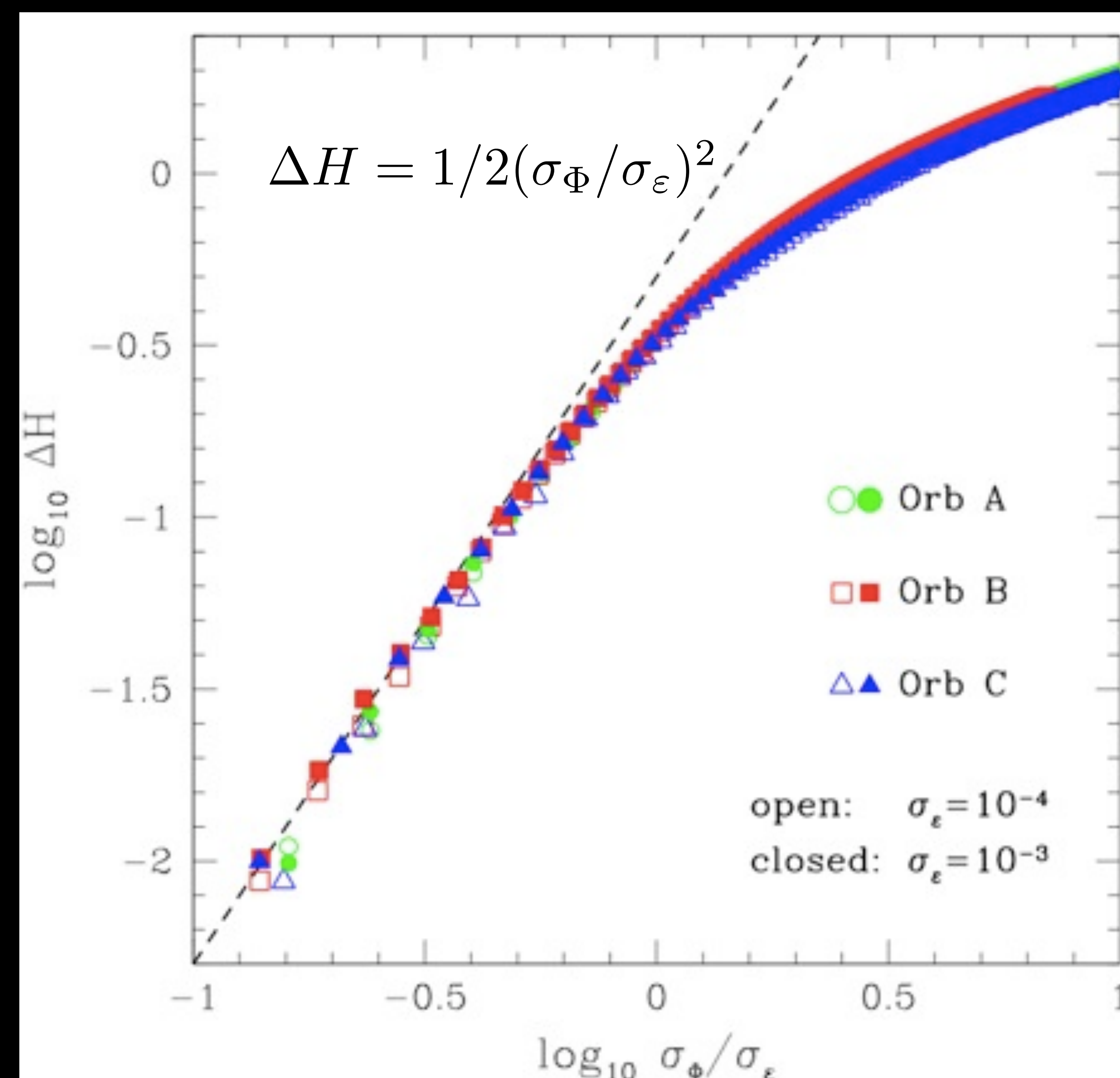

Energy biases

1. Potential parameters
2. Functional form of the potential
3. Gravity model

$$\tilde{\Phi}(r) = 2\Phi_0 \left[y + \frac{y^3}{3} + \frac{y^5}{5} + ... + \sum_{k=0}^{(N-1)/2} y^{2k+1}/(2k+1) \right] + \Phi_0 \ln d_0^2$$

$$\lim_{N\to\infty} \tilde{\Phi} = \Phi_0 \ln(r^2 + d_0^2) = \Phi$$

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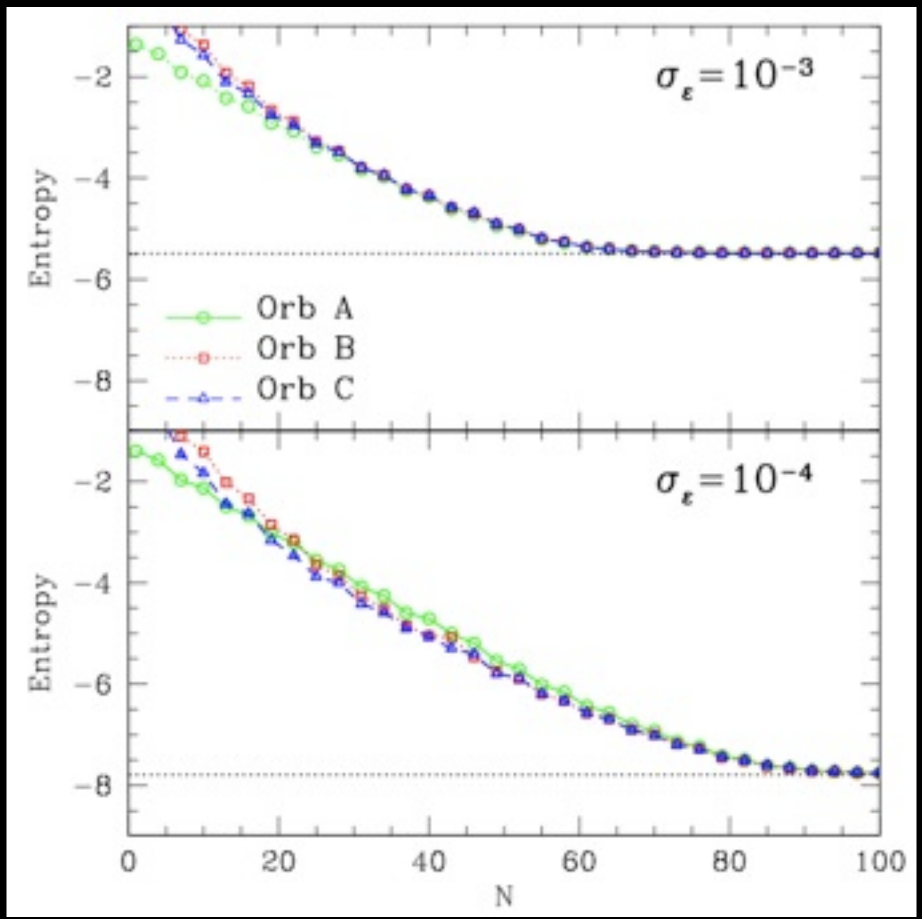

Entropy can be used to distinguish between different potential parametrizations

Energy biases

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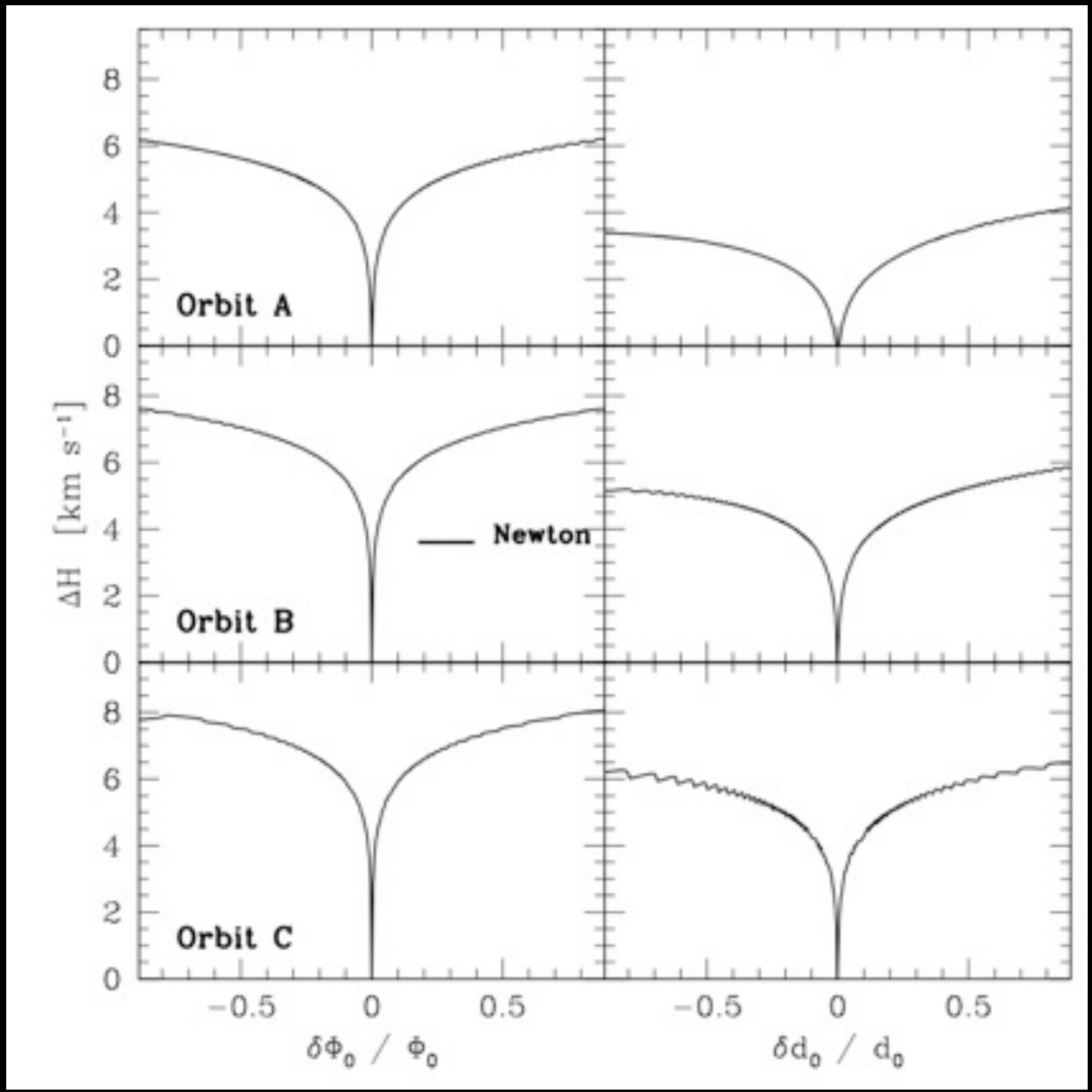

Example 1: Dirac's cosmology

$$\frac{Gm_p m_e}{e^2} \simeq 10^{-39} \simeq \frac{e^2}{m_e c^3 t};$$

$$E_D = H_0^2 t^2 \left[\frac{1}{2} \left(\frac{d\mathbf{r}}{dt} \right)^2 + \frac{G}{G_0} \Phi(\mathbf{r}) - \left(\frac{d\mathbf{r}}{dt} \cdot \frac{\mathbf{r}}{t} \right) \right] + \frac{1}{2} H_0^2 \mathbf{r}^2;$$

Lynden-Bell (1982)

at $t=H_0^{-1}$

$$\delta\Phi_D = \pm[-H_0(d\mathbf{r}/dt \cdot \mathbf{r}) + 1/2 H_0^2 \mathbf{r}^2].$$

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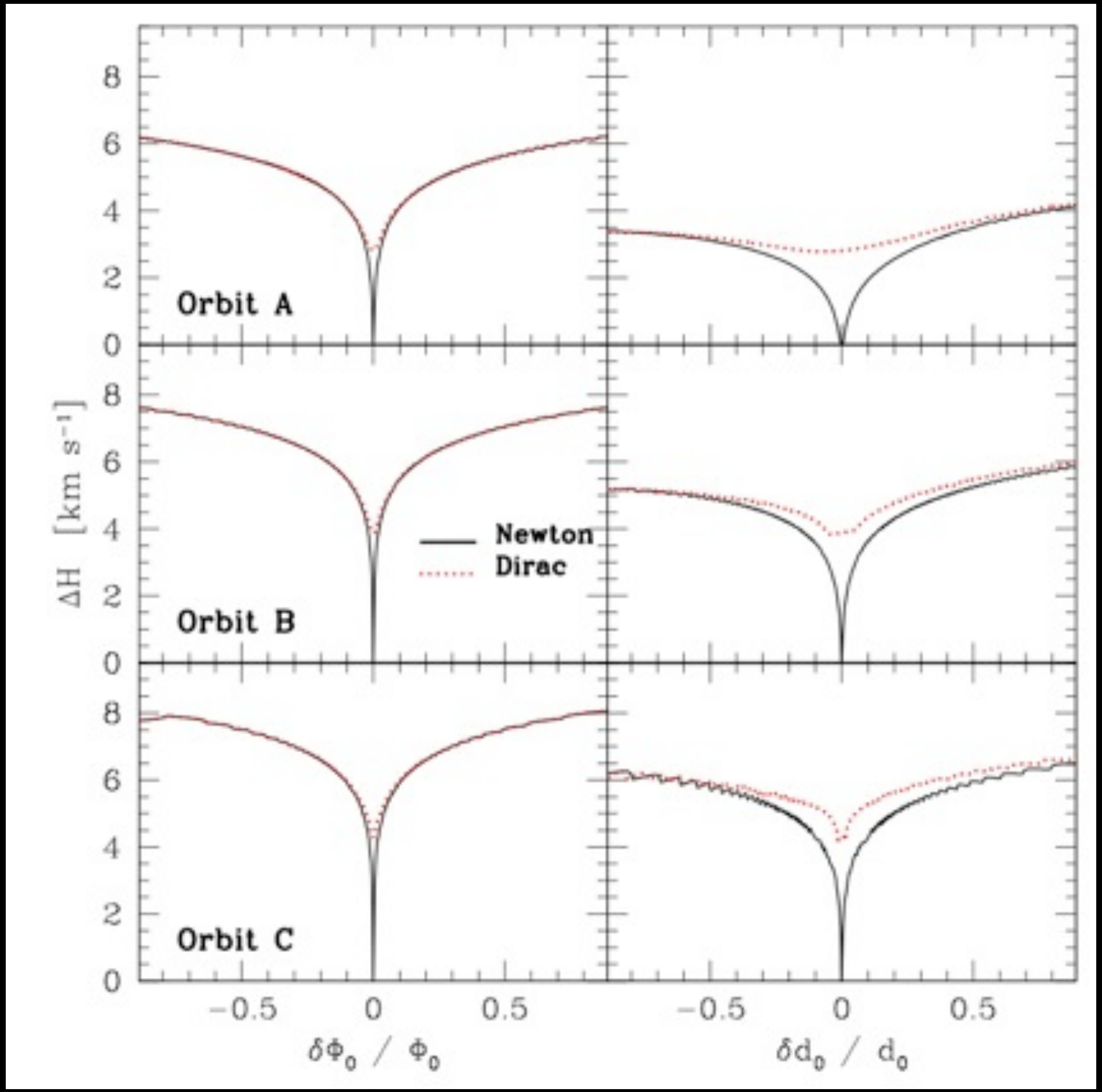

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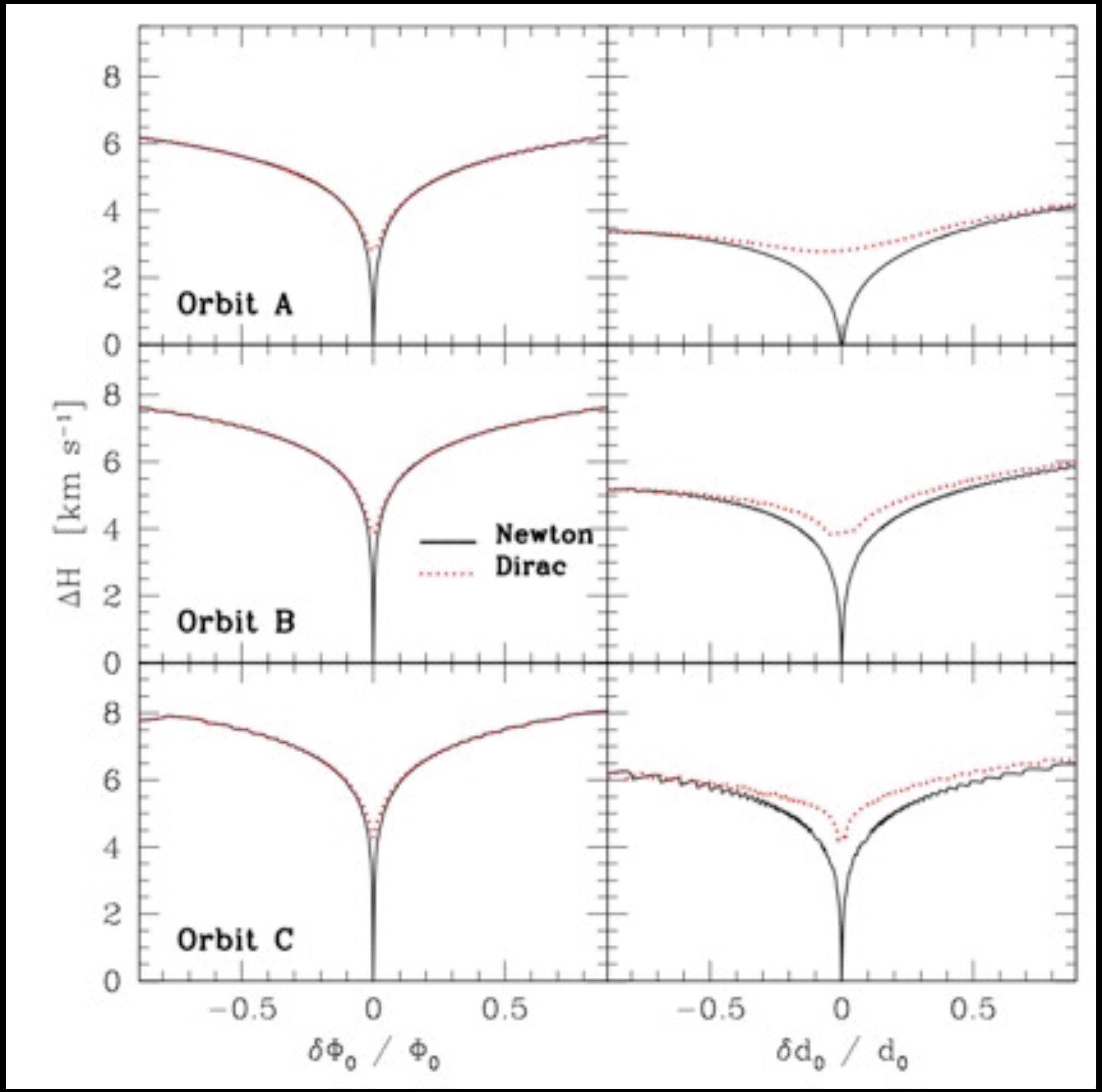

Example 2: QMOND

$$\mathbf{g}_M = \mathbf{g}_N \nu(r) \equiv \mathbf{g}_N \left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{a_0}{g_N}} \right),$$

$$g_N = -GM(< r)/r^2,$$

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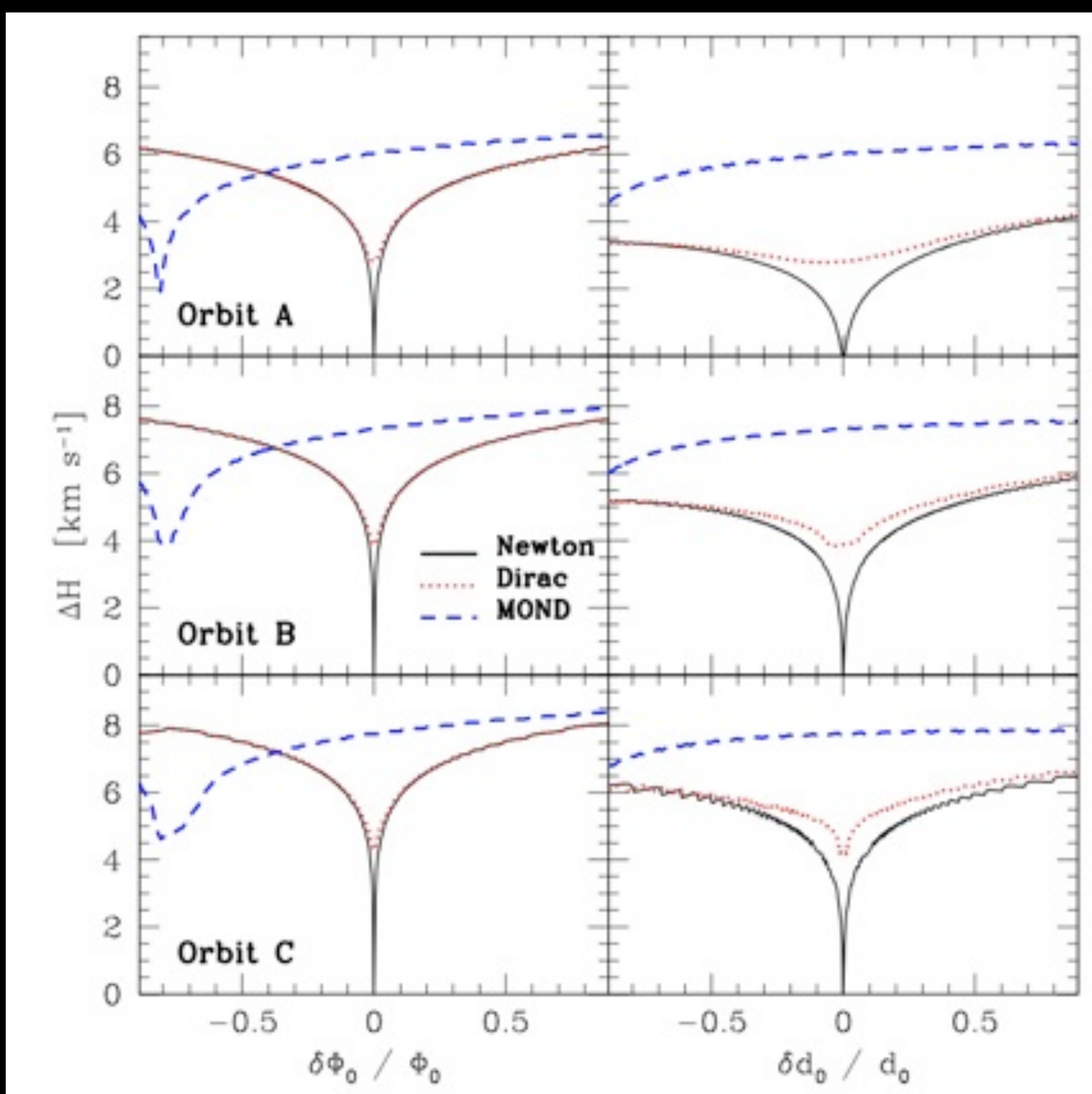

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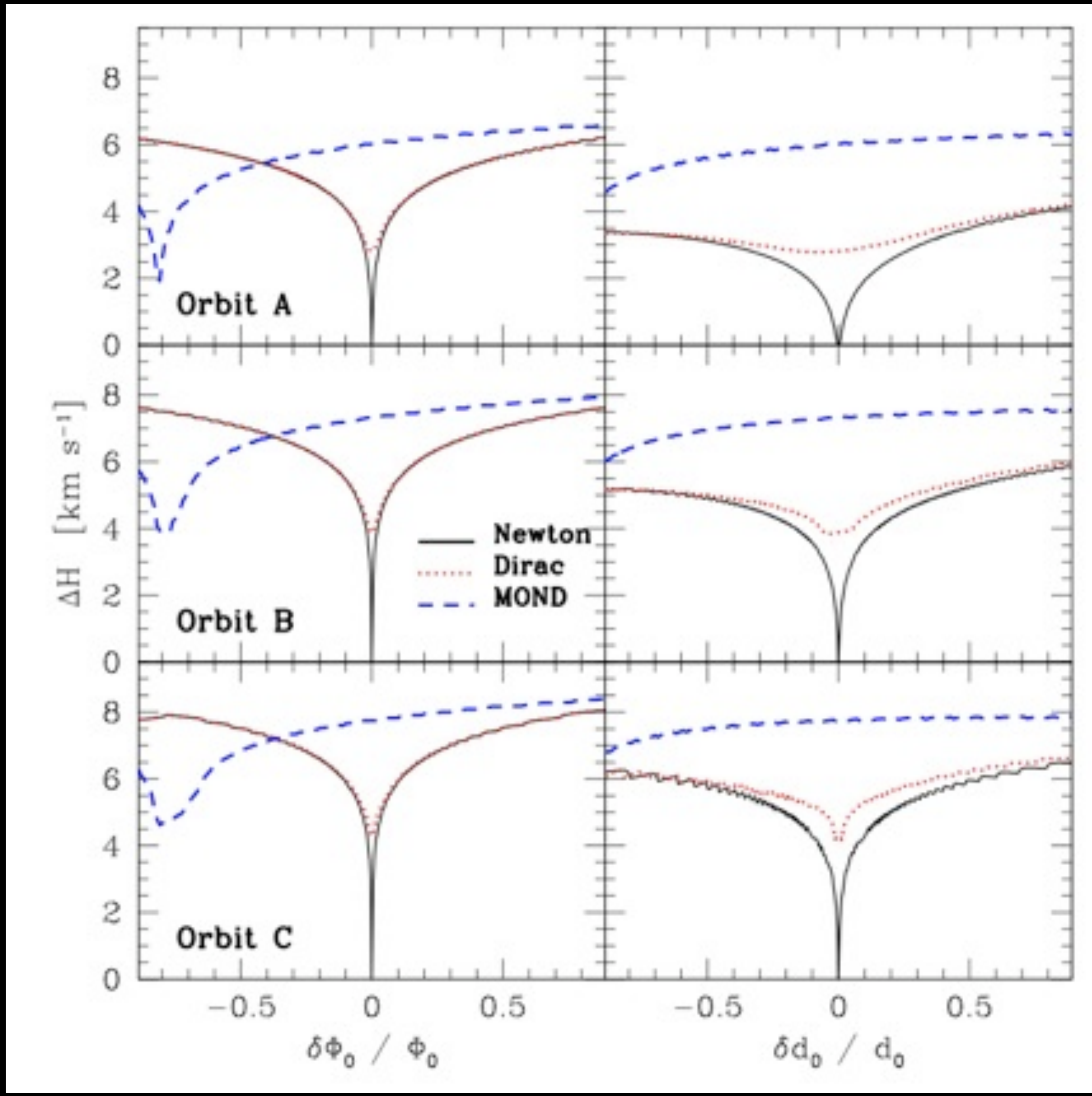

Example 3: f(R) gravity theories

$$\mathcal{A} = \int d^4x \sqrt{-g}[f(R) + \mathcal{L}_m];$$

$$f(R) = f_0 R^n$$ Ricci curvature

ΛCDM: $f(R) = R + 2\Lambda$

Cappozziello et al (2007)

$$\Phi_R = 1/2(\Phi_N + \Phi_c)$$

$$\Phi_c(r) = -4\pi G\left[\frac{1}{r}\int_0^r dr'\rho(r')r'^2\left(\frac{r}{r_c}\right)^\beta + \int_r^\infty dr'\rho(r')r'\left(\frac{r}{r_c}\right)^\beta\right].$$

$\beta = 0$ Newton

$\beta = 0.82$ Fit rotation curves with NO DM

Energy biases

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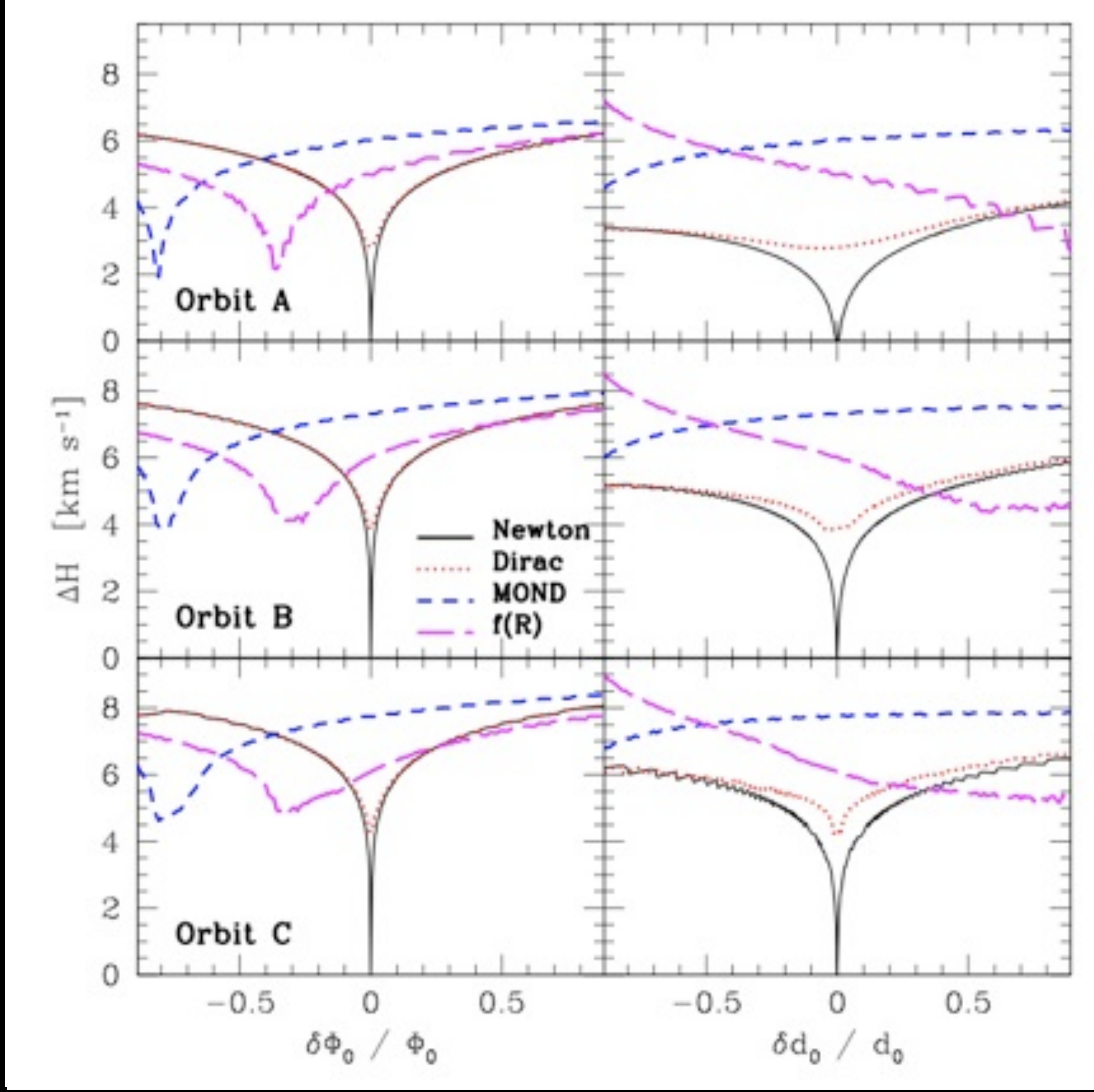

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Friday, 13 April 2012

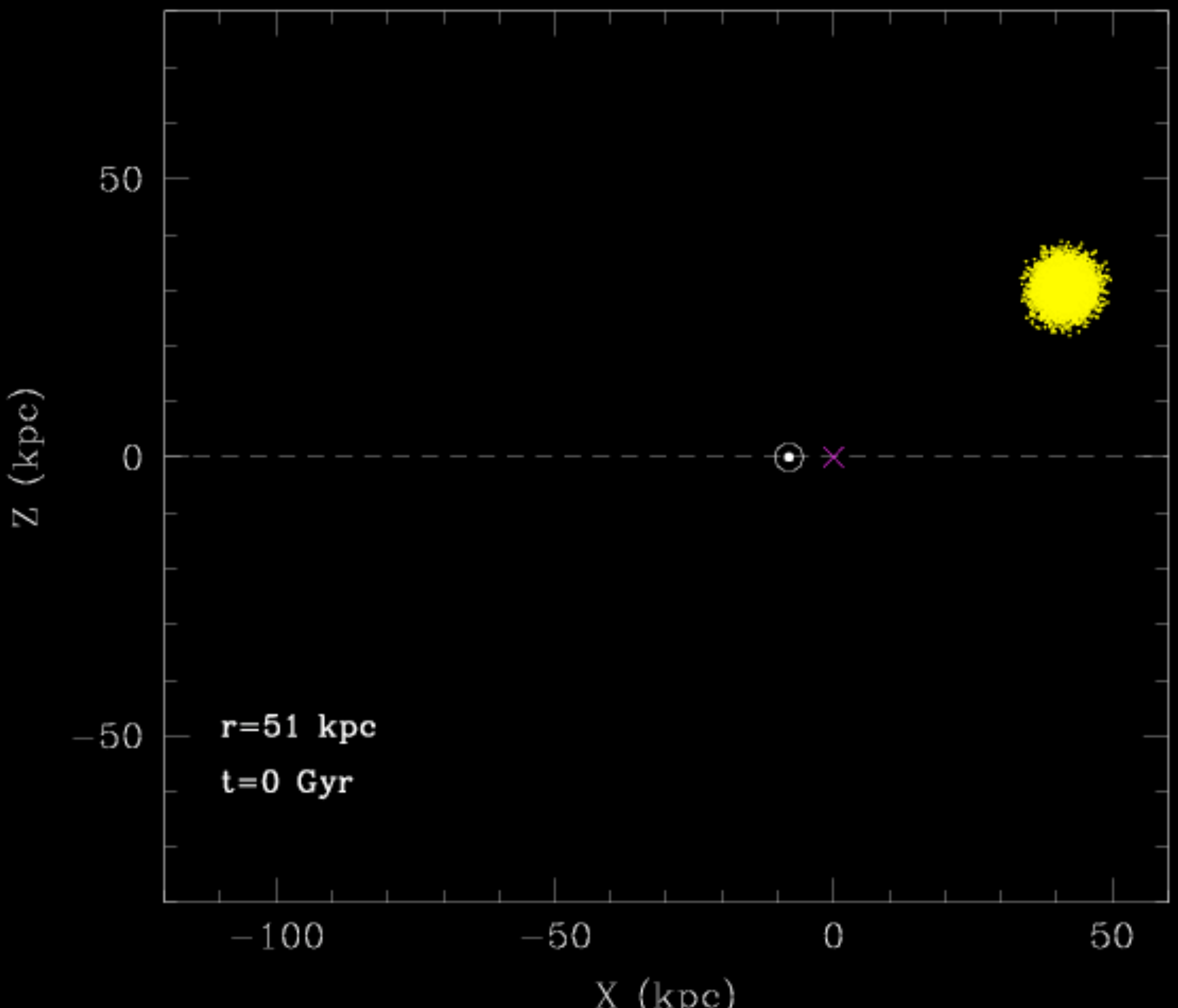
The Minimum Entropy Method

it is a simple statistical technique for constraining simultaneously the MW gravitational potential and testing different gravity theories directly from phase-space surveys and without adopting dynamical models.

1. Phase-space catalogue: $\{X,Y,Z,V_x,V_y,V_z\}_i$; $i=1,2,...,N_*$
2. Calculate $E_i=1/2(V_x^2+V_y^2+V_z^2)_i+\Phi(X_i,Y_i,Z_i)$
3. Calculate $f(E)$, H
4. Look for Φ that minimizes H

Tidal debris

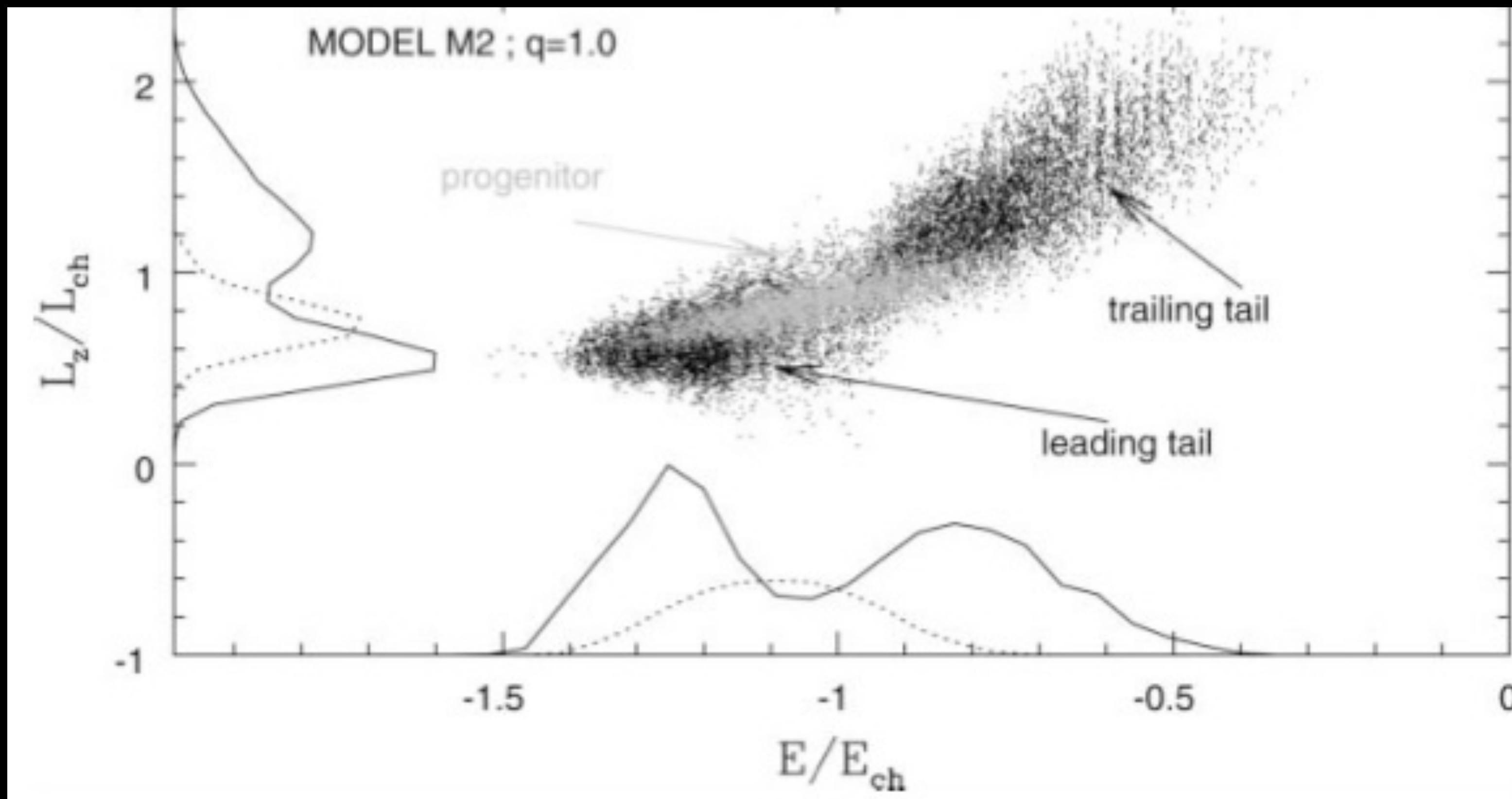
the energy distribution of tidal debris is not separable JP+06, Eyre & Binney 08



JP+10

Tidal debris

the energy distribution of tidal debris is not separable JP+06, Eyre & Binney 08



Kullback-Leiblar (or KL) divergence

$$D_i = \int f_i(\varepsilon) \ln \left[\frac{f_i(\varepsilon)}{f(\varepsilon))} \right] d\varepsilon \equiv -H_i + H_{c,i};$$

→ ***Distributions are separable if D_i=0***

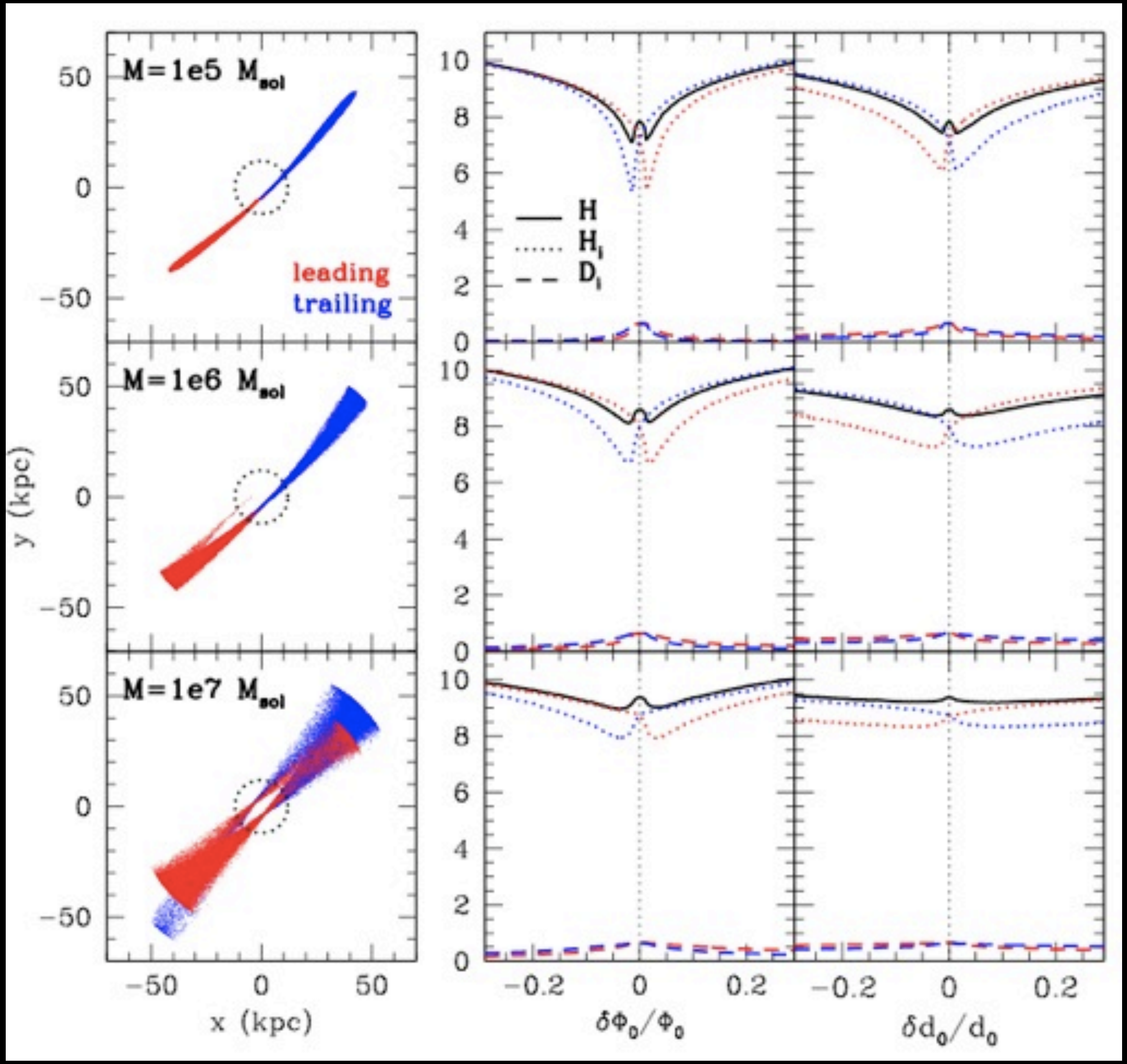
Crossed entropy

$$H_{c,i} = -\int f_i(\varepsilon) \ln f(\varepsilon) d\varepsilon$$

Tidal debris

$$H = -\int f(\varepsilon)\ln f(\varepsilon)d\varepsilon = -\alpha\int f_l(\varepsilon)\ln f(\varepsilon)d\varepsilon - (1-\alpha)\int f_t(\varepsilon)\ln f(\varepsilon)d\varepsilon$$

$$\equiv \alpha H_l + (1-\alpha)H_t + \alpha D_l + (1-\alpha)D_t \equiv \langle H\rangle_{l,t} + \langle D\rangle_{l,t};$$



$$H = \langle H\rangle_{l,t} + \langle D\rangle_{l,t};$$

minimum if δΦ=0 maximum if δΦ=0

maximum in H δΦ~0 $\langle H\rangle_l' = \langle D\rangle_l' = 0$

minimum in H δΦ~0 $\langle H\rangle_l' = -\langle D\rangle_l'$

Summary

- **“The true Milky Way potential is that that minimizes the entropy measured for stellar systems with separable energy distributions”**

- **Best targets: Tidal debris of satellites/clusters with low dynamical masses**

- **Future work: Gaia errors? MW background?**