Bridging the scales: an analytical model for Population III star formation

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Introduction

The first generation of so-called Population III (Pop III) stars, formed in extremely metal-poor primordial gas with inefficient cooling, are expected to have distinct features compared with present-day metal-enriched stars (more massive, compact, and less mass loss) and play important roles in the first billion years of cosmic history through their radiation, metal enrichment, and by seeding massive black holes (BHs). Although their detailed properties are largely unknown, direct and indirect probes (e.g., galaxy surveys by JWST, stellar archaeology, binary BH mergers, and 21-cm signal) are providing us increasingly more clues.

Self-consistent theoretical predictions of the formation rates, sites, and masses of Pop III stars are needed to fully interpret observations, but are challenging due to the large range of scales involved (from $\lesssim$ AU to $\gtrsim$ Mpc).

In order to bridge the scales, we build an analytical model to predict the final masses of Pop III stars/clusters from the properties of star-forming clouds at the intermediate scale $\sim 1-10$ pc, which covers all known modes of Pop III star formation ranging from ordinary small ($\sim 10-2000\ \mathrm{M}_\odot$) clusters in molecular cooling clouds to massive ($\gtrsim 10^4\ \mathrm{M}_\odot$) clusters containing supermassive stars (SMSs $\sim 10^4-3\times10^5\ \mathrm{M}_\odot$) from violent collapse of atomic-cooling clouds.

Analytical model

The basic idea of our model is to express the key physical processes/quantities involved in Pop III star formation (see bottom middle) as functions of the total mass of protostars M based on the results of small-scale (magneto-)hydrodynamic simulations and stellar evolution models, so the final mass $\hat{M}_\star$ can be estimated through a root-finding process.

Median evolution of star-forming discs and protostars

Scaling relations between the **median** stellar mass M, accretion rate $\dot{M}$, disc/cluster radius R, and time t [1]:

$$M \propto t^{\beta} \rightarrow \dot{M} \propto M^{1-1/\beta} \simeq M^{-0.37}$$
$$R \propto t^{\delta} \propto M^{\delta/\beta} \simeq M^{1.25}$$

$\beta = 4-3\gamma_{\rm eff}$, $\delta = 2-\gamma_{\rm eff}$ given the polytropic index of equation of state $\gamma_{\rm eff}$ ($\simeq 1.09$ for pristine H_2 cooling gas [2]).

The normalizations are set by two input parameters:

- Fraction of infalling gas accreted by protostars $\eta \sim 0.4-0.6$ [3] (The rest goes to the disc or outflows.)
- Gas inflow rate $\dot{M}_{\rm in} = \eta^{-1}\dot{M}(t=t_0)$ at the characteristic dynamical timescale $t_0 \sim 3\times10^4$ yr corresponding to the disc edge density $n_0 \sim 10^6\ \mathrm{cm^{-3}}$

Characteristic masses

The scaling relations above are assumed to hold when the evolution is governed only by gravity and hydrodynamics until star formation is terminated *quasi-instantaneously* by other factors such as stellar feedback, stellar collapse, and gas supply. In the simple case of a single star per cloud, these factors can be described by the following characteristic masses.

1. **Final mass from photo-ionization feedback** $M_{\star,\rm f}$: Protostar growth stops when the disc photo-evapolartion rate $\dot{M}_{\rm pe} \propto \dot{Q}_{\rm ion}^{1/2}R^{1/2}$ [4] reaches the accretion rate $\dot{M}$. $\dot{Q}_{\rm ion}$: emission rate of ionizing photons as a function of M
2. **Mass gained in the bloating phase** $M_{\star,\rm b} = M(t_{\rm b})$: Ionizing feedback is negligible in the early phase $t < t_{\rm b}$ when the protostar is bloated by rapid accretion with $\dot{M} > \dot{M}_{\rm crit} \sim 0.01-0.04\ \mathrm{M_\odot\ yr^{-1}}$ [5]. $\dot{M}(M_{\star,\rm b}) = \dot{M}_{\rm crit}$.
3. **Maximum mass from stellar lifetime** $M_{\star,\rm l}$: Star formation stops when the star runs out of fuel and then collapses/explodes: $M_{\star,\rm l} = M(t_\star(M_{\star,\rm l}))$, where $t_\star$ is the (nuclear burning) lifetime as a function of mass M.
4. **Maximum mass from general-relativity instability (GRI)** $M_{\star,\rm g}$: The star can collapse by GRI even at $t < t_\star$ with very high accretion rates $\dot{M} \gtrsim 0.1\ \mathrm{M_\odot\ yr^{-1}}$, reaching a maximum mass $M_{\star,\rm GR}(\dot{M})$ [6]. $M_{\star,\rm g} = M_{\star,\rm GR}(\dot{M}(M_{\star,\rm g}))$.
5. **Mass limit placed by gas supply** $M_{\rm c}$: The stellar mass cannot exceed the total mass of gas $M_{\rm c}$ available in the star-forming core/cloud, which can be estimated from $\dot{M}_{\rm in}$ as $\dot{M}_{\rm in} \sim M_{\rm c}/t_{\rm ff}$.

The single-star case

When only one star forms in the disc, its final mass can be easily obtained by combining the 5 characteristic masses:

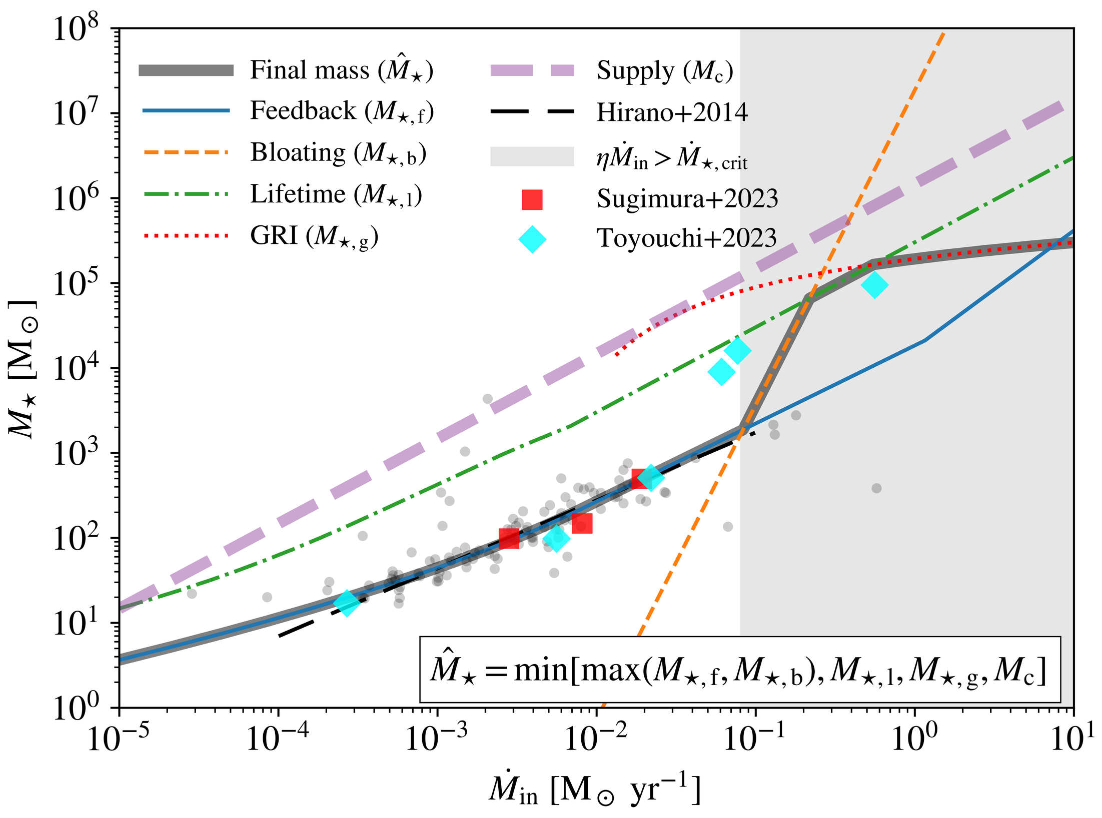

Figure 1. Final stellar mass and the characteristic masses as functions of $\dot{M}_{\rm in}$ for $\eta = 0.5$ and $\dot{M}_{\rm crit} = 0.04\ \mathrm{M_\odot\ yr^{-1}}$, in comparison with the simulation results from [7, 8, 3]. **The final mass is regulated by feedback for $\dot{M}_{\rm in} \lesssim 0.08\ \mathrm{M_\odot\ yr^{-1}}$ with $\hat{M}_\star \lesssim 2000\ \mathrm{M_\odot}$. Then for higher $\dot{M}_{\rm in}$ it goes through a transition zone with rapid increase $\hat{M}_\star = M_{\star,\rm b} \propto \dot{M}_{\rm in}^{2.7}$ dominated by the bloating phase before reaching the limits placed by lifetime and GRI at $\dot{M}_{\rm in} \gtrsim 0.2\ \mathrm{M_\odot\ yr^{-1}}$ with $\hat{M}_\star \sim 10^5\ \mathrm{M_\odot}$.**

Effects of fragmentation/multiplicity

Disc fragmentation is commonly seen in recent simulations of Pop III star formation. We explore its effects with a phenomenological multiplicity model under the following assumptions motivated by simulations [9]

- The mass distribution of protostars follows a power-law $dN/dm \propto m^{-\alpha}$.
- The fraction of total accretion rate $\dot{m}/\dot{M}$ that feeds a protostar is identical to the fraction of total stellar mass m/M occupied by the protostar.

For simplicity, we further assume that the total number of protostars N is constant, so are α and the fraction f_2 of mass occupied by the most massive protostar. Now, the first two characteristic masses are combined to consider only the feedback from non-bloating stars, while the 3rd and 4th are applied to the most massive star in the cluster.

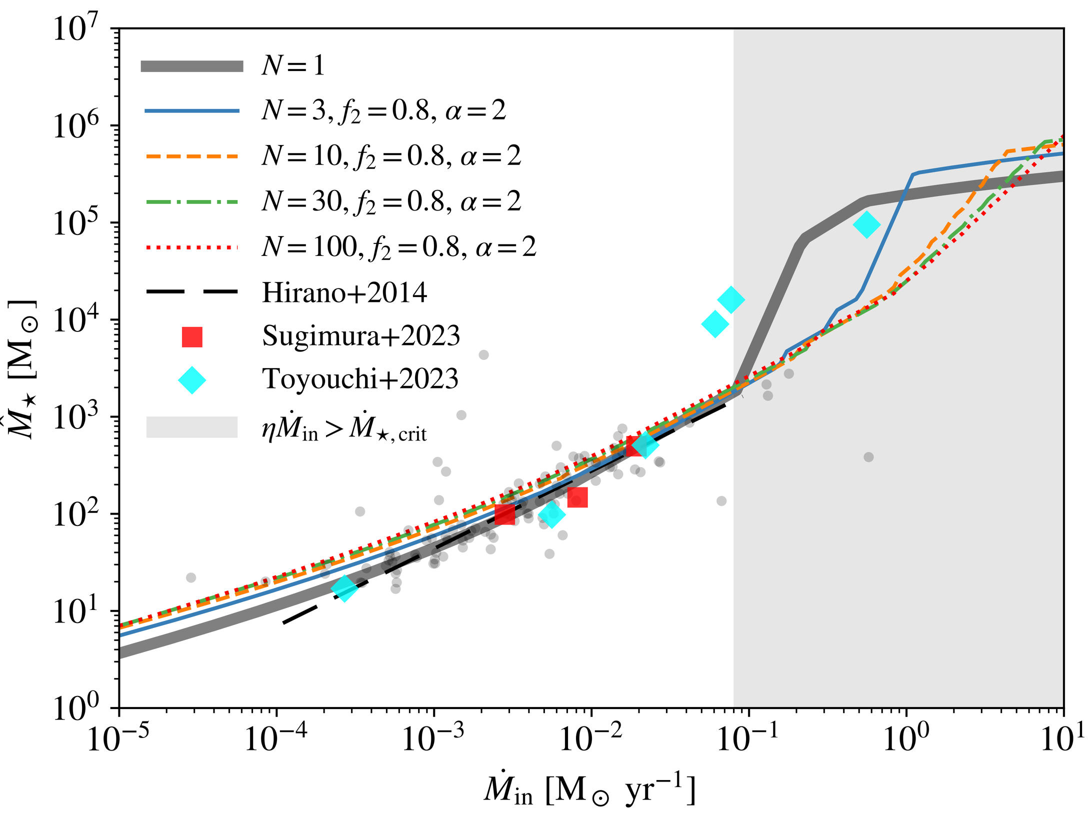

Figure 2. Similar to Fig. 1 but for the final total stellar mass of Pop III clusters with $N \sim 1-100$ stars given $f_2 = 0.8$ and $\alpha = 2$ fixed. **Formation of SMSs above $10^4\ \mathrm{M_\odot}$ becomes more difficult with stronger fragmentation (larger N), because the smallest protostars cannot accrete fast enough to enter the bloating phase even if the global accretion rate of the cluster is high.**

Physical processes and model elements

1. Gas inflow onto the star-forming disc ($\dot{M}_{\rm in}$, $M_{\rm c}$, $\gamma_{\rm eff}$).
2. Disc geometry and fragmentation, the resulting spatial, mass, and accretion rate distributions of protostars (η, $\dot{M}$, R, N, f_2, α).
3. Destruction of the star-forming disc/cloud by stellar feedback ($\dot{M}_{\rm pe}$).
4. Evolution of protostars regulated by their accretion histories ($\dot{Q}_{\rm ion}$, $\dot{M}_{\rm crit}$, $t_\star$, $M_{\star,\rm GR}$).

Application: Pop III BH seeds of high-z AGN

Pop III stars have longer been hypothesized to provide the seeds of supermassive BHs powering high-z active galactic nuclei (AGN). We apply our model to the primordial star-forming clouds in the progenitors of high-z luminous ($\gtrsim 10^{46}\ \mathrm{erg\ s^{-1}}$) AGN host haloes. Using the gas inflow rate $\dot{M}_{\rm in}$ distribution of such clouds predicted by merger trees targeting typical AGN host haloes with $M_{\rm h} = 10^{12}\ \mathrm{M_\odot}$ at $z = 6$ (corresponding to $\sim 4\sigma$ overdense regions) [10], we derive the mass distributions of Pop III clusters and stars:

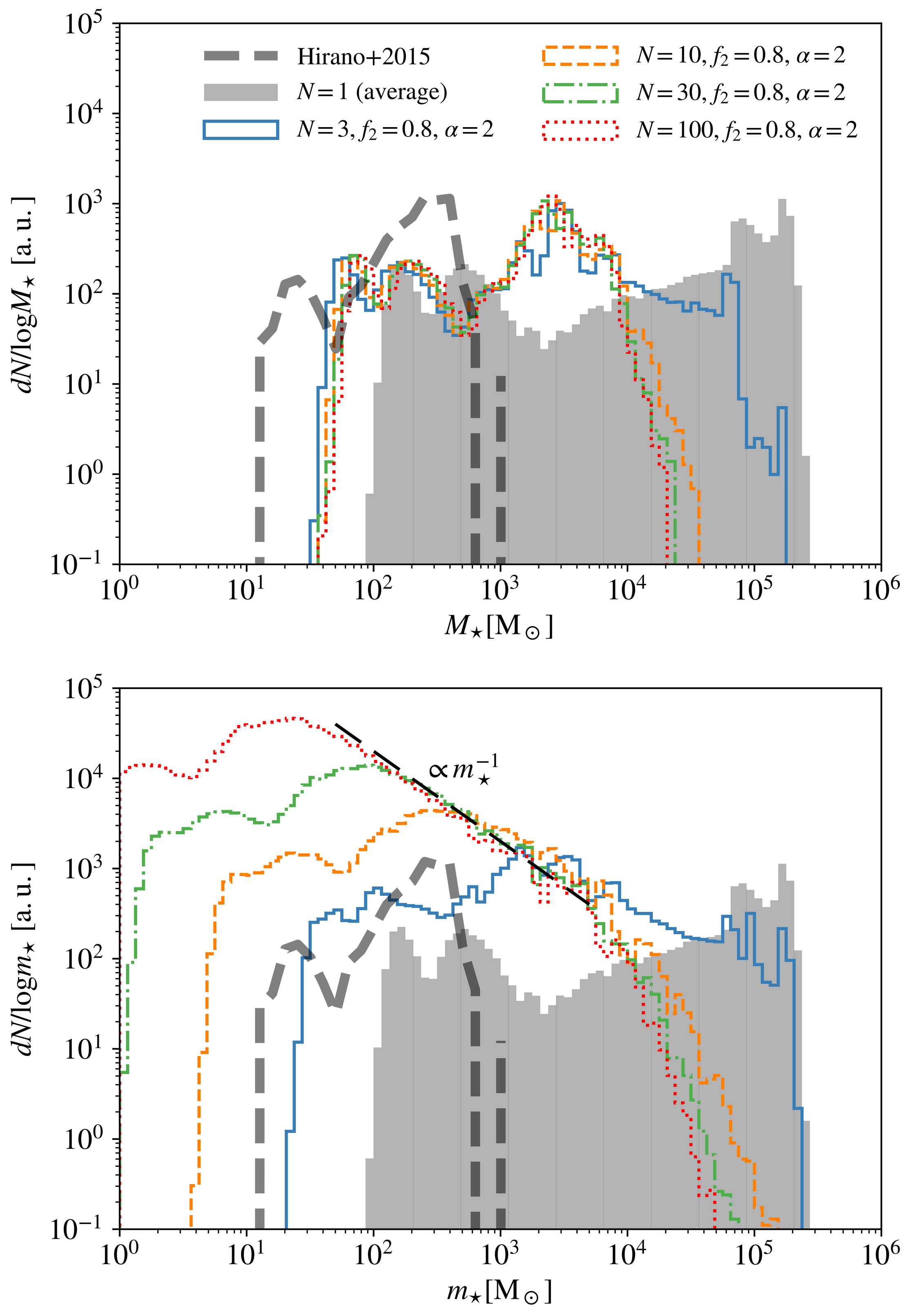

Figure 3. Mass distributions of Pop III clusters (top) and stars (bottom) for $N \sim 1-100$ with $f_2 = 0.8$ and $\alpha = 2$ fixed. The mass distribution of typical Pop III stars formed in unbiased regions from [11] is plotted as the thick dashed curve for comparison. **In the target haloes, Pop III star formation efficiency (stellar mass/baryon mass) lies in the range $\sim 10^{-3}-0.1$. The fraction of haloes hosting massive Pop III clusters ($M_\star > 10^4\ \mathrm{M_\odot}$) decreases with N but remains non-negligible ($\sim 27-70\%$). The number of SMSs per halo drops significantly from $\sim 0.4-0.7$ for $N \leq 3$ to $\sim 0.02-0.07$ for $N \sim 10-100$. The corresponding Pop III heavy BH seeds above $10^{4\ (5)}\ \mathrm{M_\odot}$ can account for up to $\sim 10\ (5)\%$ of the observed luminous AGN.**

Outlook

Our model is simple and flexible, ready to be implemented in cosmological simulations and merger trees that can marginally resolve Pop III star-forming clouds ($\delta x \sim 1-10$ pc, $n \sim 10^{3-5}\ \mathrm{cm^{-3}}$) regulated by large-scale physics (e.g., structure formation, radiation backgrounds, and baryon-dark matter streaming motion). With more comprehensive analysis of simulation data, the model can be extended to better capture the complexity of Pop III star formation, such as the stochastic and diverse growth histories of protostars, core fragmentation under enhanced turbulence, and correlations between multiplicity parameters and global cloud properties [12].

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