**Abstract**

Kepler Mission suggests that period ratios of pairs of planets show some peaks at commensurable ratios (e.g., Fabrycky et al., 2012). Some N-body simulations considering planetary migration suggest protoplanets are captured in mean-motion resonant orbits. These resonant systems are stable in the calculations. However, in some cases, resonant systems cause orbital instability and finally non-resonant systems are formed.

So, we investigated the orbital stability of first-order resonant systems to clarify what makes resonant systems unstable. We found in resonant systems, when the number of planets is more than the critical number, the crossing time decreases rapidly, while in non-resonant system, the crossing time decreases continuously. We also investigated the parameter dependence of the critical number.

**Introduction**

**Observed Planetary Systems**

KOI candidates (Batalha+ 2011, Fabrycky+ 2012)

- close to the star
- formed by orbital migration
- Pairs of planets are typically not in mean motion resonances.

Some peaks near commensurability

**N-body Simulations with Orbital Migration**

- planets also stop migration at the inner edge and the positive migration torque points KOI candidates (Batalha+ 2011, Fabrycky+ 2012)

**Discussion Summary**

Systems cause orbital instability and finally non-resonant systems are formed.

**Numerical Model**

**Formation of Resonant Systems**

orbital migration

trapped in resonances

gas depletion ($t_{\text{dep}}$ exponential decay)

Criterion of Instability

$\tau < \tau_{\text{crit}}$ or $-\tau_{\text{turb}}$ (Chambers + 1996)

Orbital Separation ($K$)

When planets are equal mass and in the same $p+1:p$ resonances, the orbital separations normalized mutual Hill space are the same, $K$.

**Planetary mass ($M_j$)**

While semi-major axes of $p+1:p$ resonances are not dependent on $M_j$, $K$ is dependent on planetary mass due to mutual Hill radius.

**Result**

**Number of Planets**

$6.5, K=6.450, 10^4 M_{\odot}$

Gas Effect

Due to the eccentricity damping by gas, planets are stabilized (Kawakita+ 2002). This keeps systems stable when $\tau < 10^{-9} \tau_{\text{dep}}$ if the timescale of gas removal is $10^7$ yr.

When $N > N_{\text{crit}}$ in resonant cases, the crossing times are equal to those of non-resonant systems.

**Orbital Separation & Palentary Mass**

The larger orbital separation or planetary mass is the longer crossing time is (non-resonance)

the larger $N_{\text{crit}}$ is (resonance).

$N_{\text{crit}}$ increase with decreasing $p$ when $M_j$ is the same.

The resonant overlap (Quillen 2011) would affect $N_{\text{crit}}$.

**Chain of Resonances**

$8:7:6, 10^4 M_{\odot}$

$N_{\text{crit}} = 6$

$N_{\text{crit}} = 6$ every other

$N_{\text{crit}} (8:7:6) > N_{\text{crit}} (7:6)$

**Discussion**

the number of planets determines whether planets are finally in resonances or not.

- planets also stop migration at the inner edge and the positive migration torque points the positive torque points (e.g., dead zone inner edge, gap by a gas planet, ...)

$N_{\text{crit}}$, is determined by the torque balance and depends on $t_{\text{dep}}$ when the eccentricity trap occurs (Ogihara+ 2010, Matsumoto+ in prep).

When planets are in 6:5 resonances, $N_{\text{crit}} = 8$ when $t_{\text{dep}} = 10^7$ yr.

$N > 8$ trapped in resonances

$N = 8$ trapped in resonances

$N < 8$ gas depletion

planets are not in resonances

$\text{t}_{\text{dep}} \geq 4.4 \times 10^5$

$\text{t}_{\text{dep}} \leq 4.4 \times 10^5$ ($\Delta r = h$)

**Summary**

Although most of observed planets are not in resonant orbits, the orbital distribution of planets has some peaks in or near resonant orbits.

We investigated the stability time of planets trapped in mean-motion resonances due to orbital migration by N-body simulations.

- the stability time of resonant systems is longer than that of non-resonant system.
- when $N > N_{\text{crit}}$, the planets in resonances become unstable.
- $N_{\text{crit}}$ increases with increasing $K$ or increasing $M_j$.
- $N_{\text{crit}}$ of planets in the chain of resonances are larger than $N_{\text{crit}}$ of planets in the same resonances.

Observed systems outside of commensurate orbits would contain the planets over the critical number and these planets cause orbital instability after the gas depletion.