Giant Star-Forming Regions

Dimitrios A. Gouliermis

Lecture #6
Physical Processes in Ionized Hydrogen Regions
Part I
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Physical Processes in HII Regions
Part I

In this Lecture

• Photoionization
• Recombination
• Photoionization Equilibrium
• Pure Hydrogen nebulae

– Literature
  • *Rybicki & Lightman*, 2004, Sec. 10.5
  • *Spitzer*, 1978, Sec. 5.1
  • Tielens, 2005, Ch. 7
Introduction

- The far ultraviolet radiation (FUV) from an O or B star ionizes its immediate neighborhood and produces an HII region. Strömgren developed the theory in 1939 for a spherical model where the HII region slowly expands into uniform HI.
- HII regions illustrate basic processes that operate in all photoionized regions of the ISM.
- Radiation with wavelengths shorter than the Lyman limit (FUV with < 916.6 Å or X-rays) photoionizes H: \[ h\nu + H \rightarrow H^+ + e^- \]
- The H\(^+\) ions recombine radiatively: \[ H^+ + e^- \rightarrow H + h\nu \]
- The balance between these two reactions determines the ionization fraction.
- Any excess photon energy above the ionization potential (IP = 13.6 eV) is given to the ejected electron and it will be exchanged in collisions with ambient electrons, thereby heating the HII region.
Processes Governing HII Regions

- **Photoionization Equilibrium**, the balance between photoionization and recombination. It determines the structure of the nebula and the rough spatial distribution of ionic states of the elements in the ionized zone.

- **Thermal Balance** between heating and cooling. Heating is dominated by photoelectrons ejected from Hydrogen and Helium with thermal energies of a few eV. Cooling is dominated by electron-ion impact excitation of metal ions followed by emission of “forbidden” lines from low-lying fine-structure levels. It is these cooling lines that give HII regions their characteristic spectra.

- **Hydrodynamics**, including shocks, ionization and photodissociation fronts, and outflows and winds from the embedded stars.
The Rosette Nebula (NGC 2237). Inside the nebula lies the open cluster NGC 2244. Its bright young stars formed about 4 Myr ago from the nebular material and their stellar winds are clearing a hole in the nebula's center, insulated by a layer of dust and hot gas. UV light from the hot cluster stars causes the surrounding nebula to glow. The nebula spans about 100 light-years across, lies about 5000 light-years away, and can be seen towards the constellation of the Unicorn (Monoceros).
Credit: Brian Lula (http://www.heavensgloryobservatory.com/)
The ISM is opaque at 911 Å (the Lyman limit) and partially transparent in the FUV and X-ray bands above 1 keV.

Ionizing photons come from:
1. Massive young stars
2. Hot white dwarfs
3. Planetary nebula stars
4. SNR shocks

Example values for an O5V star:
(Black Body peaks at 3kT or 12eV)

\[ T_{\text{eff}} \sim 46,000 \text{ K} \]
\[ F_{\text{Ly}} \sim 3 \times 10^{49} \text{ ionizing photons s}^{-1} \]
Hot Stars as Ionizing Sources

Stellar Parameters of O- and B-type Stars

<table>
<thead>
<tr>
<th>Sp. type</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$L(10^5L_\odot)$</th>
<th>$\frac{N_{\text{LyC}}}{10^{49}\text{ph. s}^{-1}}$</th>
<th>$R_\odot$ (pc)</th>
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<tr>
<td>O3</td>
<td>51 200</td>
<td>10.8</td>
<td>7.4</td>
<td>1.3</td>
</tr>
<tr>
<td>O4</td>
<td>48 700</td>
<td>7.6</td>
<td>5.0</td>
<td>1.2</td>
</tr>
<tr>
<td>O5</td>
<td>46 100</td>
<td>5.3</td>
<td>3.4</td>
<td>1.0</td>
</tr>
<tr>
<td>O6</td>
<td>43 600</td>
<td>3.7</td>
<td>2.2</td>
<td>0.88</td>
</tr>
<tr>
<td>O7</td>
<td>41 000</td>
<td>2.5</td>
<td>1.3</td>
<td>0.75</td>
</tr>
<tr>
<td>O8</td>
<td>38 500</td>
<td>1.7</td>
<td>0.74</td>
<td>0.62</td>
</tr>
<tr>
<td>O9</td>
<td>35 900</td>
<td>1.2</td>
<td>0.36</td>
<td>0.49</td>
</tr>
<tr>
<td>B0</td>
<td>33 300</td>
<td>0.76</td>
<td>0.14</td>
<td>0.36</td>
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- Smooth Blackbody curves are only an approximation of the real stellar FUV and EUV radiation.
- State of the art models for massive stellar atmospheres disagree.
Photoionization of Hydrogenic Ions

Kramers’ semi-classical formula for the photoionization cross section provides:

\[ \alpha_\nu = \alpha_1 \left( \frac{\nu_1}{\nu} \right)^3 \quad \text{with} \quad \alpha_1 = \frac{6.33 \times 10^{-18}}{Z^2} \text{ cm}^2 \]

According to quantum theory, the photoionization cross section is

\[ \alpha_\nu = \frac{7.91 \times 10^{-18}}{Z^2} \left( \frac{\nu_1}{\nu} \right)^3 g_{bf} \text{ cm}^2 \]

where \( h\nu_1 = 13.6 \ Z^2 \text{ eV} \) and \( g_{bf} \leq 1 \) is the quantum mechanical Gaunt factor for bound-free transitions from the ground state \( n = 1 \).

Both formulas are comparable, and give an inverse-cube dependence of the photoionization cross section on frequency.

The mean free path at 912 Å is very small:

\[ l_{\text{mfp}}(\nu_1) = \frac{1}{n_H^0 \alpha_1} = \frac{1.58 \times 10^{17}}{n_H^0} \text{ cm} \]
Photoionization cross section

- The ionization cross sections of H and He show a maximum at the ionization edge and drop away approximately as $\nu^{-3}$.
- Absorption of ionizing photons by $H^0$ will change the radiation field drastically when $\tau(\nu) \geq 1$ owing to this steep frequency dependence of the absorption cross section.
- The ionization cross section of oxygen shows a more complex behavior due to ionization to excited levels within the same configurations.
Photoionization Rate

The photoionization rate for one H atom is

\[ \zeta = \int_{\nu_1}^{\infty} \frac{4\pi J_\nu}{h\nu} \alpha_\nu \, d\nu = c \int_{\nu_1}^{\infty} \alpha_\nu n_\nu \, d\nu \]

The mean photon intensity, \( J_\nu \), and the photon number density, \( n_\nu \), are related through the specific energy density, \( u_\nu \), and the specific intensity, \( I_\nu \), as:

\[ n_\nu h\nu \equiv u_\nu = \frac{1}{c} \oint I_\nu \, d\omega \equiv \frac{4\pi}{c} J_\nu \Rightarrow \frac{4\pi J_\nu}{h\nu} = cn_\nu \]

The total photon number density is given as:

\[ n_\pi = \frac{1}{c} \int_{\nu_1}^{\infty} \frac{4\pi J_\nu}{h\nu} \, d\nu = \frac{4\pi J_1}{hc} I_1 \]

where \( I_1 \) is the first inverse moment of \( J_\nu \). In general, the \( n \)th inverse moment, \( I_n \), is defined as

\[ I_n \equiv \int_{\nu_1}^{\infty} \frac{J_\nu (\nu_1/\nu)^n}{J_1 (\nu_1/\nu)} \, d\nu \]
Photoionization Rate

The cross section can be expressed as a function of $\nu^{-3}$, and so the ionization rate can also be expressed in terms of these moments.

\[
\xi_\pi = \int_{\nu_1}^{\infty} \frac{4\pi J_\nu}{h\nu} \alpha_\nu d\nu \propto \frac{4\pi J_1}{h} \int_{\nu_1}^{\infty} \frac{J_\nu}{J_1} \left( \frac{\nu_1}{\nu} \right) \alpha_1 \left( \frac{\nu_1}{\nu} \right)^3 d\nu \Rightarrow \\
\xi_\pi \approx \frac{4\pi J_1}{h} \alpha_1 I_4
\]

Replacing the mean intensity by the photon number, we get:

\[
\xi_\pi \approx n_\pi \alpha_1 c \frac{I_4}{I_1}
\]

where moments $I_1$ and $I_4$ depend on the photon spectrum. A typical ratio for HII regions is $I_4/I_1 \approx \frac{1}{2}$. 
An Estimate of Photoionization Rate

We assume a O5V ionizing star. At a distance of 1 pc from it the H-ionizing photon density is

\[ n_\pi = \frac{1}{c} \frac{S_H}{4\pi R^2} = \frac{3 \times 10^{49} \text{s}^{-1}}{4\pi \cdot (3 \times 10^{18} \text{cm})^2 \cdot (3 \times 10^{10} \text{cm s}^{-1})} = 8.3 \text{ cm}^{-3} \]

\[ \xi_\pi = n_\pi a_1 c (I_4 / I_1) = 8.3 \cdot 6.3 \times 10^{-18} \cdot 3 \times 10^8 \cdot \frac{1}{2} = 7.9 \times 10^{-7} \text{ s}^{-1} \]

\( S_H \): Rate (per second) of ionizing photons produced from star.

The Ionization time is then:

\[ t_\pi = 1/\xi_\pi = 1.3 \times 10^6 \text{ s} \approx 15 \text{ days} \]
Radiative Recombination

Radiative recombination is the process by which an ion in state $i$ binds an electron to produce state ($i$-1) with the subsequent radiation of photons. The electron passes from the continuum (i.e. free) levels into the upper bound levels of the ion and then cascades down to form a ground state ion.

The process is inverse to photoionization, as the electron is captured by the ion into a bound state $n$ with emission of a photon:

$$H^+ + e^- \rightarrow H + h\nu$$

The relations between photoionization and recombination (analogous to the Einstein relations) are called Milne relations. E. A. Milne in 1924 calculated the recombination cross section using detailed balancing relations (see, e.g., Rybicki & Lightman §10.5).
Radiative Recombination

According to *Milne Relation*, the cross section for capture an electron to level \( n \), \( \alpha_{fb}(\nu) \), is given in relation to the photoionization cross section (absorption coefficient), \( \alpha_{bf}(\nu) \), as:

\[
\alpha_{fb}(\nu) = \frac{g_n}{g_1} \frac{\hbar^2 \nu^2}{m_e^2 c^2 \nu^2} \cdot \alpha_{bf}(\nu)
\]

Where \( g_n \), and \( g_1 \) are the statistical weights of the level which is doing the absorbing, i.e., into which the electron recombination is considered, and of the ion in its ground level respectively. This relation is (see *Spitzer* Ch. 5):

\[
\alpha_{fb}(\nu) = \frac{h\nu_1}{m_e c^2} \frac{h\nu_1}{1} \frac{\nu_1}{2m_e \nu^2} \frac{\alpha_1}{n^3}
\]

I.e., \( \alpha_{fb}(\nu) \) depends on electron velocity as \( \nu^{-2} \). The *recombination rate coefficient*, \( \beta_n = \langle \nu \alpha(\nu) \rangle_{th} \), is small and decreases as \( 1/\sqrt{T} \), e.g., the *total* rate coefficient, summed over \( n \) at 10,000K is \( \beta(10^4 \text{K}) \sim 4 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1} \).
Recombination Rate Coefficient

The recombination rate coefficient $\beta_n$ (units, cm$^3$ s$^{-1}$) gives the rate of direct recombinations per unit volume to level $n$:

$$n_e n_i \langle \alpha_n (\nu) \nu \rangle = n_e n_i \beta_n$$

Here $n_e$ is the e$^-$ density and $n_i$ the ion density (in this case H$^+$, i.e., proton)

Summing the rates for all levels $m \geq n$, gives the total recombination rate coefficient to level $n$:

$$\beta_n = \sum_{m=n}^{\infty} \beta_m = 2.06 \times 10^{-11} Z^2 T^{-1/2} \phi_n (h\nu_1 / kT) \text{ cm}^2 \text{ s}^{-1}$$

$\phi_n (h\nu_1/kT)$, is the recombination coefficient function, a slowly varying function of $T$, introduced and tabulated by Spitzer (Table 5.2, p. 107). The values for $n = 1 \& 2$ at 8000 K are 2.09 & 1.34, respectively, so that $\beta_1 = 5 \times 10^{-13}$ cm$^3$ s$^{-1}$ and $\beta_2 = 3 \times 10^{-13}$ cm$^3$ s$^{-1}$. 
“On the Spot” Approximation for H

The rate coefficient $\beta_1$ includes recombination to the ground state, but that process produces another ionizing photon that is easily absorbed “on the spot” at high density, as if the recombination had not occurred.

In this “on the spot” approximation, the effective recombination rate omits recombination to the ground state

$$\beta_2 = \sum_{m=2}^{\infty} \beta_m \approx 2.06 \times 10^{-13} Z^2 T_4^{-0.8} \text{ cm}^3 \text{ s}^{-1}$$

The corresponding recombination time is

$$t_{\text{rec}} = \frac{1}{n_e \beta_2} \approx 3.85 \times 10^{12} n_e^{-1} T_4^{-0.8} \text{ sec}$$

Compared with the ionization time (slide 13), in most cases $t_{\text{rec}} \gg t_\pi$, and the gas around hot stars is highly ionized.

Assumptions of the Idealized Case

• **The Nebular Approximation.** To first order all photoionizations in a nebula are from the 1s²S ground state. Transitions lifetimes are infinitely smaller than the typical photoionization time (slide 13) and thus excited H atoms have plenty of time to cascade into the 1s²S state following recombination.

• **Kinetic equilibrium.** Electron-electron collisions are very efficient with a cross-section ~ 4 orders of magnitude *larger* than the photoionization cross-section at the ionization threshold. As a result the kinetic energies of the electrons will very quickly *thermalize* into a Maxwellian velocity distribution with $T_e \approx T_{\text{kin}}$ compared to recombination or ionization.

• Compared to photoionization and e-e scattering, **recombination is a very slow process** (see previous slide) and so the electrons will have plenty of time to thermalize. A thermal distribution of electrons results in a temperature-dependent recombination coefficient.
The Eagle nebula (M16). The bright region is actually a window into the center of a larger dark shell of dust, where a cluster of stars is being formed. In this cavity tall pillars and round globules of dark dust and cold molecular gas remain where stars are still forming. Already visible are several young bright blue stars whose light and winds are burning away and pushing back the remaining filaments and walls of gas and dust. The nebula lies about 6500 light years away, spans about 20 light-years, and is visible toward the constellation of the Serpent (Serpens). Image from the 0.9-meter telescope on Kitt Peak, Arizona, USA. Credit: T. A. Rector & B. A. Wolpa, NOAO, AURA
Photoionization Equilibrium for H

The ISM is not in LTE and so the Saha equation is not valid.

Nevertheless, the time scales for dynamical changes for gas and star are much longer than \( t_{\text{rec}} \) and \( t_{\text{ion}} \). Therefore, the ionization is in a quasi-steady-state called \textit{photoionization equilibrium} where the rate of ionization \( \zeta_{\pi} \) out of ionization state \( i-1 \) equals rate of recombination into state \( i \):

\[
\zeta_{\pi} n_{i-1} = \beta n_e n_i \\
\frac{n_i}{n_{i-1}} = \frac{\zeta_{\pi}}{\beta n_e} = \frac{t_{\text{rec}}}{t_{\pi}}
\]

where \( \beta = \beta_1 \) or \( \beta_2 \), depending on the optical depth. For H we assume \( \beta = \beta_2 \) and \( n_{\pi} = n_{\pi}(h\nu > 13.6 \text{ eV}) \), and we have (recall Slide 12):

\[
\frac{n_{\text{H}^+}}{n_{\text{H}^0}} = \frac{\zeta_{\pi}}{\beta_2 n_e} = \frac{n_{\pi} c \alpha_{\nu_1} I_4}{\beta_2 x_e n_{\text{H}}}
\]
Pure Hydrogen Nebula

A static, homogeneous, pure H nebula ionized by photons from a single star. The photoionization equilibrium condition for this nebula describes the balance between photoionization of H\(^0\) (the nebular approximation) and recombination of electrons and protons back into H\(^0\):

\[
 n_{H^0} \int_{\nu_1}^{\infty} \frac{4\pi I_\nu}{h\nu} \alpha_\nu d\nu = n_e n_p \beta_1(T)
\]

The equation of radiative transfer for photons with \(\nu \geq \nu_1\)

\[
 \frac{dI_\nu}{ds} = -n_{H^0} \alpha_\nu I_\nu + j_\nu
\]

\(I_\nu\) : specific intensity of the radiation field

\(j_\nu\) : local emission coefficient (erg cm\(^{-3}\) s\(^{-1}\) sr\(^{-1}\) Hz\(^{-1}\))

The radiation field, \(I_\nu\), consists of two parts

\[
 I_\nu = I_{\nu s} + I_{\nu d} = (\text{stellar}) + (\text{diffuse})
\]
Pure Hydrogen Nebula

The stellar radiation field at a given location in the nebula, \( r \), is

\[
4\pi J_{\nu s} = \frac{L_{\ast}}{4\pi r^2} e^{-\tau_{\nu}(r)} \quad \text{with} \quad \tau_{\nu}(r) = \int_0^r n_{H_0} \alpha_\nu \, ds = \frac{\alpha_\nu}{\alpha_1} \tau_1(r)
\]

The equation of photoionization equilibrium is then:

\[
n_{H_0} \frac{1}{r^2} \int_{\nu_1}^{\infty} \frac{L_{\ast}}{h\nu} \alpha_\nu e^{-\tau_{\nu}(r)} d\nu = n_e n_p \beta_1(T)
\]

For \( \nu \geq \nu_1 \), the emission from diffuse ionizing photons \( J_{\nu d}(T) \) is strongly peaked at \( \nu = \nu_1 \), and the diffuse emission is:

\[
n_{H_0} \int_{\nu_1}^{\infty} \frac{4\pi J_{\nu d}}{h\nu} d\nu \approx n_e n_p \beta_{1s}(T)
\]

Since \( \beta_{1s} < \beta_1 \), the diffuse ionizing radiation field is always weaker than the stellar radiation field.
Pure Hydrogen Nebula

There are two relevant approximations:

- **Optically Thin Nebulae.** Here $I_{\nu d} \approx 0$, and we only consider the ionizing radiation from the photoionizing star.

- **Optically Thick Nebulae.** In this approximation none of the ionizing photons can escape the nebula, meaning that all ionizing photons in the diffuse radiation field eventually get absorbed elsewhere in the nebula. This leads to the “on the spot” approximation in which each new ionizing photon emitted following recombination into the ground state is absorbed physically close to where it was created.
Pure Hydrogen Nebula

The “on the spot” approximation allows us to simplify the equation of photoionization equilibrium to

\[ n_{H_0} \frac{1}{r^2} \int_{\nu_1}^{\infty} \frac{L_*}{h\nu} e^{-\tau_\nu (r)} d\nu = n_e n_p \beta_2 (T) \]

where the total recombination coefficient \( \beta_2 \) is introduced:

\[ \beta_2 (H_0, T) = \beta_1 (H^0, T) - \beta_{1^2S} (H^0, T) = \sum_{n=2}^{\infty} \beta_n (H^0, T) \quad [\text{cm}^3 \text{ s}^{-1}] \]

This equilibrium condition means that the following conditions will prevail in an optically thick nebula:

- Photoionization by the stellar radiation field is balanced by recombination into excited states of H.
- Recombinations directly into the 1s\(^2\)S ground state emit ionizing photons that are quickly reabsorbed by the nebula, and so have no net effect on the overall ionization balance.
Photoionization Equilibrium

\[ n_{H}^{0} \int_{\nu_1}^{\infty} 4\pi N_{*}(\nu) \alpha_{\nu} e^{-\tau_{\nu}(r)} d\nu = n_{e} n_{p} \beta_{2}(T) \]

- \( n_{H}^{0} \): Volumetric neutral H density
- \( 4\pi N_{*}(\nu) \): Local photon field due to the star
- \( \alpha_{\nu} \): Ionization cross section
- \( \tau_{\nu} \): Optical depth
- \( n_{e}, n_{p} \): Volumetric electron and proton densities
- \( \beta_{2} \): Recombination rate coefficient for levels \( n > 1 \)

In order to solve the photoionization equilibrium condition in a nebula, two inputs are required:

1. **The stellar spectrum**, or Luminosity, usually derived from model stellar atmospheres.
2. **The density distribution** in the nebula: \( n_{H}(r) = n_{e}(r) + n_{p}(r) \)
Ionization Parameter

The H^+/H^0 ratio depends on the ratio of photon density (strength of the radiation field) to the particle density. This fact is a general property of external energy sources and is recognized by the ionization parameter \( U = n_\pi / n_H \). The H^+/H^0 ratio of slide 20 can now be rewritten as:

\[
\frac{n_{H^+}}{n_{H^0}} = \frac{U}{U_H}, \quad \text{where} \quad U_H = \frac{\beta_2 x_e}{\alpha_{\nu_i} c (I_4 / I_1)} = 1.37 \times 10^{-6} \frac{x_e}{T_4^{0.7} (I_4 / I_1)}
\]

A typical value in an HII region is \( U \sim 10^{-2.5} >> U_H \). The degree of ionization \( x_e \) is usually \( x_e = n_e / n_H \leq 1.2 \) (<10% He), and therefore hydrogen in HII regions is usually fully ionized.

Similar equations apply to all elements using appropriate values of \( n_\pi \) and \( U_Z \) (replacing \( U_H \)).
Summary

• Processes that govern the physics of HII Regions are *Photo-ionization & Recombination*. As a first-order approximation we assume that the nebula consists of pure Hydrogen.

• *Photoionization equilibrium*, the balance between photo-ionization and recombination, describes the quasi-static state of ionization in such a nebula.

• *“On the spot” approximation*. The diffuse photons produced by recombinations to the ground level will be re-absorbed very close to where they were produced (on the spot). So, effectively the recombinations directly to the ground level do not count, because they are exactly balanced by the photons they produce.

• *Kinetic equilibrium*. The kinetic energies of the emitted electrons will very quickly *thermalize* into a Maxwellian velocity distribution through electron-electron collisions.