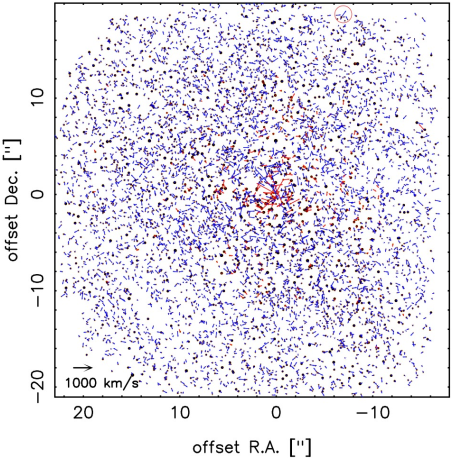
Lessons from the Galactic centre

John Magorrian

“Dynamics meets kinematics tracers”, Thu 12 Apr 2012

The problem

Schoedel et al (2009) measure (x, y, v_x, v_y) for sample of 6000 stars within 1pc of Galactic centre:



What's the mass distribution $\rho(r)$?

Take published PMs at face value.

Motivation

We understand how to find f given Φ.

Can I find Φ by marginalising over f for a non-toy problem?
(No, not in this talk.)

Compare two different methods:

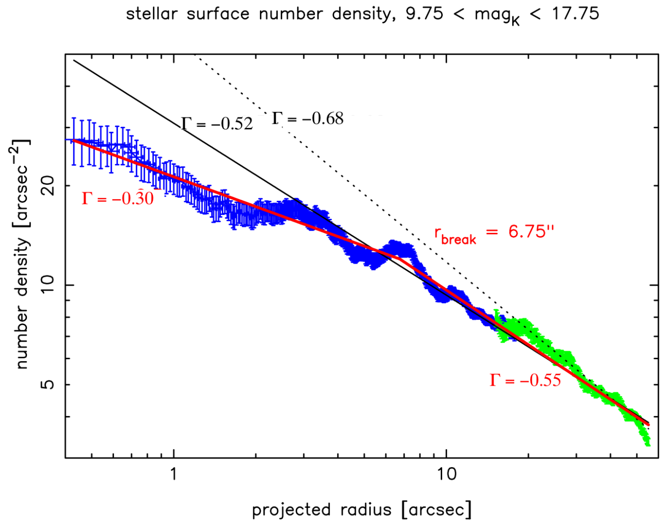
- simple Jeans models
- full-on OS method

Bonus: independent measurements of $M_\bullet$ (S stars).

I. Simple models using the Jeans equations

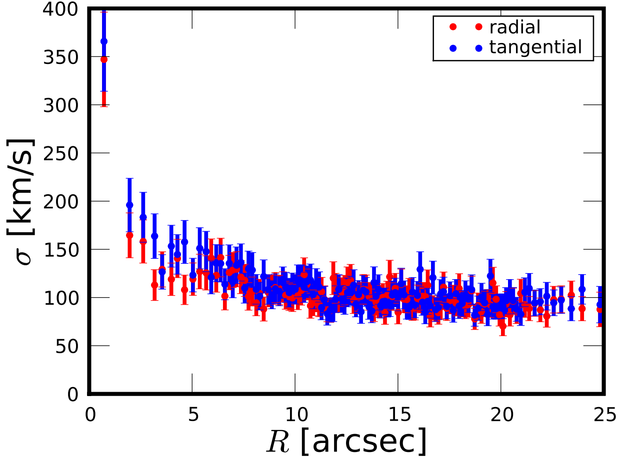
Galactic centre in context
(Schödel et al. 2007)

Surface density profile from NACO (10” = 0.4 pc) and ISAAC:



Galactic centre in context
(Schödel et al. 2009)

Ignore rotation. Binned $\sigma_R(R)$ and $\sigma_\phi(R)$ from PM data:



Simple Jeans models of the kinematics

Assumptions: Mass distribution is

1. spherical
2. in steady state
3. smooth.

More assumptions: Stars (late-type only!)

1. are drawn fairly from number density distribution

$$j(r) \propto r^{\alpha} \left(1 + \frac{r}{r_0}\right)^{-1.8-\alpha} \quad \text{with} \quad r_0 = 1\text{pc}$$

2. isotropic velocity distribution.

Jeans models

Given trial $M_\bullet$ and mass density $\rho(r)$:

1. Calculate enclosed mass $M(< r)$;
2. Integrate Jeans equation to find intrinsic (isotropic) velocity disperson:

$$j(r)\sigma^2(r) = \int_r^\infty j(r') \frac{GM(< r')}{r'} \mathrm{d}r';$$

3. Project this $j\sigma^2$ along the line of sight;
4. Compare to binned dispersions

This gives me $\chi^2(M_\bullet, \rho)$

Note: linear relationship between ρ and σ_p^2:

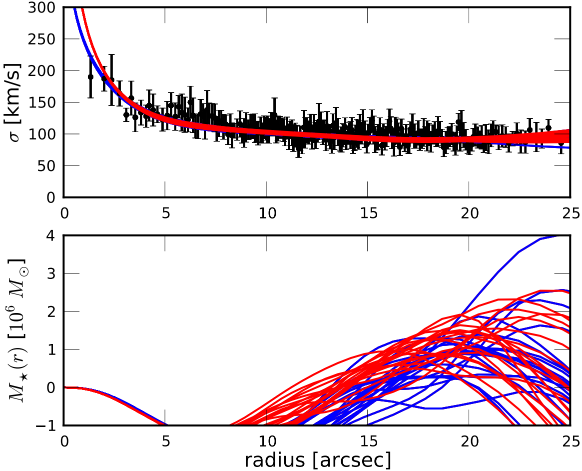
$$\sigma_\mathrm{p}^2 = P\rho!$$

“Non-parametric” stellar potentials

(Following Magorrian & Ballantyne 2001)

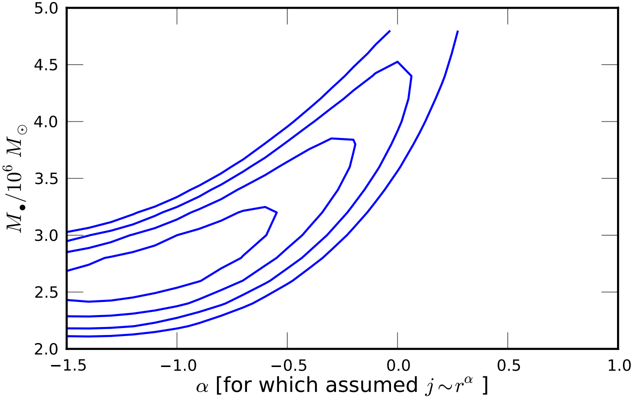
Invert $\sigma_p^2 = P\rho$, with smoothness penalty on $\rho(r)$.

Results for $M_\bullet = 3.6 \times 10^6\, M_\odot$ and $j \sim r^0$ and $j \sim r^{-1}$



Scan over isotropic Jeans models

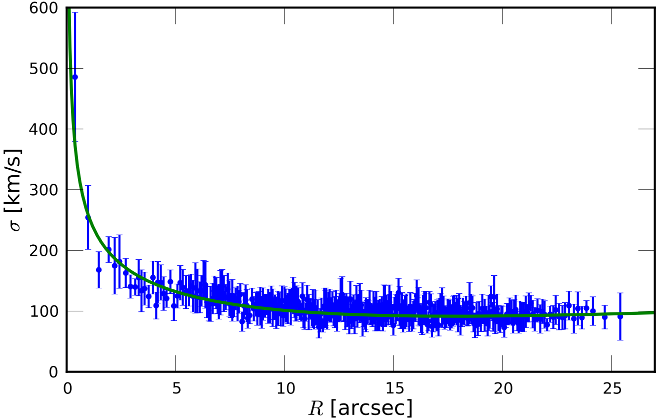
χ^2 as a function of assumed $M_\bullet$ and number-density slope



Assumptions: isotropy; smoothing on ρ; binning to get $\sigma_{\rm p}$

Scan over isotropic Jeans models

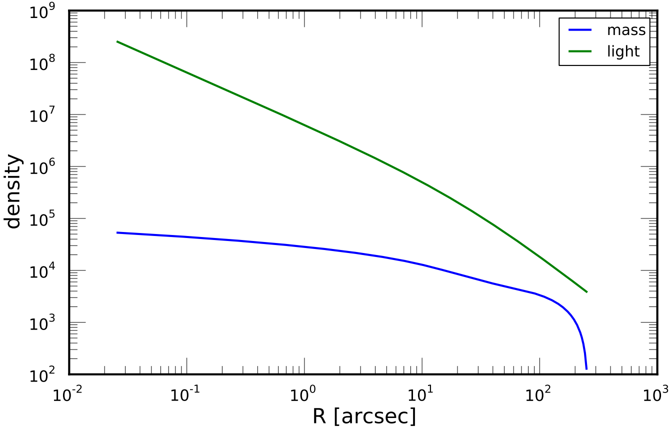
Best-fit model $M_\bullet = 2.8 \times 10^6\, M_\odot$



Assumptions: isotropy; smoothing on ρ; binning to get σ_p

Scan over isotropic Jeans models

Best-fit model $M_\bullet = 2.8 \times 10^6\, M_\odot$



Assumptions: isotropy; smoothing on ρ; binning to get σ_p

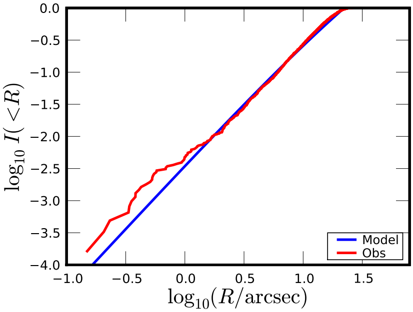
Conclusions from isotropic Jeans models

1. BH mass $\sim 2.8 \times 10^6\, M_\odot$
   - less than the accepted $M_\bullet \simeq 4 \times 10^6 M_\odot$.
2. $M_\star \sim 2 \times 10^6 M_\odot$ within 1 pc, having
3. flat core in mass density profile, $\rho \sim r^\alpha$, $\alpha \sim 0$.

Models forced to have $M_\bullet \simeq 4 \times 10^6 M_\odot$ have hole in $\rho(r)$!

Limitations of isotropic Jeans models

1. More information to be extracted than just $\sigma_R(R)$, $\sigma_\phi(R)$
2. The NSC is slightly anisotropic: $\frac{\langle v_R^2 \rangle}{\langle v_\phi^2 \rangle} = 0.91$
3. We don't really know $j(r)$ well:

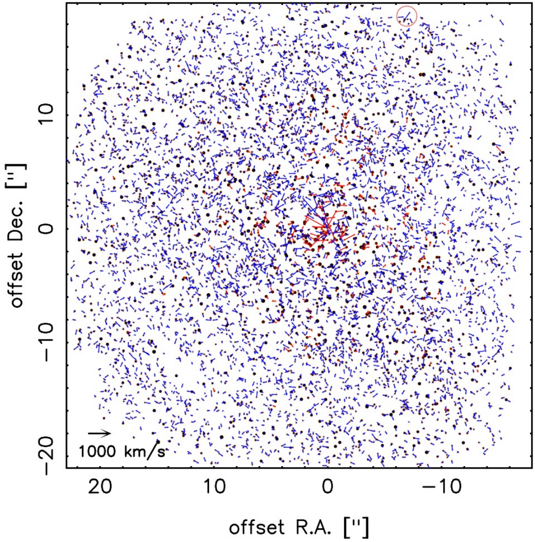


Affects predicted $\sigma(R)$ profiles.

II.**Orbit-superposition models**

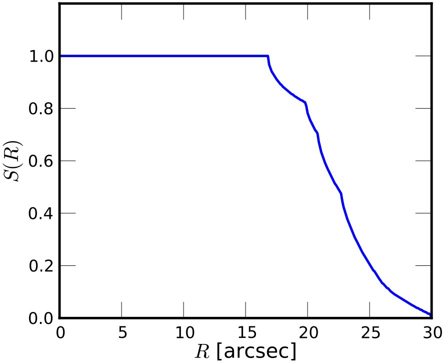
Selection function

Kinematical survey has limited spatial extent:



Selection function

Multiply model likelihoods by **selection function**



For model with pdf $f(x)$, likelihood of measuring $x = x_0$ is

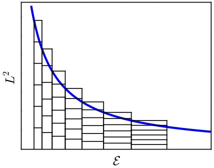
$$p(x_0|f, S) = \frac{f(x_0)S(x_0)}{\int f(x)S(x)\,\mathrm{d}x}.$$

Spherical orbit-superposition models
(aka Schwarzschild models)

Galaxy = Potential Φ + orbits in Φ.

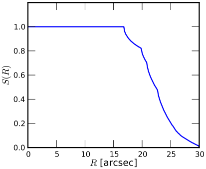
Given trial potential $\Phi(M_\bullet, M_\star, \alpha)$:

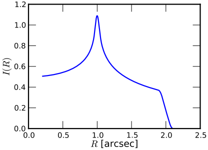
1. Partition phase space into blocks, weights $\boldsymbol{w}$, $\sum_j w_j = 1$:

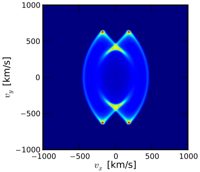


[DF $f(\mathcal{E}, L^2)$ is correctly normalised probability distribution]

2. Calculate $P_{ij} = p(\mathrm{Obs}_i \mid \mathrm{block}_j, S)$ and $I_j = p(\mathrm{Anything} \mid \mathrm{block}_j, S)$ for selection function S







3. Given this Φ, find weight vector $\boldsymbol{w}$ that maximises

$$p(D|\Phi \boldsymbol{w} S) = \prod_{i=1}^{n_{\rm obs}} \left[\frac{\sum_j P_{ij} w_j}{\sum_j I_j w_j} \right]^{n_i}$$

   subject to $\sum_j w_j = 1$.
   n_i is the number of stars observed in the $i^{\rm th}$ "bin."

4. Assign **(Bzzzt)**

$$p(\Phi|D) = \max_{\boldsymbol{w}} p(D|\Phi \boldsymbol{w}).$$

   [That is, take best $\boldsymbol{w}$ as representative of Φ.]

III. Finding the best-fit model (technical details)

Expectation–maximisation algorithm

Problem

Find weight vector $\boldsymbol{w}$ that maximises

$$p(D|\Phi \boldsymbol{w} S) = \prod_i \left[\frac{\sum_j P_{ij} w_j}{\sum_j l_j w_j} \right]^{n_i}, \qquad (1)$$

subject to $\sum_j w_j = 1$.

Solution: If we had a **mixture model** with

$$p'(D|\Phi \boldsymbol{w}') = \prod_i \sum_j P'_{ij}(\Phi) w'_j \quad \text{and} \quad \sum_j w'_j = 1, \qquad (2)$$

then we could use the EM algorithm to find best $(\Phi, \boldsymbol{w}')$.

So, turn (1) into (2) by taking $w'_j = l_j w_j / \sum_k l_k w_k$ and $P'_{ij} = P_{ij}/l_j$.

Expectation–maximisation algorithm

Full EM algorithm varies both $\boldsymbol{w}$ and Φ.
Calculating $P_{ij}(\Phi)$ and $I_j(\Phi)$ is expensive, so I hold Φ fixed.

Resulting algorithm

$$w_j'^{\text{new}} = w_j'^{\text{old}} \frac{1}{NI_j} \sum_i \frac{n_i}{\sum_k P_{ik} w_k} P_{ij},$$

from which

$$w_j = \frac{w_j'}{\sum_k w_k'}.$$

Nothing more than Richardson–Lucy with an extra I_j^{-1} factor...

IV. Results from orbit-superposition models

Assumptions behind the orbit superposition models

Model assumptions

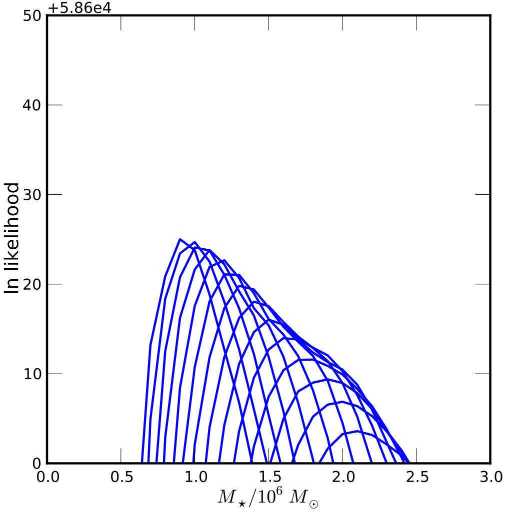
- spherical, non rotating, in equilibrium
- Mass profile $\rho \sim r^{-\alpha}$ (Recycling α, sorry...)
  - Free parameters in Φ: $M_\bullet$, $M_\star(< 1\mathrm{pc})$, α
- $n_\mathcal{E} \times n_L = 50x10$ orbit blocks
- simple selection function.

Not included in the models

- any assumption about $j(r)$
- any assumption about isotropy
- any binning whatsoever (except for the DF orbit blocks...)

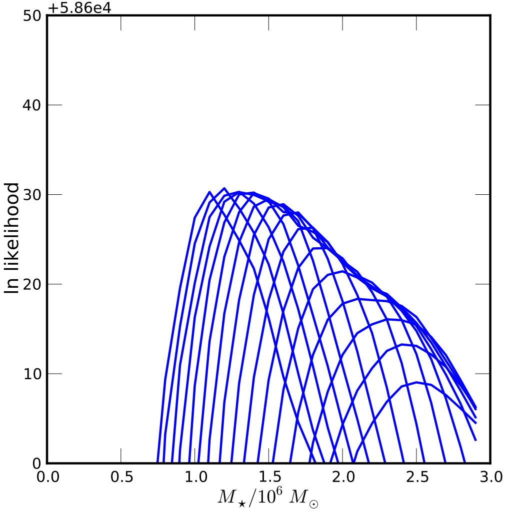
Results from orbit superposition models

Results for $M_\bullet = 3.6 \times 10^6\, M_\odot$: Models want $\rho \sim r^{-\alpha}$ with $\alpha < 0$



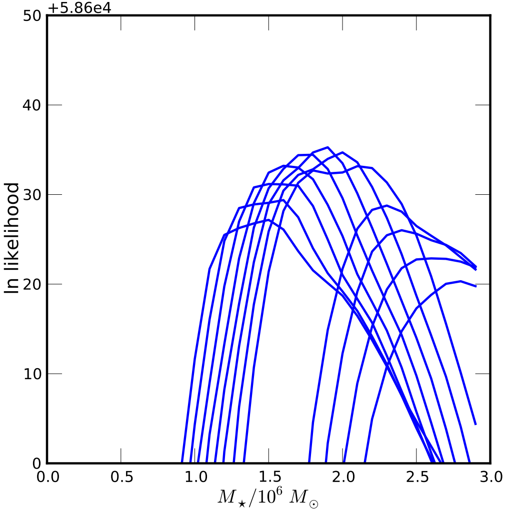
Results from orbit superposition models

Results for $M_\bullet = 3.2 \times 10^6\ M_\odot$:



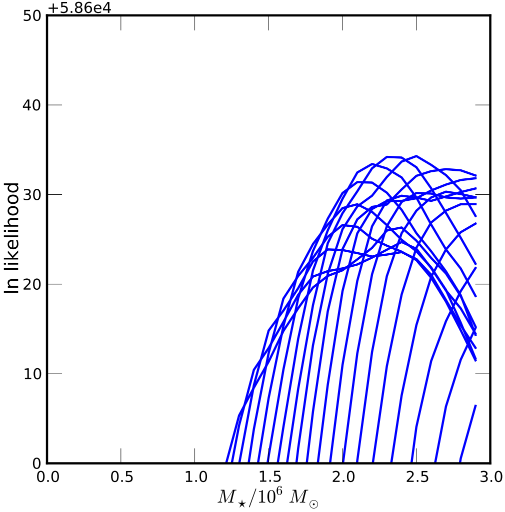
Results from orbit superposition models

Results for $M_\bullet = 2.8 \times 10^6\, M_\odot$:



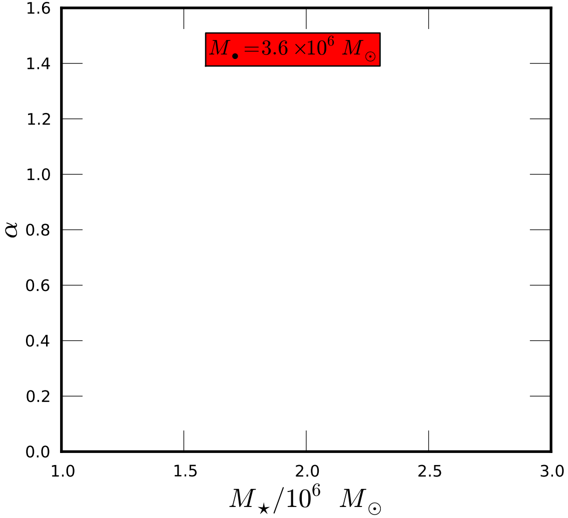
Results from orbit superposition models

Results for $M_\bullet = 2.4 \times 10^6\, M_\odot$:



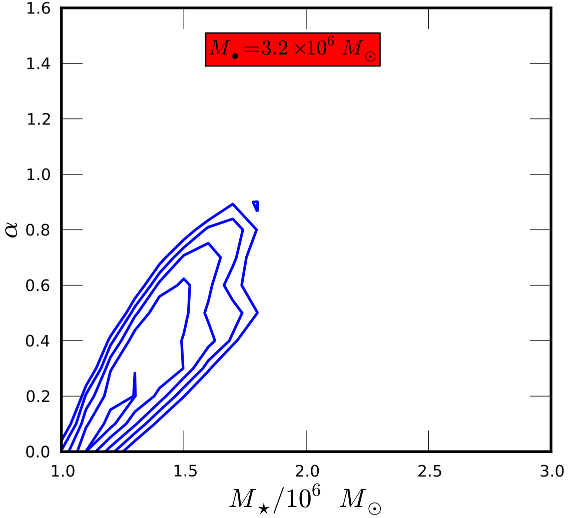
Results from orbit superposition models

Contours spaced at $\Delta \log \mathrm{p}(\Phi|D) = 1$ (i.e., "$\Delta\chi^2 = 2$"):



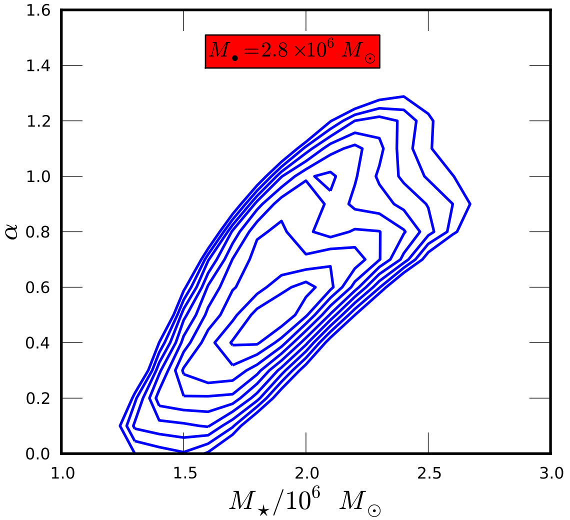
Results from orbit superposition models

Contours spaced at $\Delta \log \mathrm{p}(\Phi|D) = 1$ (i.e., "$\Delta\chi^2 = 2$"):



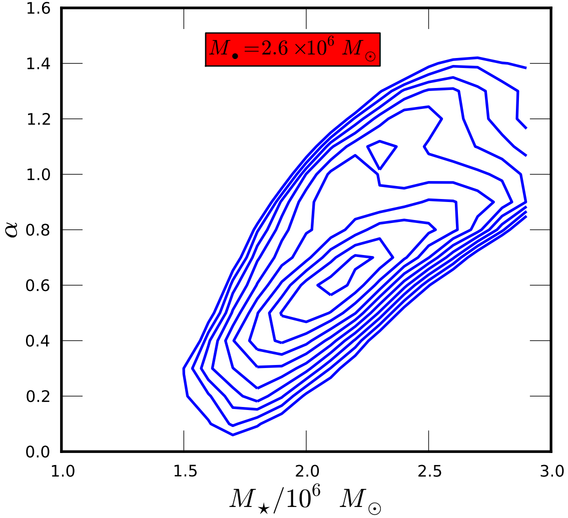
Results from orbit superposition models

Contours spaced at $\Delta \log \mathrm{p}(\Phi|D) = 1$ (i.e., "$\Delta\chi^2 = 2$"):



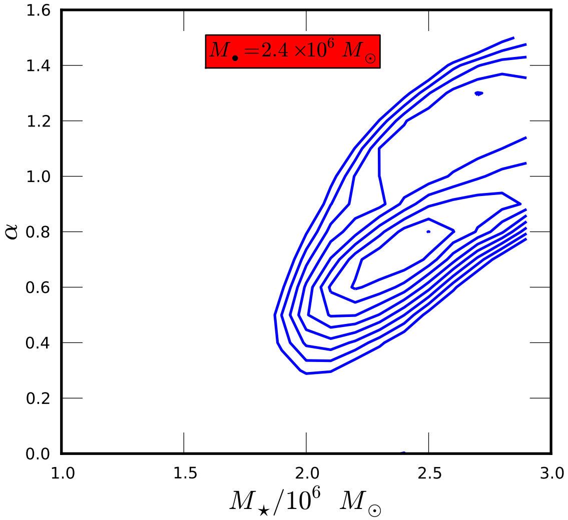
Results from orbit superposition models

Contours spaced at $\Delta \log \mathrm{p}(\Phi|D) = 1$ (i.e., "$\Delta\chi^2 = 2$"):



Results from orbit superposition models

Contours spaced at $\Delta \log \mathrm{p}(\Phi|D) = 1$ (i.e., "$\Delta\chi^2 = 2$"):



Summary of OS models

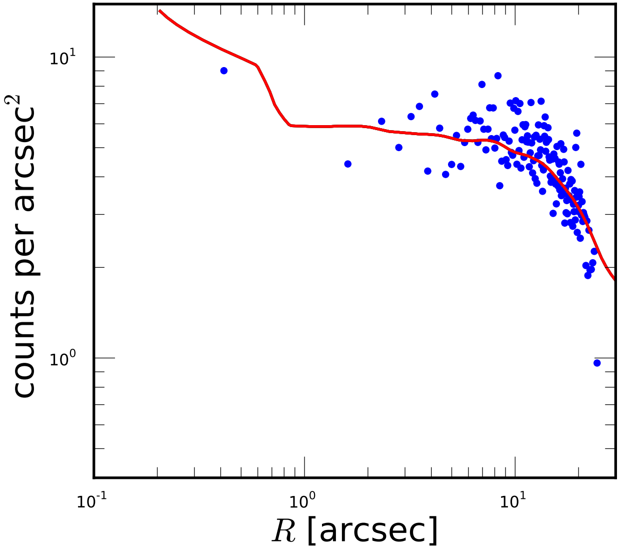
Best-fitting orbit-superposition model has:

- $M_\bullet = \underbrace{2.6}_{\pm 0.1\text{ish}} \times 10^6 M_\odot$, around which
- $\rho \sim r^{-0.6}$ having
- $M_\star = 2.1 \times 10^6 M_\odot$ within 1 pc.

Broadly consistent with Jeans.

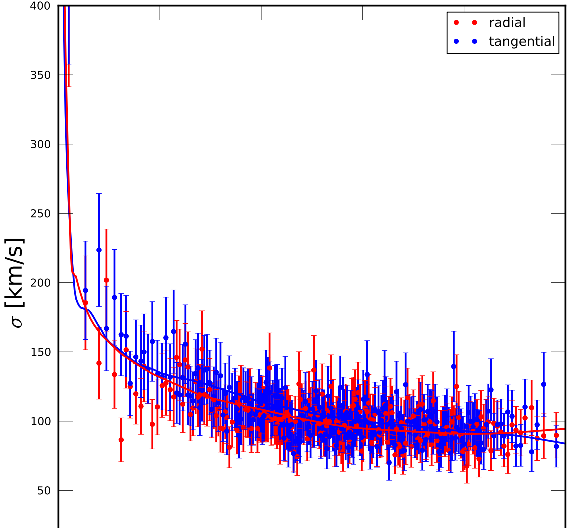
What does the best-fit model look like?

In projection



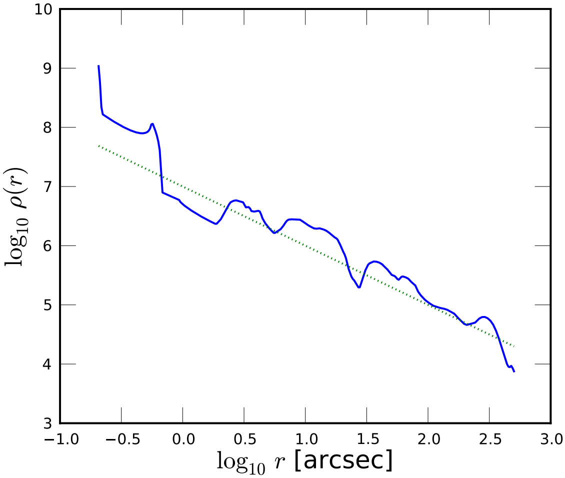
What does the best-fit model look like?

In projection



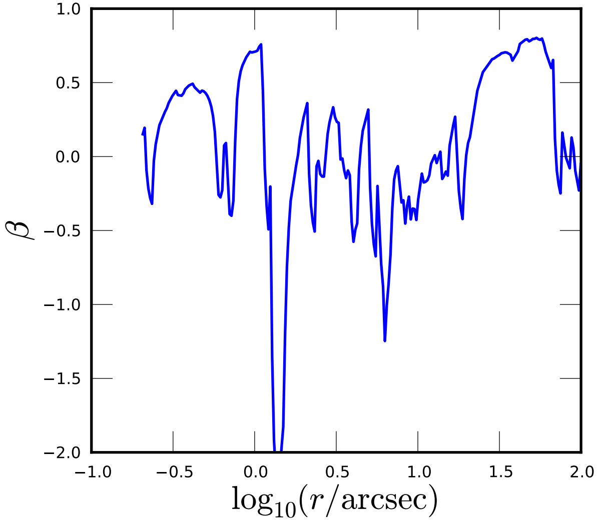
What does the best-fit model look like?

3d density (dotted: mass, solid: light)



What does the best-fit model look like?

Anisotropy parameter



Summary

OS models imply $\sim$ isotropic cluster in which mass follows light around central $M_\bullet \simeq (2.6 \pm 0.1) \times 10^6 M_\odot$.

My own Jeans analysis broadly agrees.
So do independent pre-2003 analyses (for $M_\bullet$ at least).
The S stars don't... (post 2003)

Possible resolutions:

- I don't know where Sgr A* is.
- Observational selection effects aren't as simple as I've assumed. (Bellini talk...)
- Cluster isn't spherical, non-rotating and in equilibrium.
  - e.g., contamination by disc of early-type stars?

Summary

OS models imply $\sim$ isotropic cluster in which mass follows light around central $M_\bullet \simeq (2.6 \pm 0.1) \times 10^6 M_\odot$.

My own Jeans analysis broadly agrees.
So do independent pre-2003 analyses (for $M_\bullet$ at least).
The S stars don't... (post 2003)

Possible resolutions:

- I don't know where Sgr A* is.
- Observational selection effects aren't as simple as I've assumed. (Bellini talk...)
- Cluster isn't spherical, non-rotating and in equilibrium.
    - e.g., contamination by disc of early-type stars?

Lessons

If the S-stars result didn't exist, I'd preach:

1. Still haven't marginalised f.
2. Best-fit OS model fits data too well.
3. We're looking for $O(1)$ changes in log likelihood ~ 59000
   - calculate individual likelihoods as accurately as possible.
4. Choice of Φ relies on inspiration.

Sobering observation

This *looks* like a relatively clean problem:

- simple geometry
- easy-to-interpret observations
- simple selection function.