Overview

We develop a wavelet-based cross-correlation (WCC) method to study the correlation between structural changes in molecular clouds as a function of scale. The method compares a pair of maps observed in different traces or at different velocity ranges.

Advantages of the WCC method:
- Allows to measure the correlation coefficient and structural offset between two maps as a function of scale.
- Allows to weight individual pixels by their observational significance.
- Is robust against the noise.

Application of the WCC to simulated fBm (fractional Brownian motion) maps reveals that:
- Cross-correlation coefficient can strongly depend on the scale.
- Correlation coefficient and offset can be recovered robustly regardless of noise.

Analysis of the G333 molecular line maps (12CO and C18O) shows:
- A large scale gradient in the structural distribution. This could indicate a density structure where every core shows a large density tail towards the South-West mainly seen in 12CO.

The WCC can be used to trace the correlated structural changes between different maps of a molecular cloud at scales representing the structural and physical importance such as chemical and phase transitions.

Wavelet cross-correlation (WCC)

Wavelet transform is proven to be a powerful tool for the scaling analysis in galaxies, interstellar clouds (Stutzki et al. 1998; Frick et al. 2001; Ossenkopf et al. 2008, hereafter O08). Convention of the wavelet filter \( \psi(r) = (r - c_0) \) with an image \( \phi(r) \) filters the image on a scale \( l \) given by the wavelet:

\[
F(l, r) = \int f(r) \psi(l, r, dr.
\]

Optimal wavelet filter for molecular clouds is found to be the Mexican-hat filter \( \psi(l) \) which provides the correct power spectral slope and spectral features:

\[
\psi(l) = \phi(l) + \phi(-l) = \frac{4}{\sqrt{\pi}} \frac{\sqrt{r}}{l^{3/2}} \exp \left( -\sqrt{r/l^3} \right) \exp \left( -\sqrt{r/l^3} \right)
\]

where \( \psi(l) \) and \( \phi(l) \) are the core and annulus of the filter, and \( r = 1.5 \) is the diameter ratio between the annulus and the core.

The \( l \)-variance \((D)\) of the filtered map \( F(l, r) \) and its weights \( \psi(l, r) \), indicating the significance of each pixel of the image, shows characteristics scales and the power spectral slope in the individual maps.

To study the cross-correlation of the amount of structure between two maps, \( f_1 \) and \( f_2 \), on different scales, we introduce the wavelet-based cross-power spectrum.

\[
C(l, \tau) = \frac{\int \psi(l, r) \delta F(l, r) \xi F(l, r) \delta G(l, r + \tau, dr}{\int \psi(l, r) \delta F(l, r) \xi F(l, r) \delta G(l, r)}
\]

where

\[
\delta F(l, r) = F(l, r) \xi F(l, r) \delta G(l, r) = G(l, r) - c(l, r)
\]

and

\[
\delta G(l, r) = G(l, r) - c(l, r)
\]

Simple tests

We start testing the WCC for two simulated circular structures having Gaussian intensity profiles with amplitude: 1 and standard deviations of 3 pix and 5 pix, which are offset by 5 pix along Y-axis (Fig. 2, top panel). The size of the Gaussians is traced by the maxima in the \( D \)-variance spectra (10-15 pix; 2nd panel). Correlation coefficient strongly depends on scale becoming large above the dominant structure scales. Amplitude and direction of the offset map (bottom panel) are correctly recovered on those scales.

We also compared two fBm maps with spectral index of 3 (Fig. 3, top panel) where the second map is the filtered (maximum filter with a size of 15 pix) version of the first map, which mimics the opacity of optically thick lines. Correlation is negligible at small scales and it becomes significant at \( l \geq 30 \) pix (3rd panel). The correlation coefficient turns negative at scales below the mutual offset. The offset vector (length and direction) is exactly recovered for \( l \geq 8 \) pix (bottom panel).

Realistic tests

In reality the offset can vary on different spatial scales. To test this, we generate a fBm map with S/N=10 (Fig. 4, top left panel) and use the Fourier shift theorem to shift the large scales (\( l \geq 10 \) pix) by 18 pix in the South-West direction (top right panel). The \( D \)-variance spectrum has a bump at small scales due to low S/N in large scales. The correlation gradually decreases from 10 to 20 pix (3rd panel) due to the shift of a structure at scales \( l > 10 \) pix. At scales of 2, 20 pix the structure sizes exceed the offset so that we find a gradual increase of the correlation coefficient at large scales.

Application to GMC G333

We applied the WCC to the maps of the giant molecular cloud (GMC) G333 observed in \(^{12}\)CO and \(^{13}\)CO (O08, Lu et al. 2009, Fig. 5, top panels). Their \( D \)-variance spectra have pronounced structure on scales of about 30 pix and 50 pix respectively and the structure is strongly correlated at scales > 5 pix (3rd panel). Large differences between the maps only occur at small scales affected by noise. The structure is offset at scales larger than 40 pix with amplitude of ~ 5 pix along the filament, where all structures in \(^{12}\)CO are shifted to the South-West relative to \(^{13}\)CO. This could indicate a large-scale column density gradient enhancing the structure in \(^{12}\)CO in that direction.

Figure 2. Top panels. Maps of Gaussian intensity profiles with amplitude: 1 and standard deviations of 3 pix and 5 pix, which are offset by 5 pix along Y-axis (Fig. 2, top panel). The size of the Gaussians is traced by the maxima in the \( D \)-variance spectra (10-15 pix; 2nd panel). Correlation coefficient strongly depends on scale becoming large above the dominant structure scales. Amplitude and direction of the offset map (bottom panel) are correctly recovered on those scales.

Figure 3. Application for original fBm map (left) and filtered map with a maximum filter of a size of 15 pix (right), using the Fourier shift theorem to shift the large scales (\( l \geq 10 \) pix) by 18 pix in the South-West direction (top panel). The \( D \)-variance spectrum has a bump at small scales due to low S/N in large scales. The correlation gradually decreases from 10 to 20 pix (3rd panel) due to the shift of a structure at scales \( l > 10 \) pix. At scales of 2, 20 pix the structure sizes exceed the offset so that we find a gradual increase of the correlation coefficient at large scales.

Figure 4. Application for molecular line maps of the GMC G333 observed in \(^{12}\)CO and \(^{13}\)CO (O08, Lu et al. 2009, Fig. 5, top panels). Their \( D \)-variance spectra have pronounced structure on scales of about 30 pix and 50 pix respectively and the structure is strongly correlated at scales > 5 pix (3rd panel). Large differences between the maps only occur at small scales affected by noise. The structure is offset at scales larger than 40 pix with amplitude of ~ 5 pix along the filament, where all structures in \(^{12}\)CO are shifted to the South-West relative to \(^{13}\)CO. This could indicate a large-scale column density gradient enhancing the structure in \(^{12}\)CO in that direction.

Figure 5. Application for molecular line maps of the GMC G333 observed in \(^{12}\)CO and \(^{13}\)CO (O08, Lu et al. 2009, Fig. 5, top panels). Their \( D \)-variance spectra have pronounced structure on scales of about 30 pix and 50 pix respectively and the structure is strongly correlated at scales > 5 pix (3rd panel). Large differences between the maps only occur at small scales affected by noise. The structure is offset at scales larger than 40 pix with amplitude of ~ 5 pix along the filament, where all structures in \(^{12}\)CO are shifted to the South-West relative to \(^{13}\)CO. This could indicate a large-scale column density gradient enhancing the structure in \(^{12}\)CO in that direction.