

# Planet Formation

lecture by Roy van Boekel (MPIA)

suggested reading:

“LECTURE NOTES ON THE FORMATION AND EARLY  
EVOLUTION OF PLANETARY SYSTEMS”

by Philip J. Armitage

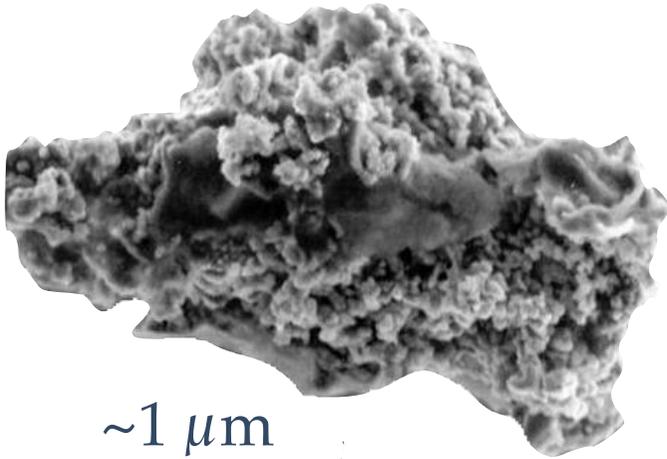
<http://arxiv.org/abs/astro-ph/0701485>

with material from

Kees Dullemond, Christoph Mordasini, Til Birnstiel

# the scales

from



$\sim 1 \mu\text{m}$

to



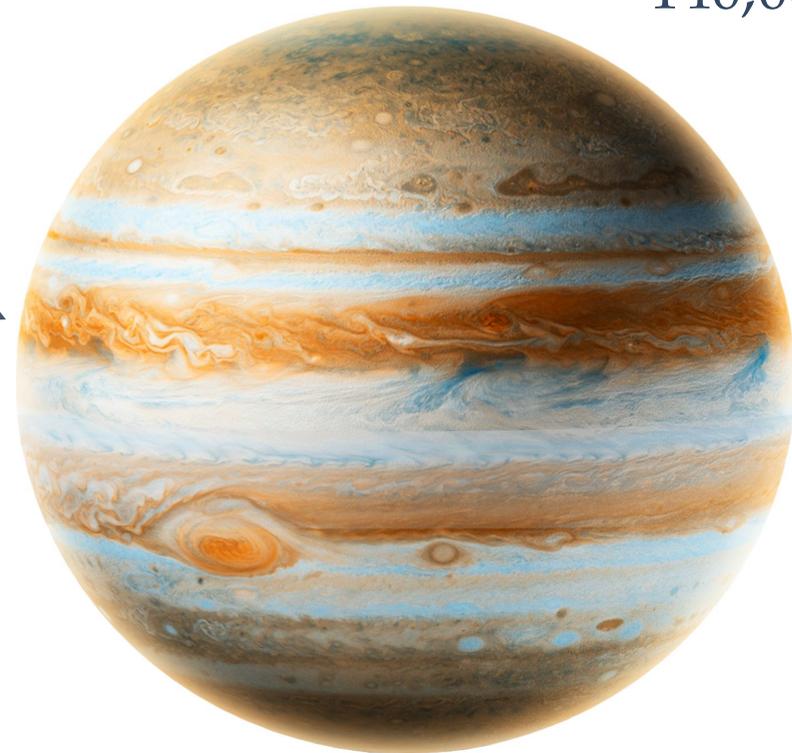
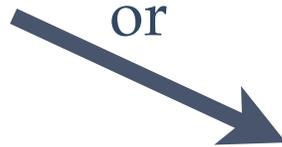
$\sim 13,000 \text{ km}$

in



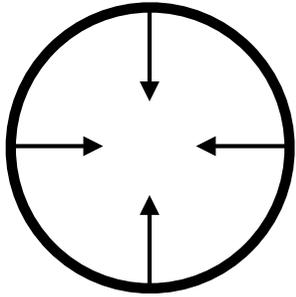
$\sim 30,000,000,000 \text{ km}$  ( $\sim 100 \text{ AU}$ )

or

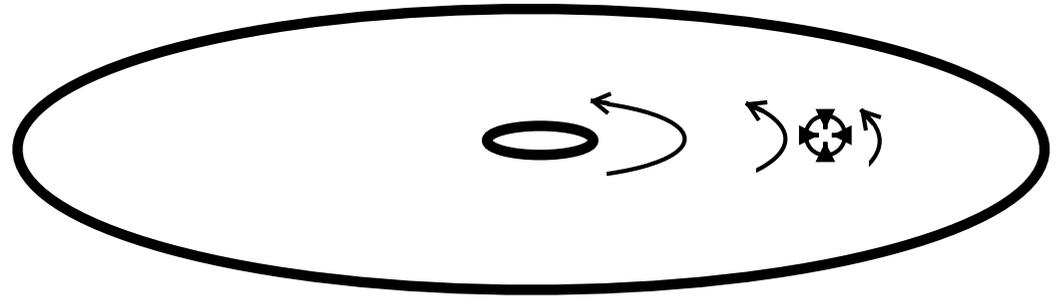


$\sim 140,000 \text{ km}$

# star formation vs. planet formation



- ~spherical collapse, “own gravity” dominant
- rotation, shed *very much* angular momentum
- cooling
- (elemental) composition ~interstellar/solar
- scales:  $\sim 0.1$  pc  $\rightarrow$   $\sim 1 R_{\text{Sun}}$



- stellar gravity usually dominant
- Keplerian shear, much less net angular momentum loss
- cooling!
- composition can (locally) be highly enriched in heavy elements/“dust”
- scales:  $< 1$  to  $\sim$ several AU  $\rightarrow$   $\sim 1 R_{\text{Earth}}$  to  $\sim 1 R_{\text{Jupiter}}$

# Minimum Mass Solar Nebula

“What is the minimum amount of disk material to make the solar system planets?” Basic idea:

- (1) Consider the disk region from which each planet (given current mass / location) would accrete material
- (2) Consider the amount of refractory elements in each planet (add volatiles/ices beyond iceline)
- (3) assume disk bulk composition is solar; add H+He gas accordingly

Result (Hayashi 1981):  $\Sigma = 1.7 \times 10^3 \left( \frac{r}{\text{AU}} \right)^{-3/2} \text{ g cm}^{-2}$ . (4)

$\Sigma$  is gas surface density (~total dens., H+He gas ~99% of mass)

*This estimate relies on several assumptions and is only an approximate result*

# Minimum Mass Solar Nebula

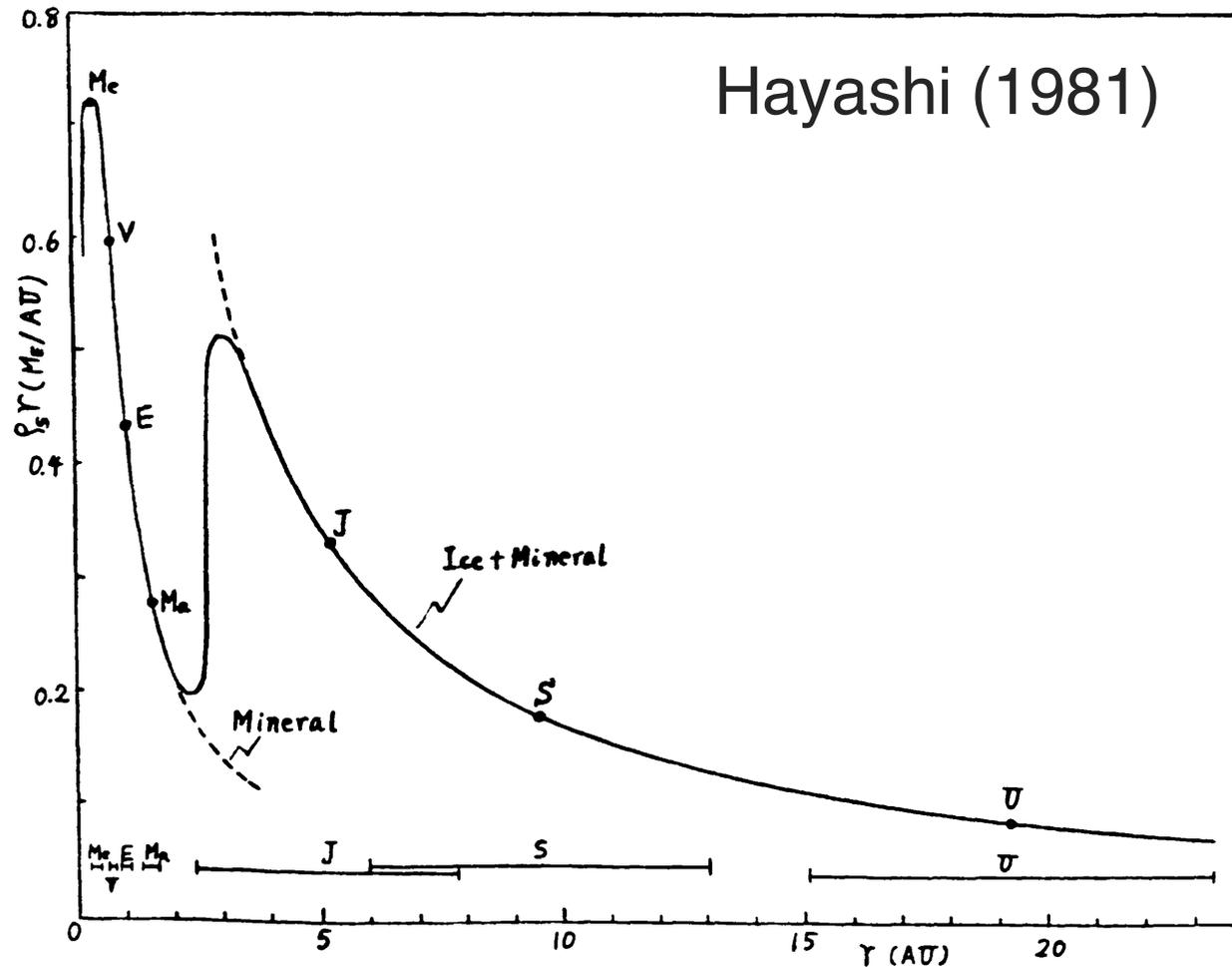
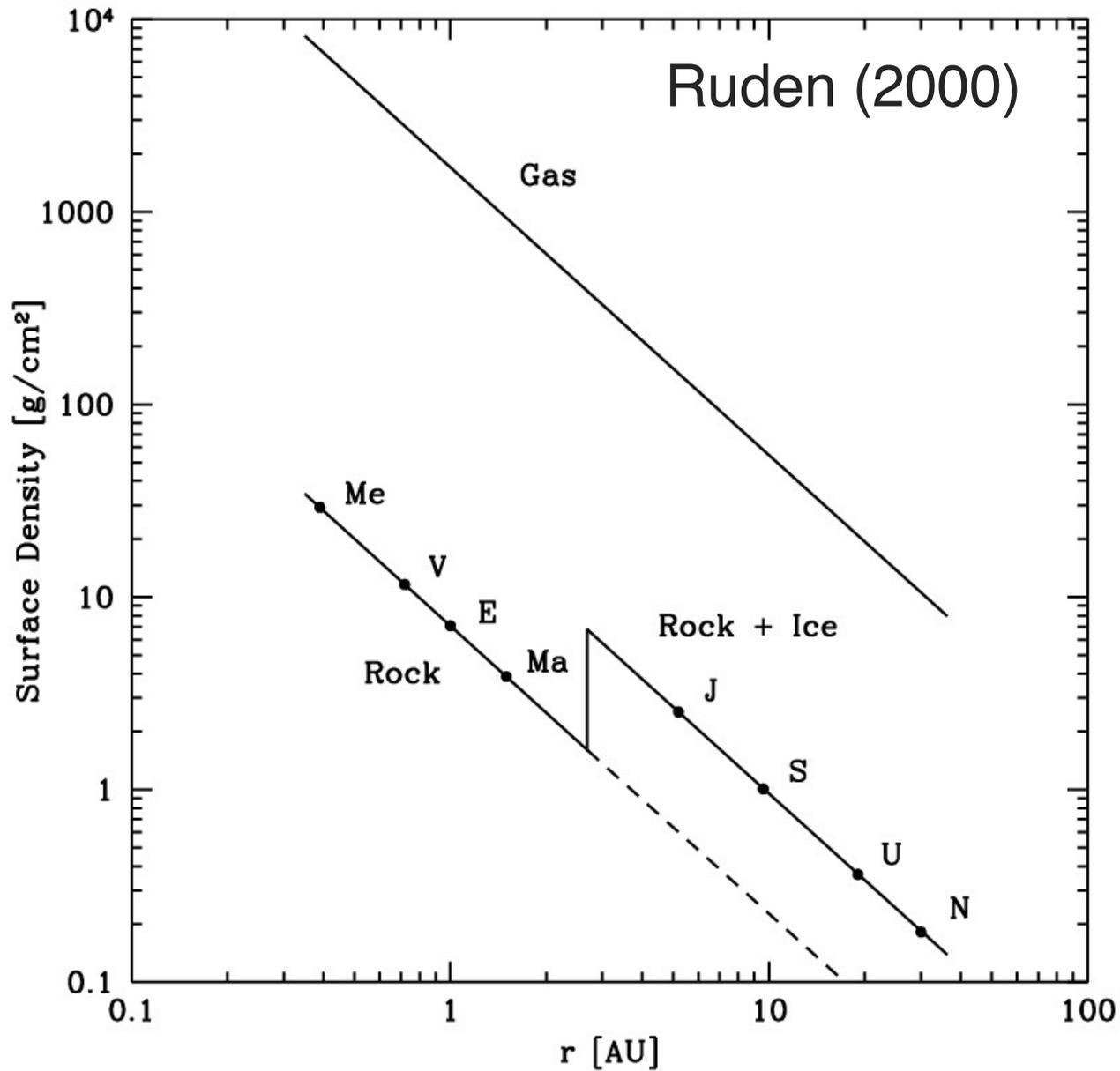


Fig. 1. Distribution of dust mass and ranges of planetary perturbation. The ordinate is the surface density  $\rho_s$  of dust mass multiplied by the distance  $r$ . The ranges of planetary perturbation, indicated by the segments in the lower region of the diagram, correspond to  $a \pm (7h+ea)$  where  $a$ ,  $e$  and  $h$  are the semi-major axis, the eccentricity and the Hill radius of the present planets.

# Minimum Mass Solar Nebula

more recent rendition, ~equivalent to previous plot



# (Giant) Planet Formation

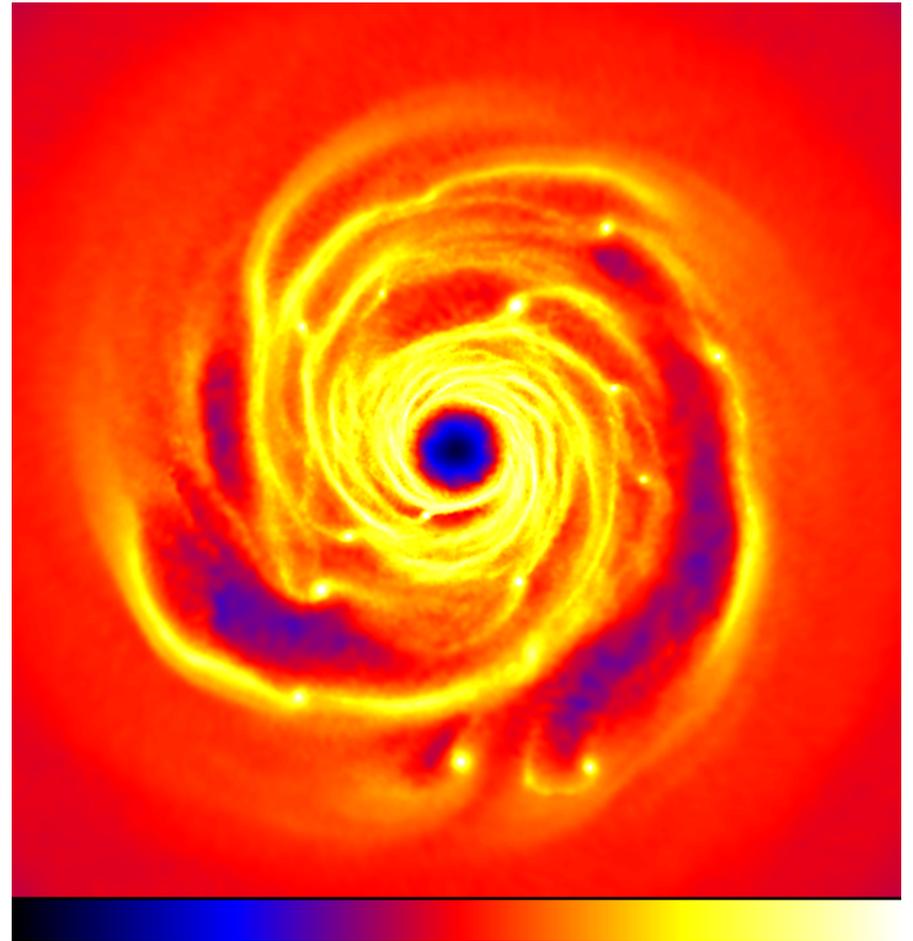
Two main theories of planet formation:

- (1) *Core accretion*: formation of solid core (terrestrial planet), if core sufficiently massive ( $\sim 8 M_{\oplus}$ ) subsequent accretion of gas (gas giant planet).
- (2) *Gravitational Instability*: direct collapse from gas phase (only gas giant planets, relatively far out in the disk)

# Gravitation Instability (GI)

Main idea:

- (1) instability causes initial overdensity, subject to self-gravity
- (2) if these “clumps” can get rid of potential energy faster than pressure and differential rotation smooth them out again, they can collapse
- (3) fast process; planet composition  $\sim$  (local) disk bulk composition



W.K.M. Rice, P.J. Armitage, M.R. Bate & I.A. Bonnell, *MNRAS*, 339, 1025 (2003)

# Gravitation Instability (GI)

Safronov-Toomre Criterion for disk **stability** against isothermal collapse in keplerian disk:

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} > Q_{\text{crit}} \simeq 1 \quad (213)$$

$Q$  is “Toomre parameter”,  $c_s$  is sound speed [ $\text{cm s}^{-1}$ ],  $\Omega$  is orbital frequency [ $\text{rad s}^{-1}$ ],  $G$  = gravitational constant [ $\text{cm}^3 \text{g}^{-1} \text{s}^{-2}$ ],  $\Sigma$  is surface density [ $\text{g cm}^{-2}$ ].

Disk will stable against fragmentation where  $Q > Q_{\text{crit}}$ . Typical value  $Q_{\text{crit}} \approx 1$ . **Having  $Q < Q_{\text{crit}}$  is necessary but not sufficient condition for fragmentation;**

# Gravitation Instability (GI)

Example:

$h/r = 0.05$  at 10 AU around solar-type star;

$h/r = c_s/v_\phi \rightarrow$  sound speed  $c_s \approx 0.5 \text{ km s}^{-1}$ . ( $v_\phi$  is orbital velocity)

To get  $Q < 1$  we require  $\Sigma > 1500 \text{ g cm}^{-2}$ .

Compare to “minimum mass solar nebula”:  $\Sigma \approx 54 \text{ g cm}^{-2}$  @ 10 AU  
(using normalization of Hayashi (1981))

$\rightarrow$  *GI works only for very massive disks*

# GI: resulting planets

spatial scale most unstable to collapse:

$$\lambda_{\text{crit}} = 2c_s^2 / (G\Sigma)$$

resulting planet mass if such a disk region would collapse (for our example with  $\Sigma = 1500 \text{ g cm}^{-2}$ , at 10 AU with  $c_s = 0.5 \text{ km s}^{-1}$ ):

$$M_p \sim \pi \Sigma \lambda_{\text{crit}}^2 \sim \frac{4\pi c_s^4}{G^2 \Sigma} \sim 6 M_J \quad (215)$$

—> *GI produces very massive planets. Works better in outer disk*

# Importance of cooling

(additional requirement on top of Toomre criterion)

Collapsing fragment  $\rightarrow$  release of gravitational energy, needs to be radiated away sufficiently quickly for collapse to proceed.

Cooling time:

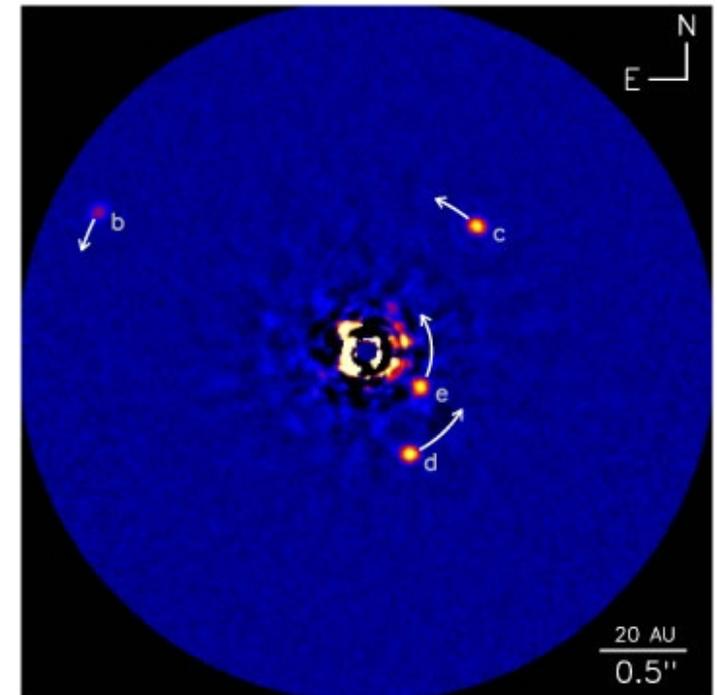
$$t_{\text{cool}} = \frac{U}{2\sigma T_{\text{disk}}^4} \quad (217)$$

where  $U$  is the thermal energy content of the disk per unit surface area. For an ideal gas EOS ( $\gamma = 5/3$ ) we get:

- $t_{\text{cool}} \gtrsim 3\Omega^{-1}$  — the disk fragments.
- $t_{\text{cool}} \lesssim 3\Omega^{-1}$  — disk reaches a steady state in which heating due to dissipation of gravitational turbulence balances cooling.

# Is this (GI) how it goes?

- (1) vast majority of planets (solar system, exoplanets discovered mainly with transit & radial-velocity techniques) are less massive than GI predicts
- (2) In solar system, gas/ice giants are enriched in heavy elements; hard to reconcile with GI.
- (3) some “direct imaging” observations found (young) planets at large distances. Possibly these formed through GI (“jury is still out”).

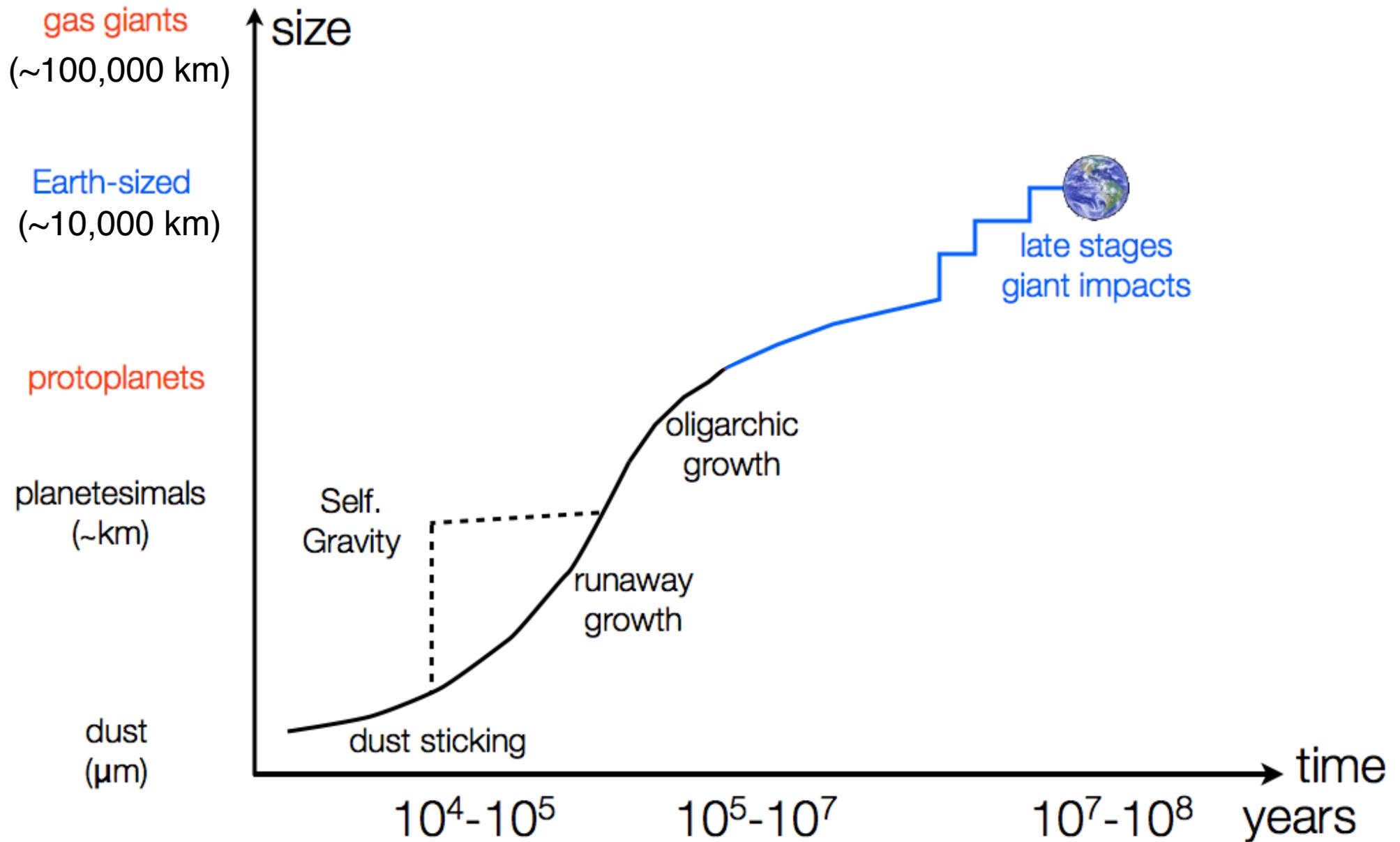


# core-accretion (CA)

Basic idea:

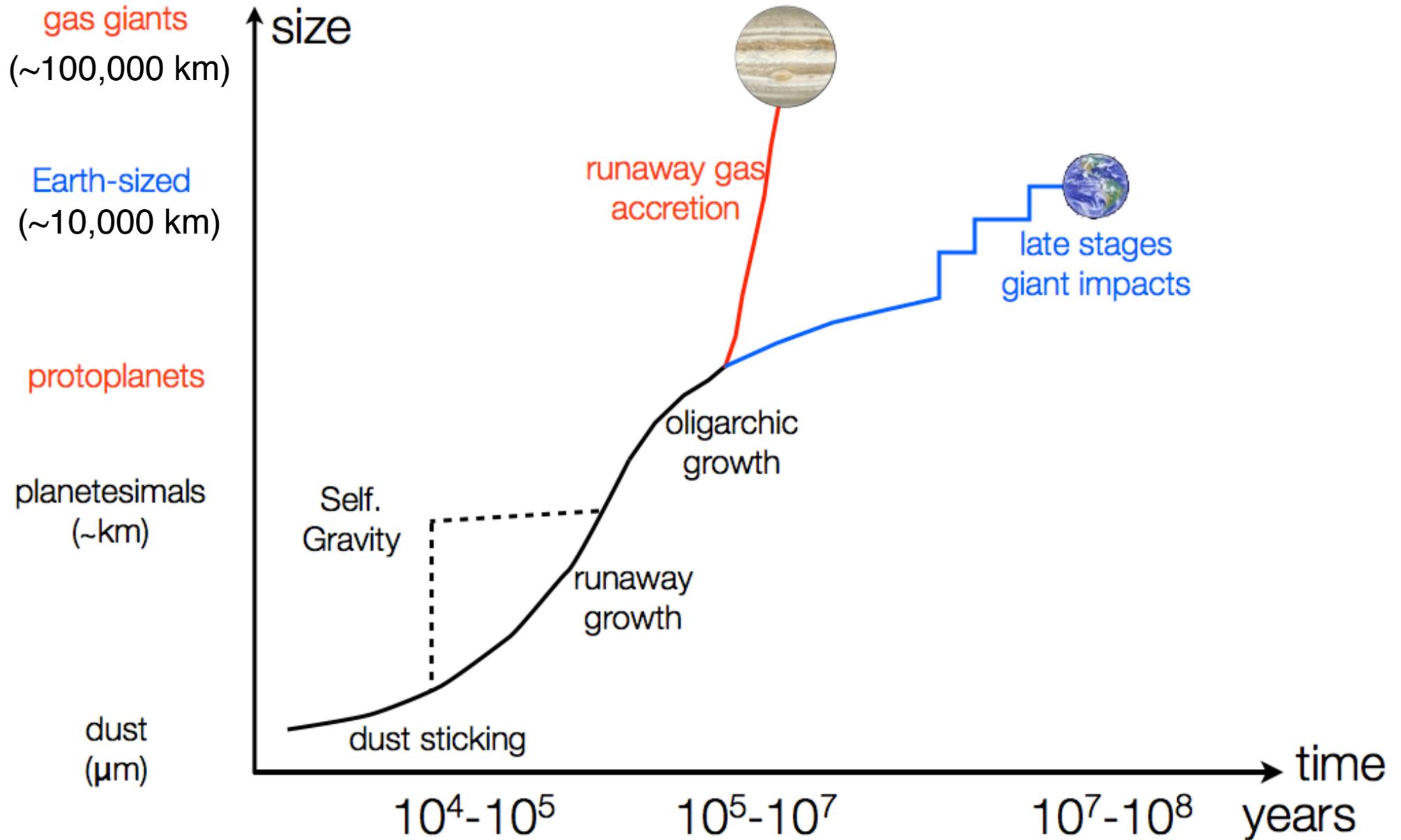
- (1) solid “refractory” material comes together to form larger bodies. Beyond “snowline” this includes (water-) ice.
- (2) bodies grow by low-velocity collisions, until they get so large that their mutual gravity becomes important for the dynamics.
- (3) gravity-assisted growth, initially in a “run-away” fashion, later in an “oligarchic” fashion, to rocky/icy planets
- (4) if rocky/icy core reaches sufficient mass (surface gravity) to retain H+He gas ( $5-10 M_{\text{earth}}$ ) *while the disk is still gas-rich*, then gas accretion ensues. If  $\sim 30 M_{\text{earth}}$  is reached, run-away gas accretion forms a  $\sim$ Jupiter mass planet

# rough timeline

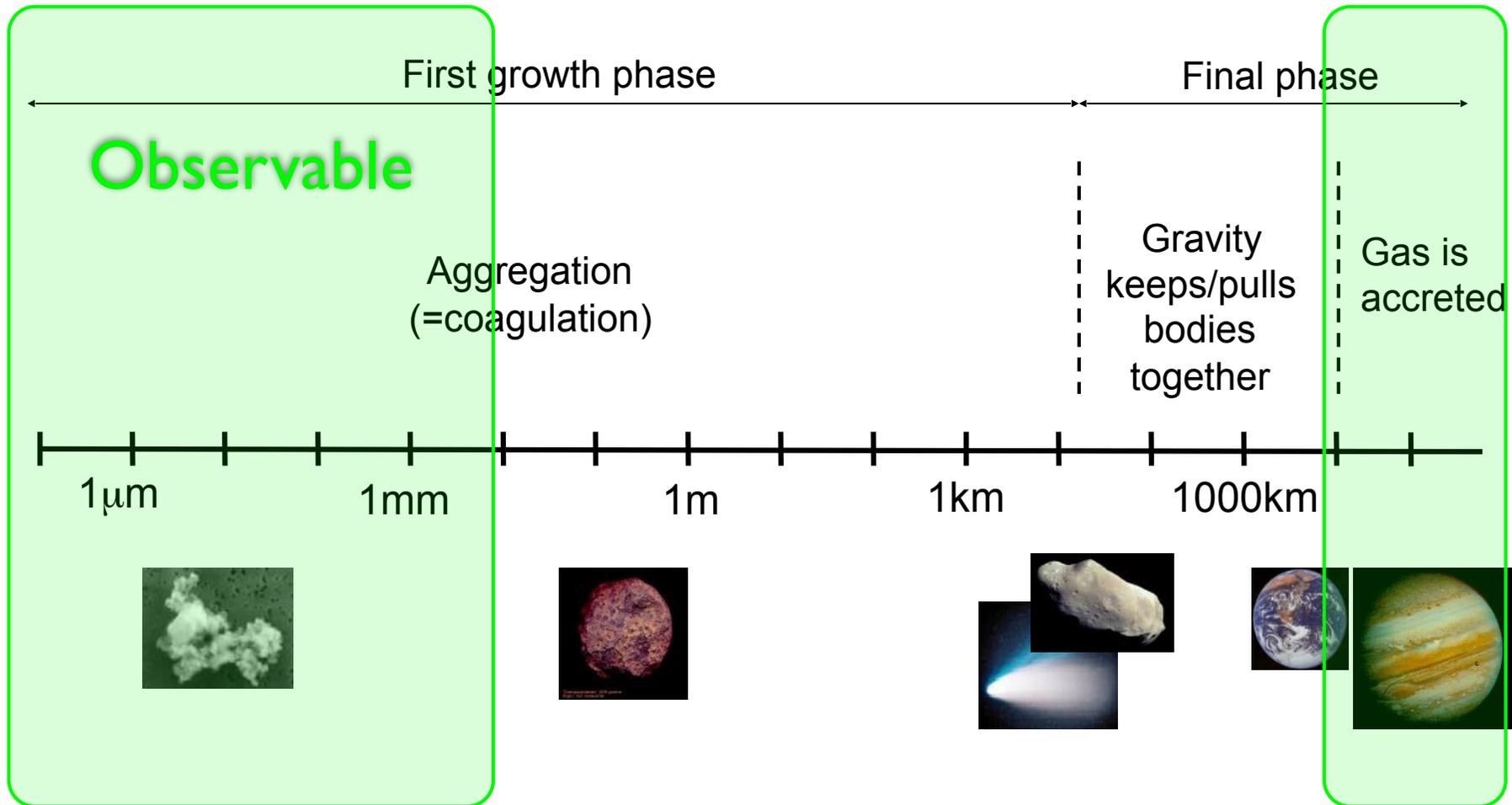


# Formation of gas giants

(if sufficient gas is present)



# The long road from dust to planets



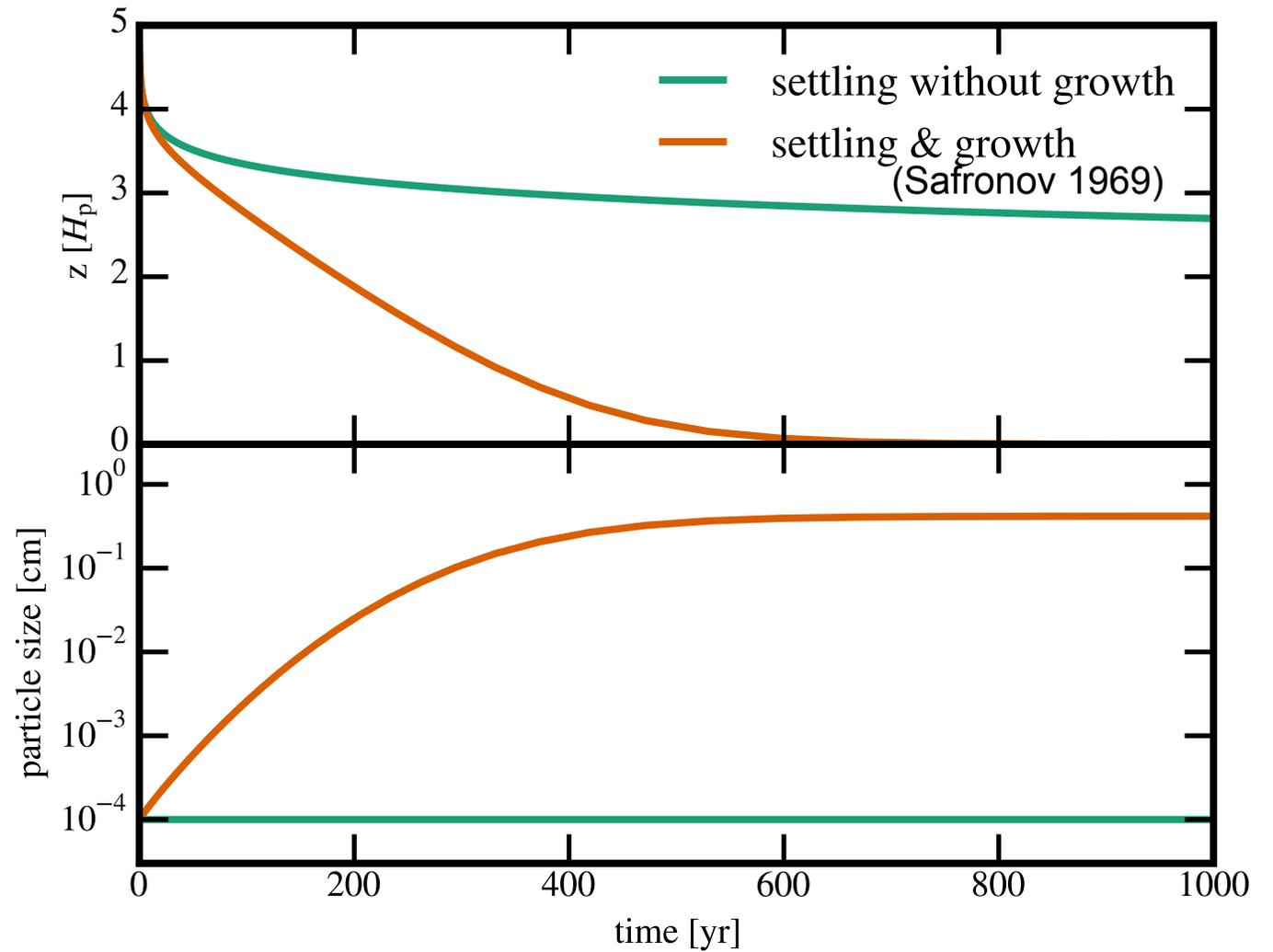
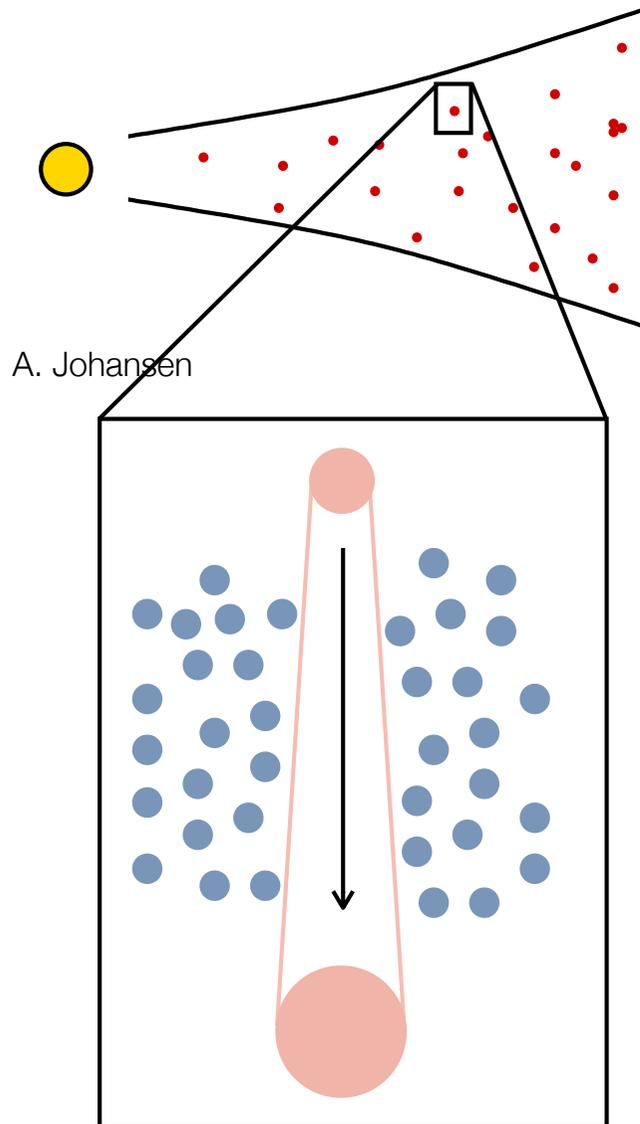
Covers 13 orders of magnitude in size = 40 (!! ) orders of magnitude in mass

# initial growth

Assumed initial situation: small dust particles, e.g.  $0.1 \mu\text{m}$ , mixed homogeneously with gas. (ignores dust growth in cloud/core/collapse phase).

- (1) small dust grains well coupled to gas; small relative motions (brownian motion dominant until  $\sim 1 \mu\text{m}$ ,  $\Delta v \approx 100 \mu\text{m/s}$ ; turbulence-driven motions dominant for larger grains, with  $\Delta v$  up to several  $10 \text{ m/s}$  for  $\sim \text{cm}$  particles
- (2) “touch and stick” at low  $\Delta v$  ( $\lesssim 1 \text{ m/s}$  for pure silicates,  $\lesssim 10 \text{ m/s}$  if particles are icy), destruction / erosion at high  $\Delta v$
- (3) vertical component of gravity, “dust settling”, accelerates growth. ***Also radial drift → problem!***

# vertical settling



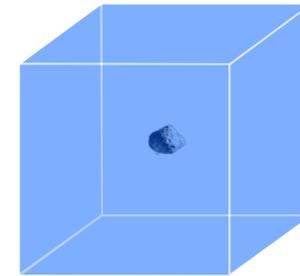
T. Birnstiel

# gas-dust coupling

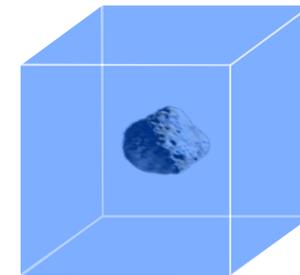
$$\frac{t_{\text{stop}}}{t_{\text{orb}}} = t_{\text{stop}} \cdot \Omega_{\text{K}} \equiv \text{St} \quad (\text{Stokes number})$$

T. Birnstiel

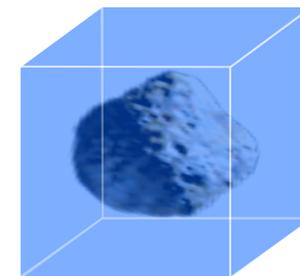
$$\text{St} \ll 1 \quad \text{i.e.} \quad \tau_{\text{fric}} \ll \tau_{\text{orb}}$$



$$\text{St} \sim 1 \quad \text{i.e.} \quad \tau_{\text{fric}} \simeq \tau_{\text{orb}}$$



$$\text{St} \gg 1 \quad \text{i.e.} \quad \tau_{\text{fric}} \gg \tau_{\text{orb}}$$



*It's (mostly) not size that matters - it's the Stokes number!*

# gas-dust coupling

“Epstein” drag regime, particle radius  $a \ll \lambda$ , where  $\lambda$  is mean free path of gas molecule.

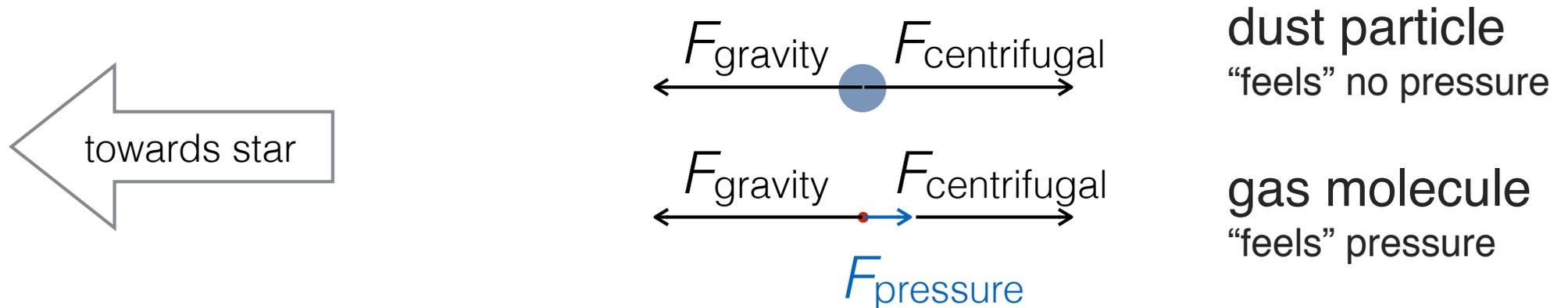
$$\mathbf{F}_{\text{drag}} = -\frac{4\pi}{3} \rho_g a^2 v_{\text{th}} \mathbf{v}$$

$\rho_g$  is gas density,  $\mathbf{v}$  is the velocity of the particle relative to the bulk motion of the gas,  $v_{\text{th}}$  is the thermal velocity of the gas, given by :

$$v_{\text{therm}}^2 = \frac{8kT}{\pi \mu m_H}$$

where  $\mu$  is the mean molecular weight in AMU (typically  $\mu=2.3$ ) and  $m_H$  is the mass of a hydrogen atom.

# radial drift problem



- (1) Gas pressure radially decreasing (in continuous disk).
- (2) Pressure gradient force supports gas disk (but not the dust)
- (3) Force Equilibrium leads to *sub-Keplerian gas rotation*
- (4) Head-wind removes dust angular momentum
- (5) Orbital decay;

# relative particle velocities

- very small particles ~completely coupled to gas (they go wherever the gas goes), Stokes  $\ll 1$ ; small  $\Delta v$ .
- very large particles ~completely de-coupled from gas (they “don’t care” about the gas), Stokes  $\gg 1$ ; small  $\Delta v$  (all ~keplerian), unless gravitational stirring becomes important (later on)
- Intermediate range where particles are semi-coupled (gas affects velocities differently for different sizes), Stokes  $\sim 1$ ; large  $\Delta v$ . Relative velocity several 10 m/s.

# “meter-sized barrier”

(note: more “barriers” exist; e.g. “bouncing barrier”)

two-fold “barrier” for reaching “boulder” sizes (large enough to ~de-couple from gas)

- (1) **drift**: In a MMSN, the decay time for particle with maximum drift velocity (“Stokes parameter”  $\sim 1$ ) from:
- 1 AU :  $\sim 200$  yrs ( $\sim 3$  m particle)
  - 5 AU:  $\sim 1,400$  yr ( $\sim 1$  m particle)
  - 100 AU:  $\sim 30,000$  yrs ( $\sim 10$  cm particle)

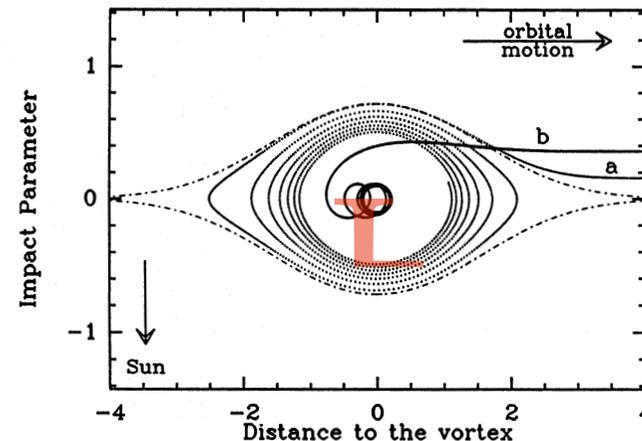
(note: numbers are for specific set of assumptions, only indicative of order of magnitude).

- (2) **fragmentation** relative velocities up to  $\sim$ several 10 m/s around  $St \sim 1$ , destructive collisions.

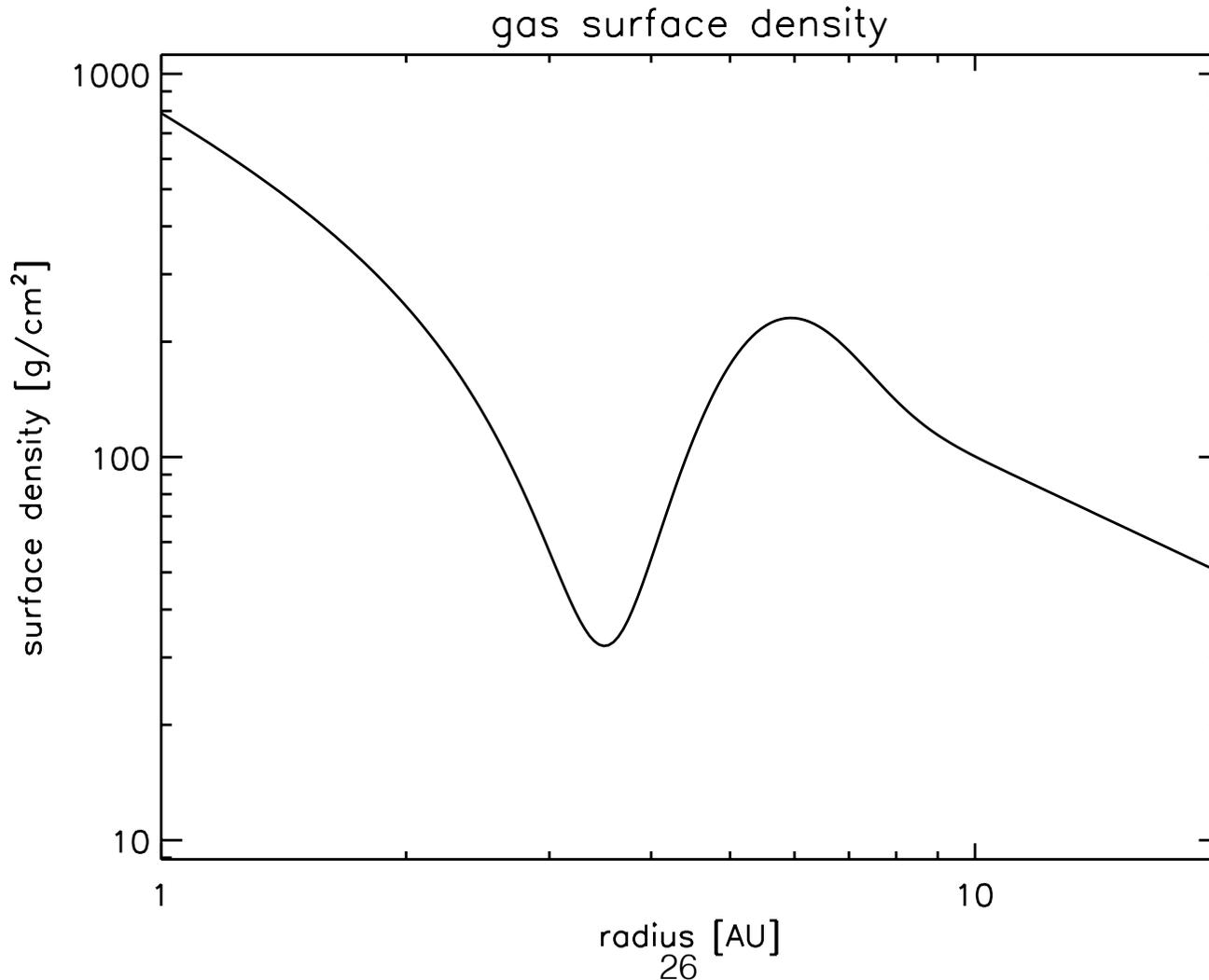
# overcome m-sized barrier

(1) **Gravitational Instability planetesimal formation:** if dust settles in **very** thin disk ( $H_{\text{dust}}/H_{\text{gas}} \sim 10^{-4}$ ) that is also nearly perfectly free of turbulence (relative velocities  $< \sim 10$  cm/s at 1 AU), then dust disk may fragment into clumps that collapse under own gravity (“Goldreich & Ward mechanism”). **Considered unlikely** (turbulence prohibits these circumstances to be reached).

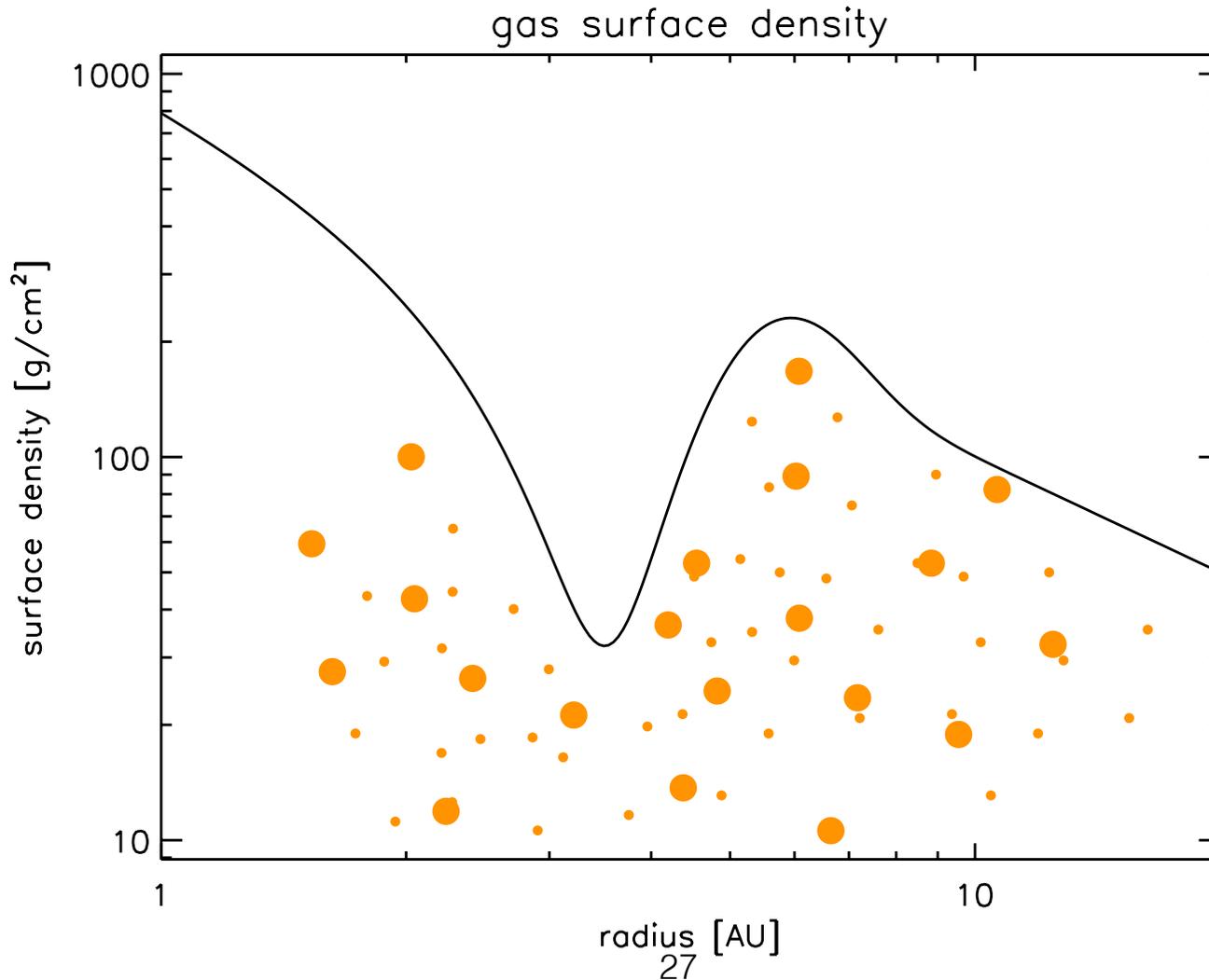
(2) **gravo-turbulent planetesimal formation:** the turbulence itself causes local enhancements of dust density, vortices can “trap” dust



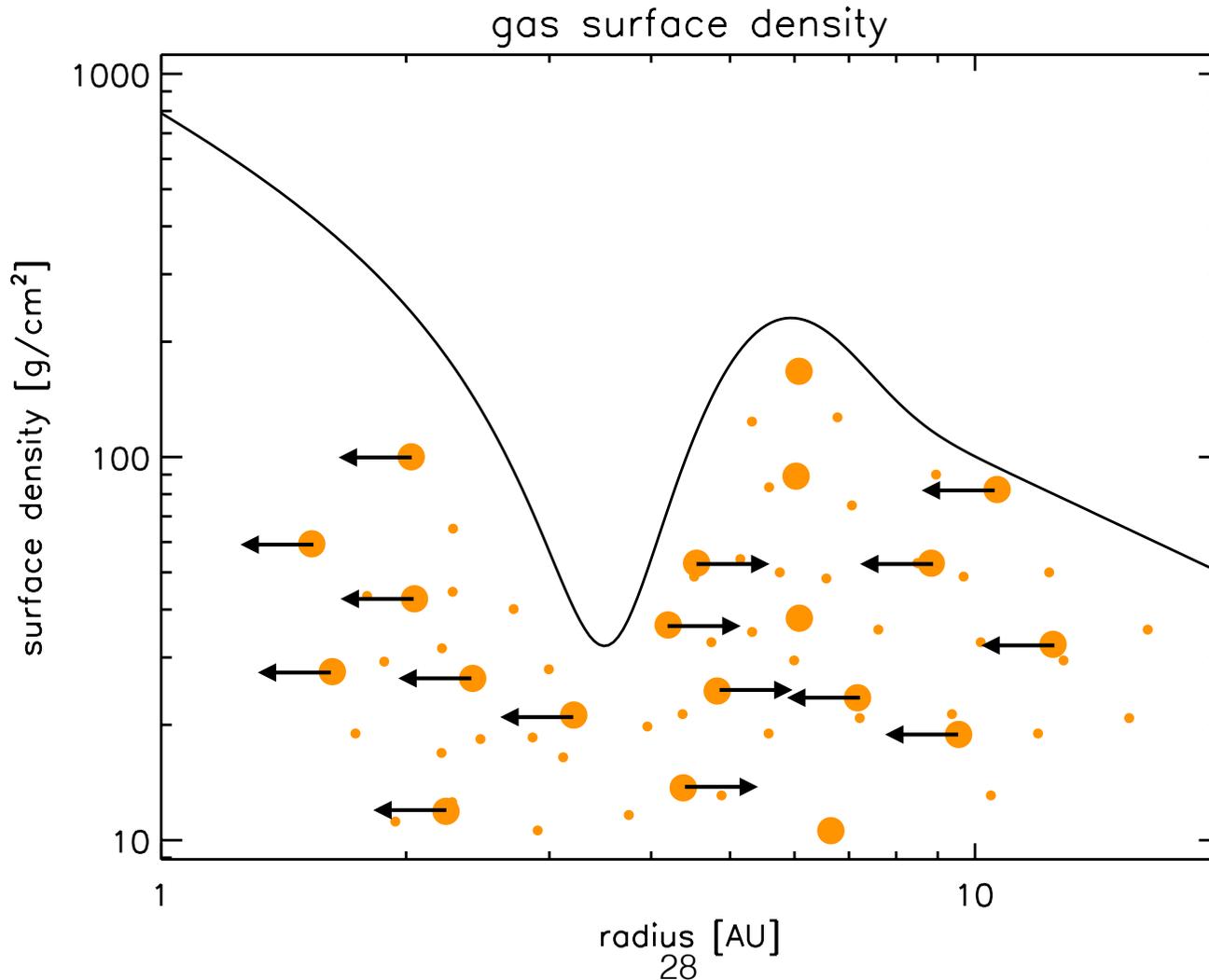
# why dust piles up in regions of high (gas) density



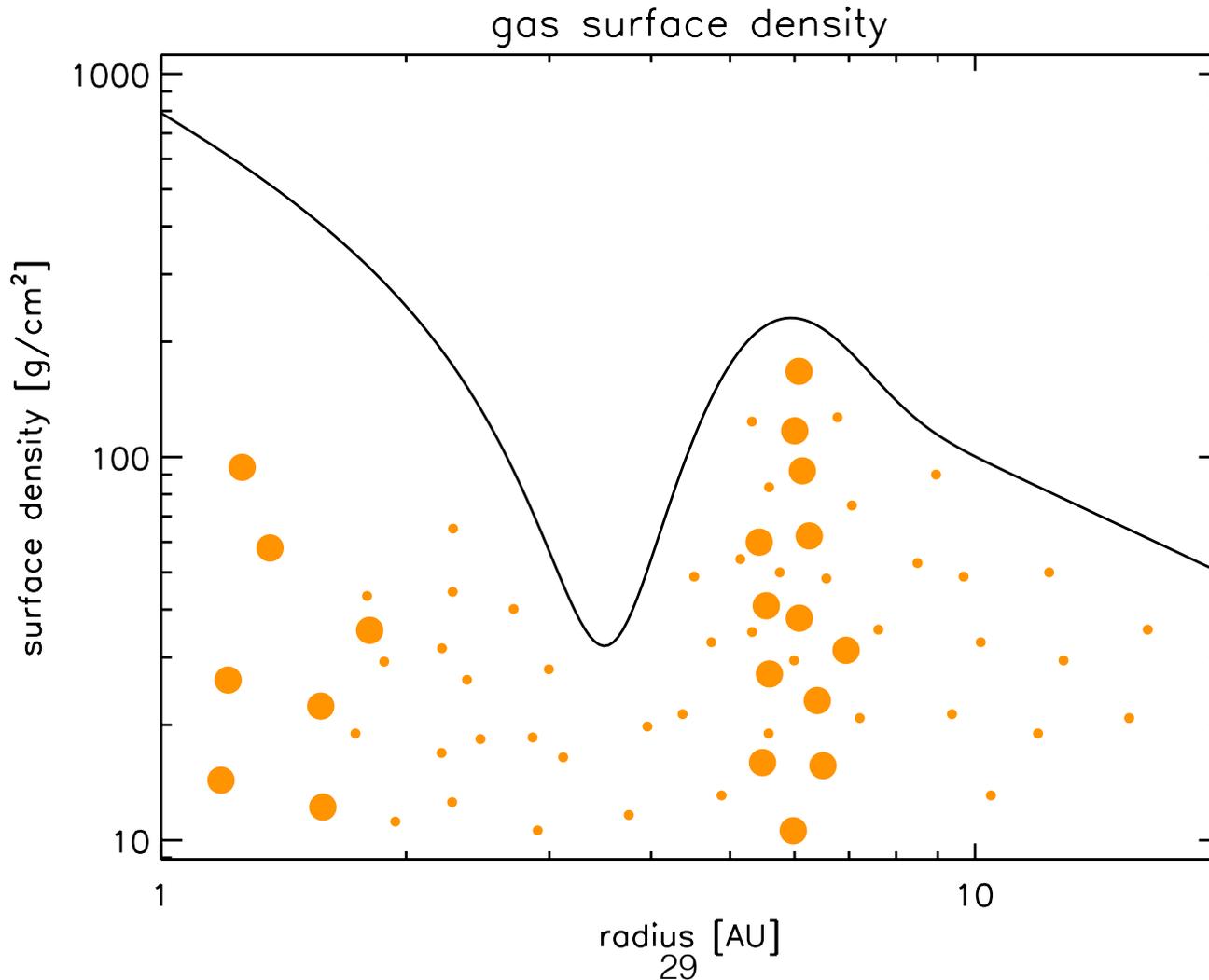
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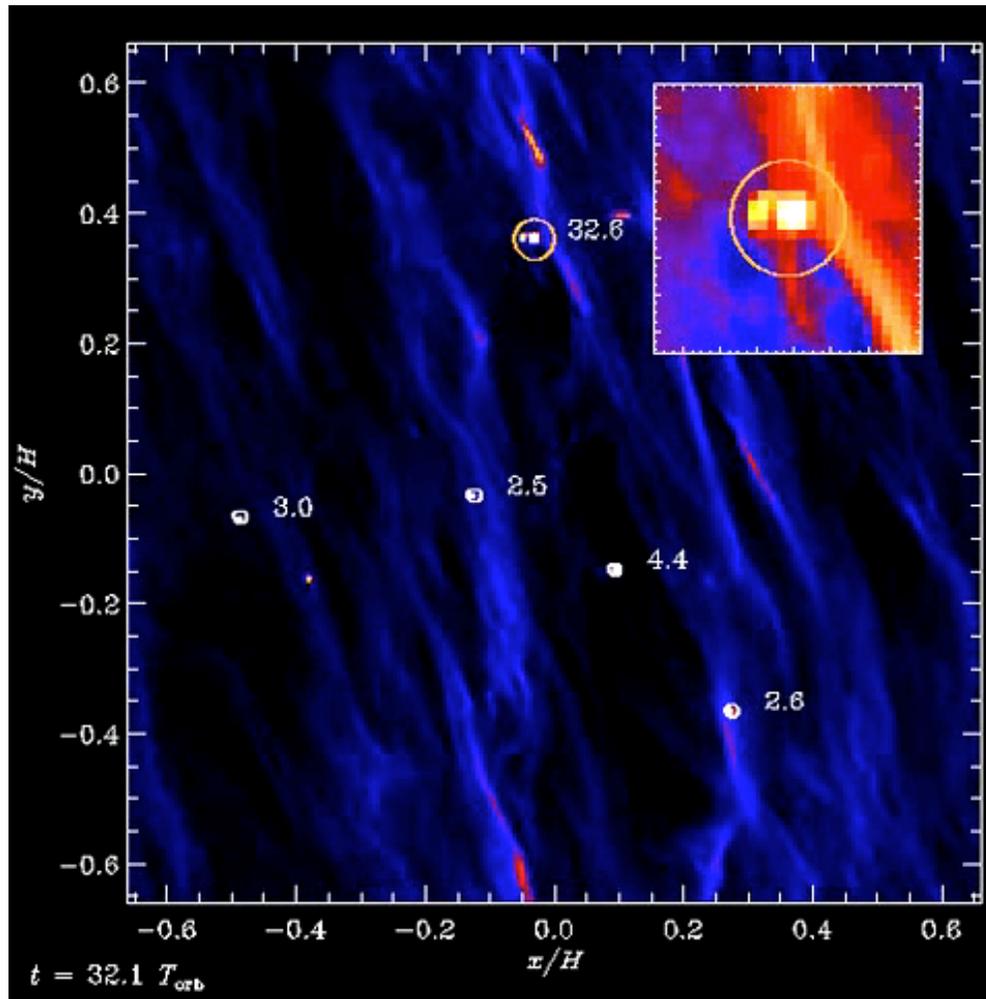
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# why dust piles up in regions of high (gas) density



# gravo-turbulent planetesimal formation

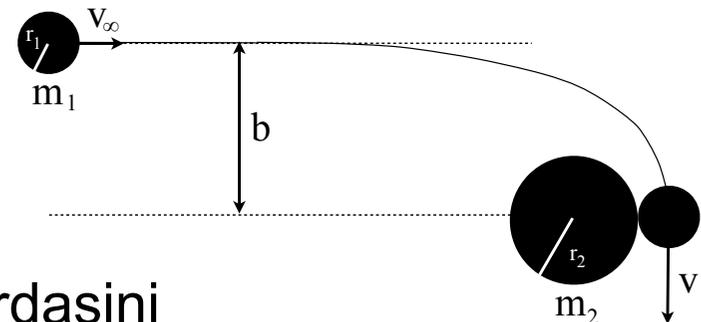


Johansen, Klahr, Henning

- (1) MRI turbulence
- (2) particles gather in overdensities, local enhancements in solids;
- (3) gravity of concentrated dust sustains overdensity, attract more solids
- (4) simulations produce ~few to ~35 Ceres mass bodies in just 13 orbits

# further growth

- (1) particles are  $\gg 1$  m, all move on  $\sim$ keplerian orbits.
- (2) velocities are “damped” by gas  $\rightarrow$  low  $\Delta v$
- (3) growth by low velocity collisions, “mergers”. Slow, “orderly” growth
- (4) initially gravity of growing bodies is not important, but once bodies reach  $\sim$ km sizes, “gravitational focusing” starts to become important and growth goes faster.

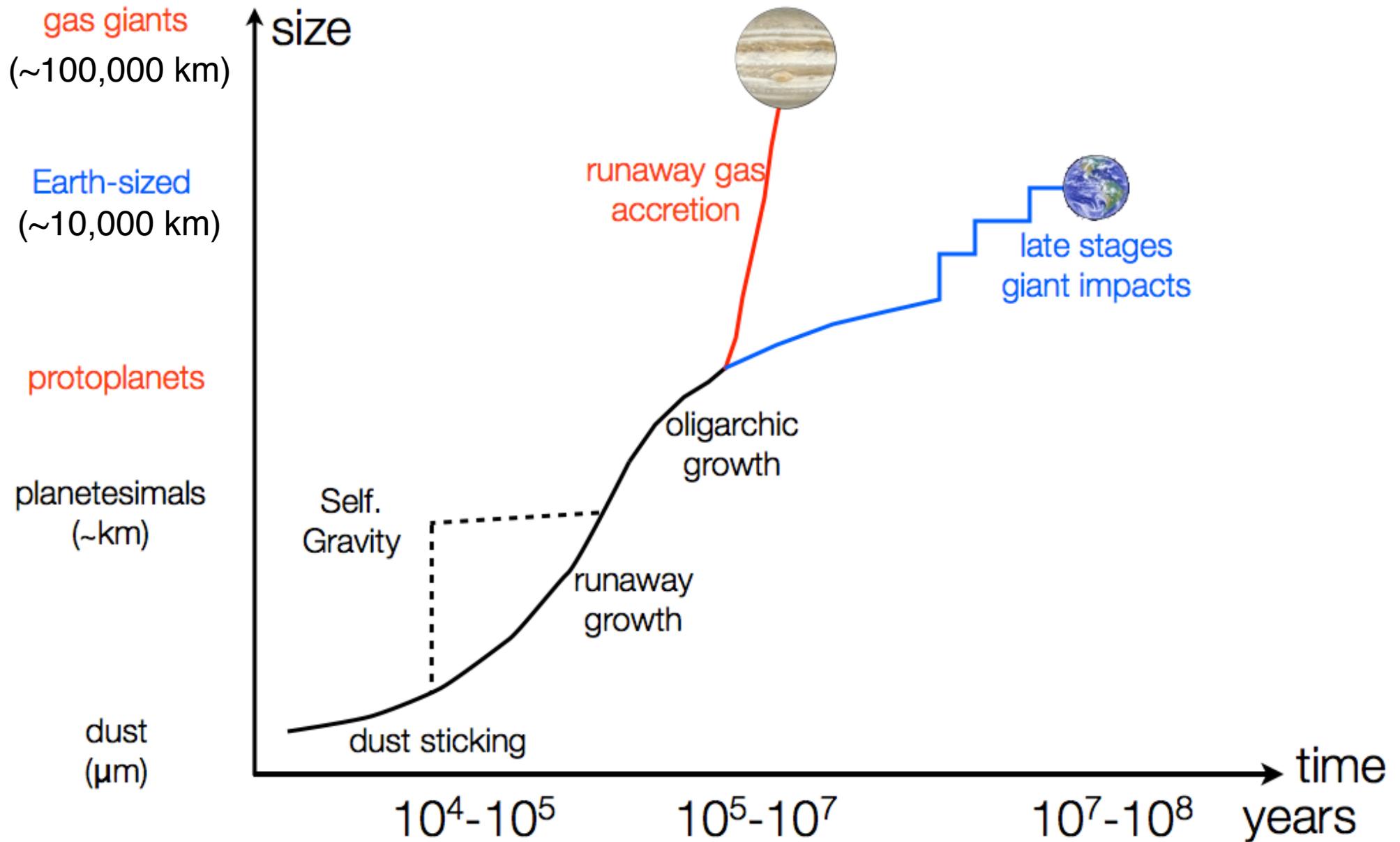


C. Mordasini

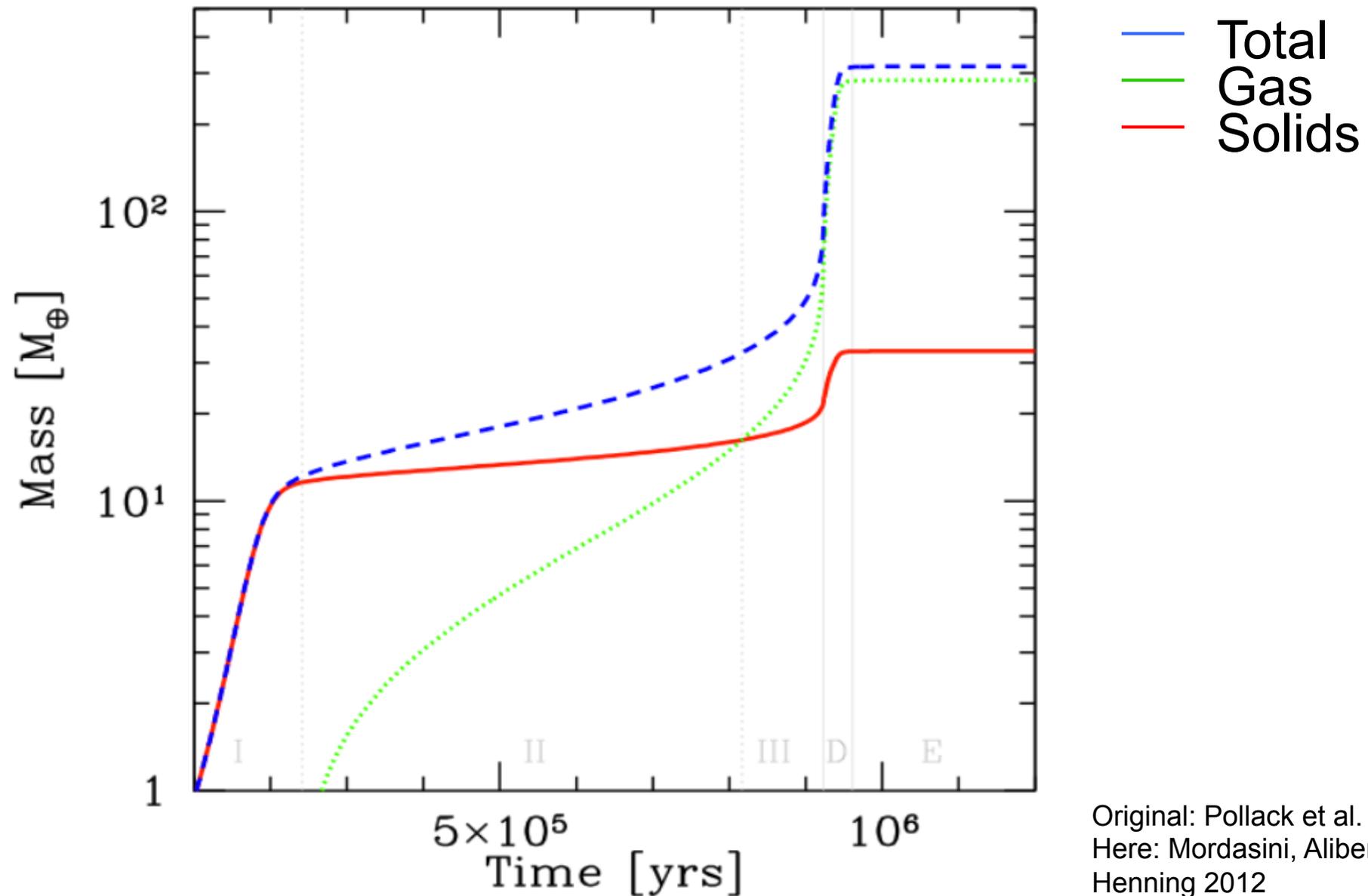
# further growth (II)

- (5) gravity-assisted, accelerated growth. For “dynamically cold” disk, growth rate  $dM/dt \propto M^{4/3}$   $\rightarrow$  largest bodies grow fastest, run-away growth where “winner takes all”
- (6) largest body accretes most planetesimals within its gravitational sphere of influence.
- (7) largest bodies start to gravitationally disturb (“excite”) smaller ones, planetesimal disk gets “dynamically hot” and gravitational focusing is less effective.
- (8) “oligarchic growth” where dozens of  $\geq 0.1 M_{\text{Earth}}$  bodies dominate their local environment and slowly grow further
- (9) complex N-body interactions; many “oligarchs” merge in “giant impacts” (last one in our case is thought to have resulted in Earth-Moon system).

# On to final planets

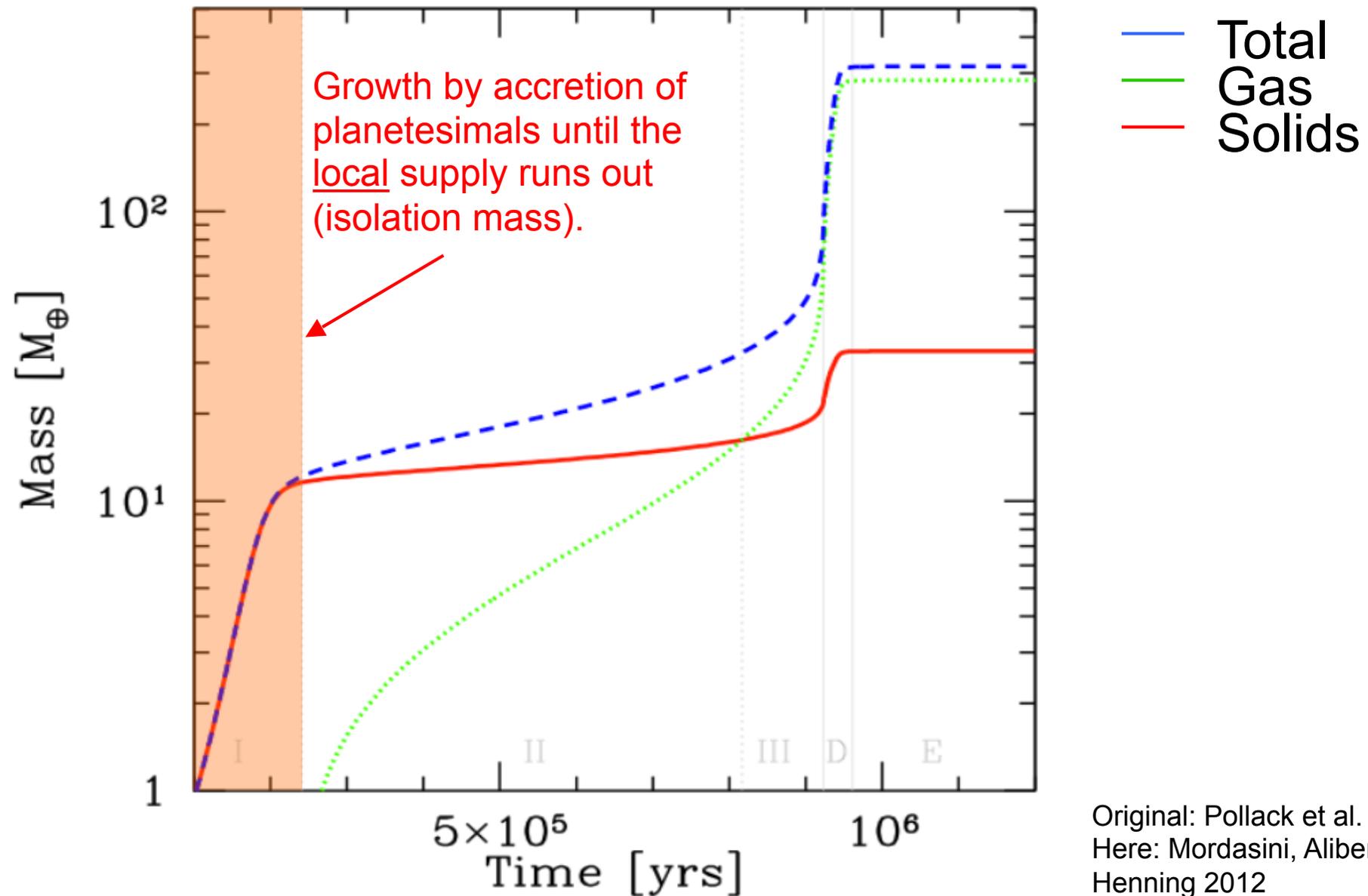


# Formation of a Gas Giant Planet



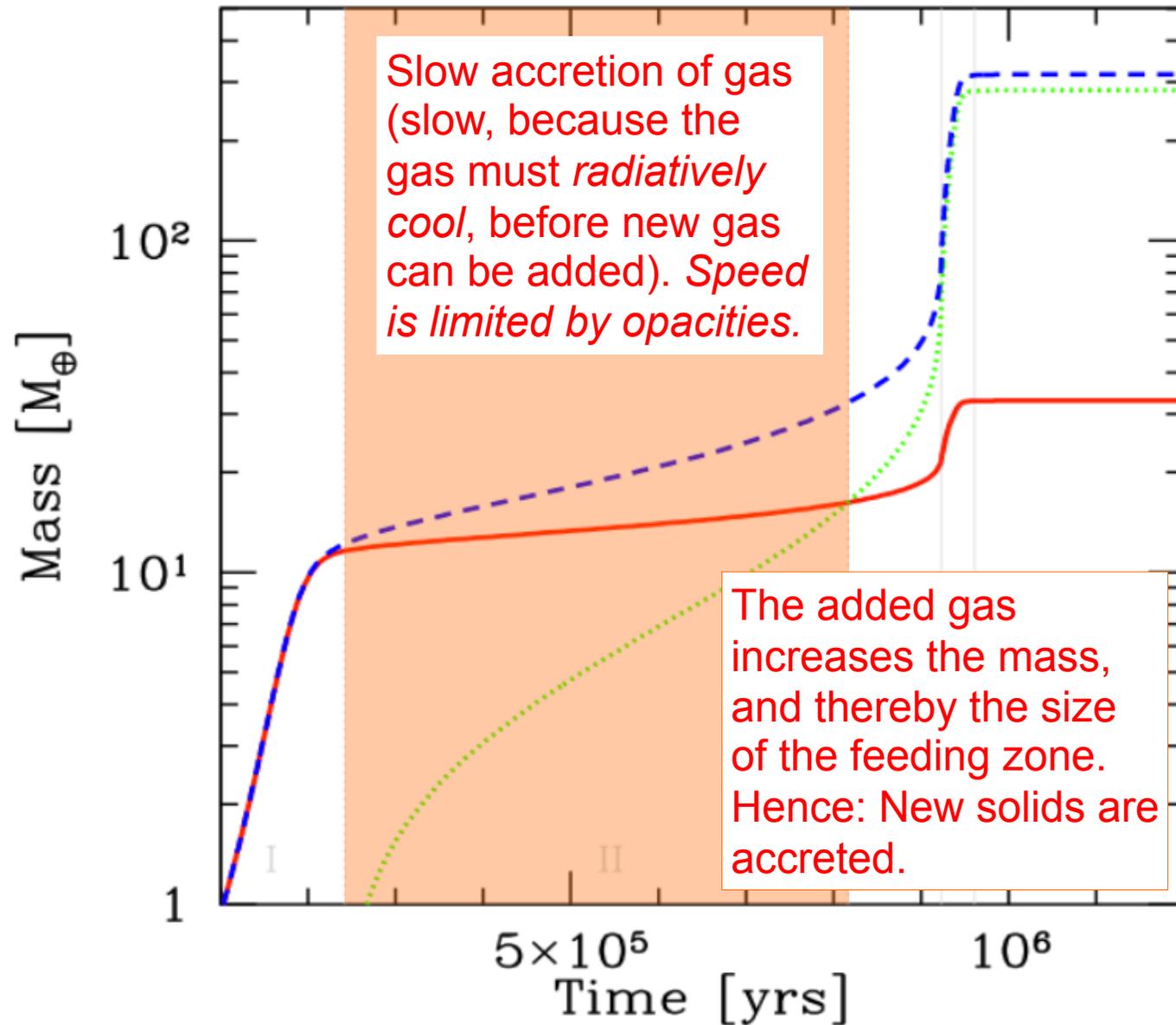
Original: Pollack et al. 1996;  
Here: Mordasini, Alibert, Klahr &  
Henning 2012

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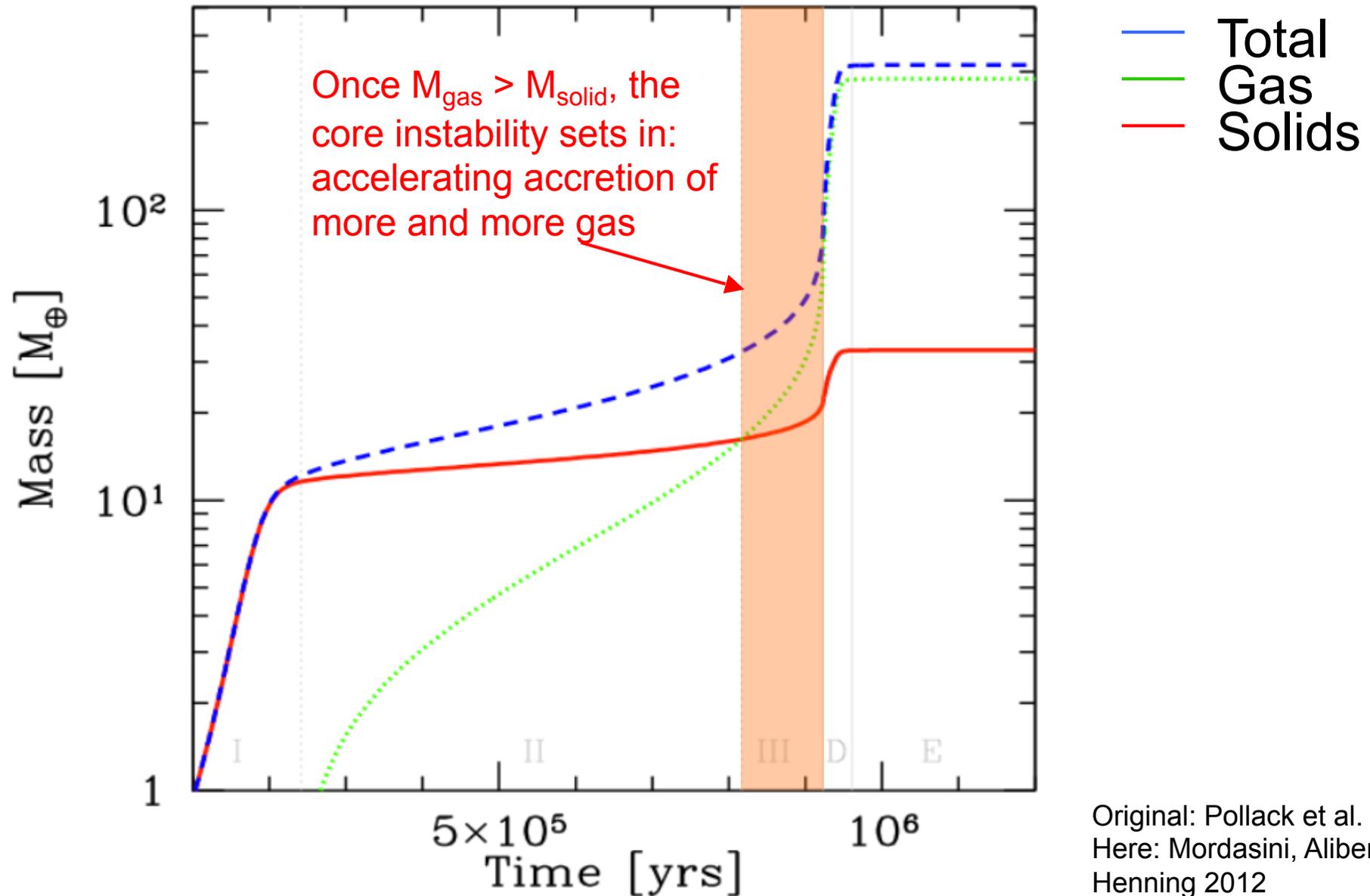


— Total  
— Gas  
— Solids

If planet migrates, it can sweep up more solids, accelerating this phase.

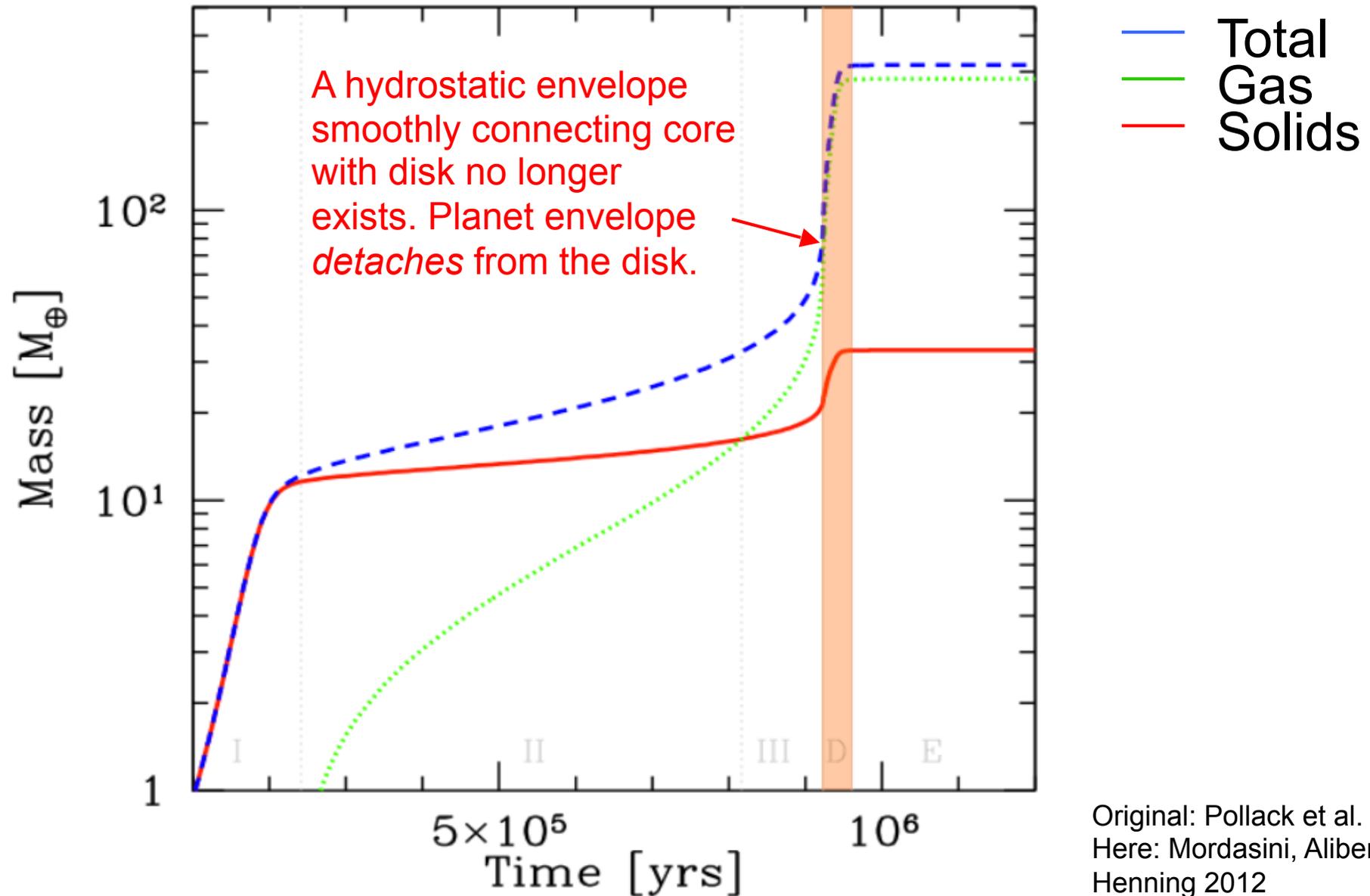
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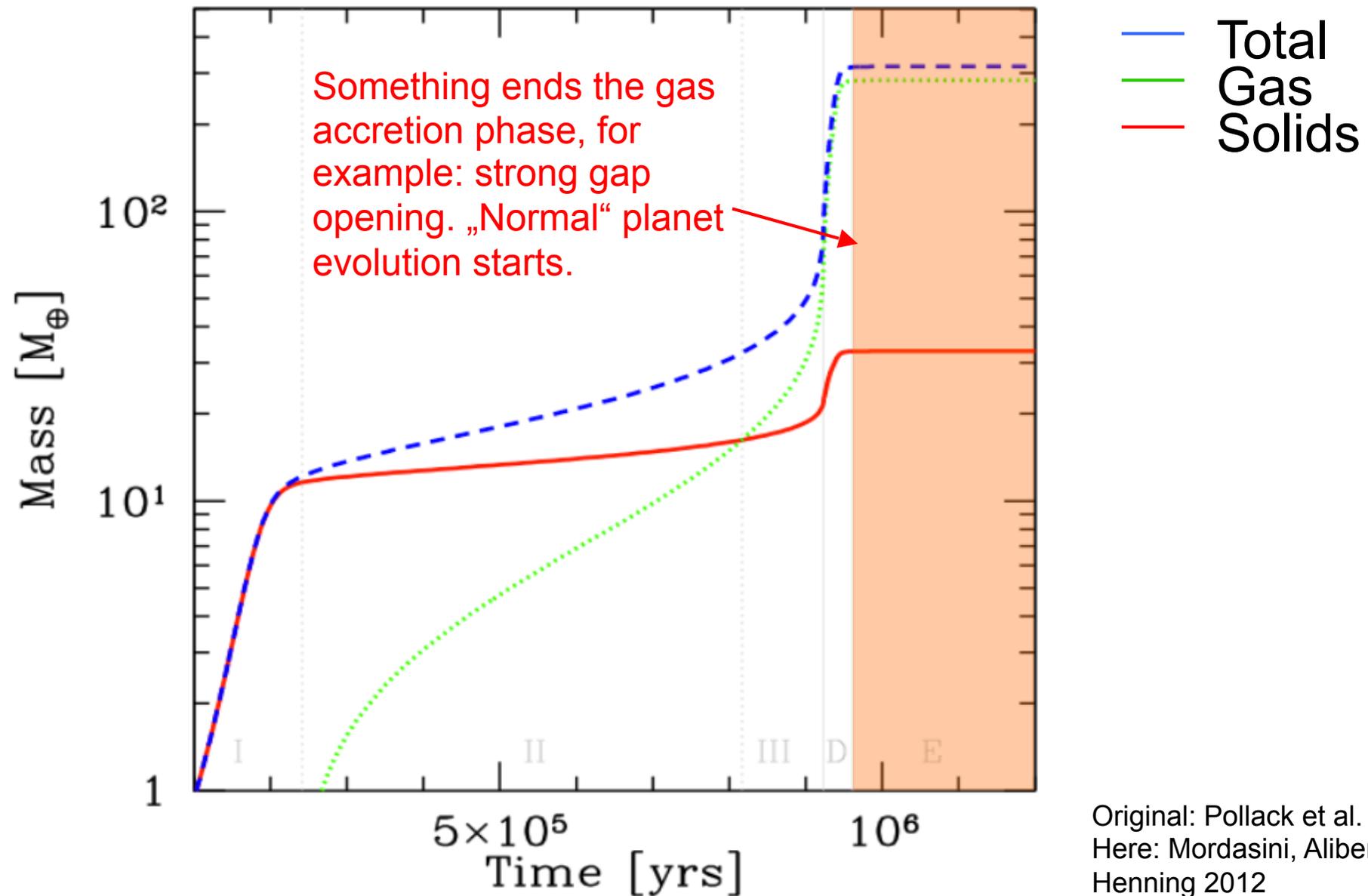


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# Formation of a Gas Giant Planet



# Formation of a Gas Giant Planet



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Henning 2012

# Is this (CA) how it goes?

CA is a complex, “multi stage” problem, still only partially understood theoretically and many phases poorly/not constrained observationally. But: huge progress in modeling; “barriers” resulting from over-simplification go away if better physical models are applied.

- (1) CA can produce rocky planets
- (2) CA can yield strongly enriched planets (GI much less so)
- (3) CA can yield ice giants (GI cannot do this)

Core Accretion is the currently favored scenario. Not much doubt that the basic idea is right. Unclear whether GI, in addition, is at work in outer regions of massive disks

# migration

(forming) planets interact gravitationally with the disk (and other planets), and may move from where they form(ed), sometimes a lot

- (1) type I migration: relatively low-mass planets (e.g.  $\sim 1 M_{\text{earth}}$ ) do not significantly alter surface density profile  $\Sigma(R)$  but material concentrates asymmetrically in resonances and exerts torque causing migration
- (2) type II migration: high-mass planets ( $\sim 1 M_{\text{jup}}$ ) open gaps and launch strong spiral arms that exert torque.
- (3) Planet-planet interaction can significantly alter orbits of planets on timescales of  $\gg 1$  orbit

