

The material in the B lectures is extra and probably will not be covered.

## 1. Introduction and probability

- course goals, content, structure, requirements
- probability as a means of expressing uncertainty
- basic rules of probability; Bayes' theorem; Bayesian reasoning and inference
- permutations and combinations. R functions: `choose`, `gamma`, `factorial`, `combinations`, `permutations`,
- introduction to R: concept, data structures (`vector`, `matrix`, `data.frame`), syntax. Basic R functions for manipulation such as `c`, `cbind`, `rbind`, `[]`, `sum`, `length`, `max`, `which.max`, `range`, ..., and for mathematics such as `log`, `log10`, `exp`, `sin`, `atan`, ...
- basic R statistical functions such as `mean`, `median`, `var`, `sd`, `mad`, `quartile`, `IQR`, ...
- data inspection and R plots: scatter plots, histograms. R functions: `plot`, `points`, `lines`, `abline`, `boxplot`, `hist`, `truehist{MASS}`, `mfrow`, `mfcoll`, `par`, ...

## 2. Statistical models and probability distributions

- introduction to statistical models (modelling data)
- Bayesian hypothesis testing (model comparison)
- continuous probability distributions: distribution and density functions; expectation values
- working with the Gaussian distribution in R. R functions: `dnorm`, `pnorm`, `qnorm`; `qqnorm`, `qqline`, `qqplot`, `cumsum`
- summarizing probability distributions: expectation values, mean, variance, moments, skew, kurtosis. R functions: `polygon`
- random numbers and random number generators. R functions: `set.seed`, `sample`, `runif`, `rnorm`
- multiple variables: covariance. R functions: `cor`, `cov`
- multivariate (joint) probability distributions: conditional and marginal distributions. R functions: `mvrnorm`

## 3. Estimation, errors and uncertainty

- estimation
- the central limit theorem
- repeated measurements and  $\sqrt{n}$  reduction in error; standard error in the mean
- measurement errors vs. uncertainties
- random and systematic errors

- principle of propagating errors: general derivation
- propagation of errors in one, two, many variables; fractional errors; weighted average
- confidence bounds and intervals intervals
- gamma distribution (in homework). R functions: `dgamma`, `pgamma`, `qgamma`, `rgamma`

#### 4. Orthodox hypothesis testing

- degrees of freedom
- concept of orthodox hypothesis testing
- one sample tests
- Gaussian z statistic, t statistic, Student t-test and t distribution. R functions: `TDist`: `dt`, `pt`, `qt`, `rt`; `t.test`
- two sample tests
- p-values and the problem with orthodox hypothesis testing

#### 4B. Supplement to Orthodox hypothesis testing

- Fisher F test: F statistic and F distribution. R functions: `df`, `pf`, `qf`, `rf`; `factor`, `var.test`
- relation between the  $\chi^2$  and F distributions
- summary of tests for Gaussian data
- non-parametric tests: KS test, Wilcoxon test. R functions: `ks.test`, `wilcox.test`
- pairwise tests (matched samples)

#### 5. Linear models and regression

- least squares estimation of parameters of a 1D linear fit
- uncertainty estimates on the model parameters
- confidence intervals on predictions
- hypothesis testing with linear models
- linear model fitting with measurement errors
- measures of goodness of fit
- why least squares?
- linear regression in R: R functions: `lm`, `attributes`, `predict`
- single parameter nonlinear regression
- goodness of fit and the  $\chi^2$  distribution and test. R functions: `Chisquare`: `dchisq`, `pchisq`, `qchisq`, `rchisq`
- multi-parameter linear regression

## 5B. Categorical regression

- types of variables: categorical data
- analysis of variance (ANOVA). R functions: `lm`, `aov`, `tapply`, `xyplot`
- ANOVA as linear regression
- multi-way ANOVA
- analysis of covariance (ANCOVA)
- comparing regression models using `anova`

## 6. Binomial and poisson distributions

- binomial distribution for binary processes. R functions: `dbinom`, `pbinom`, `qbinom`, `rbinom`
- logistic model and logistic regression
- Poisson distribution for counting processes. R functions: `dpois`, `ppois`, `qpois`, `rpois`
- Examples: radioactive decay; large telescopes
- the generalized linear model (GLM). R functions: `glm`, `jitter`
- modelling binominal and Poisson data using GLMs

## 7. Likelihood-based (Bayesian) parametric modelling

- data modelling with parametric models
- the prior, likelihood and posterior (probability density functions)
- conjugate priors; the beta distribution. R functions: `dbeta`, `pbeta`, `qbeta`, `rbeta`
- Is this coin fair? A Bayesian analysis (i.e. binomial likelihood, beta prior)

## 8. Bayesian modelling using Monte Carlo methods for sampling and integration

- why we need efficient sampling
- why uses of integration: marginalization, expectation, evidence, prediction
- Monte Carlo integration
- Monte Carlo sampling: rejection sampling, importance sampling, MCMC: Metropolis–Hastings
- example of MCMC: fitting a straight line with unknown noise.
- new R functions: `density`, `%*%`, `drop`, `as.matrix`, `plotCI{glots}`, `rmvnorm{mvtnorm}`, `apply`
- included own R functions: `make.covariance.matrix`, `metrop` (uses function passing, and `noquote`, `formatC`) and example-specific `log.post`
- MCMC with `MCMCmetrop1R{MCMCpack}` in R (optional)

## 9. Maximum likelihood and density estimation

- maximum likelihood; least squares,  $\chi^2$
- density estimation
- histograms. R functions: `truehist{MASS}`, `hist`
- kernel density estimation: Gaussian and nearest neighbour kernels, nearest neighbours. R functions: `density`
- two-dimensional kernel density estimation. R functions: `kde2d`, `image`, `persp`

## 9B. Nonparametric and nonlinear regression

- the concept of regularization in regression and model fitting
- linear regression in matrix form (i.e. multidimensional linear regression)
- nonparametric regression: ridge regression; in the exercises R functions: `solve`, `ksmooth`, `locpoly{KernSmooth}`
- nonlinear regression. R functions: `nls`

## 10. Use and abuse of statistics

- Discovery of the Higgs boson? What p-values mean
- Simpsons paradox
- The stopping paradox, or the dependence of p-values on irrelevant information
- The three doors problem
- The three prisoners problem
- The danger – in orthodox hypothesis testing – of over-testing
- Do plane crashes come in clusters? Poisson processes