The Sundiver

Coryn Bailer-Jones
Max Planck Institute for Astronomy, Heidelberg

1. Goal
You are in a spacecraft in a circular orbit about the Sun and want to leave the solar system. Your spacecraft has engines that deliver a change in velocity of $\Delta v$, and a solar sail of lightness number $\lambda$. How can you use these to maximize your velocity at infinity?

2. Solution
It seems obvious to open the sails and to use all the $\Delta v$ to boost away from the Sun. But a sail is more effective the closer it is to the Sun, plus a given $\Delta v$ is more efficient the faster the spacecraft (the Oberth effect). So it might make more sense to first use some $\Delta v$ to dive closer to the Sun. But how much?

To solve this analytically we keep the sail pointed at the Sun, which ensures that orbits are Keplerian. As shown on the right, the spacecraft starts off (1) with $r_i=1$ au (and so velocity $v_i=29.8$ km/s). A fraction $f$ of the fuel is then used in a retrograde burn at A to dive (2) towards the Sun (Hohmann transfer orbit). At perihelion (P) the sail is deployed and the rest of the fuel is used in a prograde burn to escape (3). The objective is to find the value of $f$ that maximizes the velocity at infinity (equivalently at $r=r_i$) for given $\lambda$ and $\Delta v$.

3. Results
The figure above shows how the velocity of the spacecraft back at $r=r_i$ varies with $f$ for various $\lambda$ for $\Delta v=10$ km/s. If $\lambda$ is small we should use $f=0$, i.e. no dive. But for larger $\lambda$ we should use all the propellant to do the deepest dive possible. The figure below shows the velocity at infinity as a function of $\Delta v$ for various $\lambda$. The solid blue lines show the full dive ($f=1$); the dashed lines the corresponding results for no dive ($f=0$).

Solar sails

A solar sail is propelled by the pressure of photons from the Sun. It is characterized by its lightness number, the ratio of the radiation acceleration to the gravitational acceleration

$$\lambda = \frac{L_\odot}{2\pi G\rho c^2} = \frac{1.5 \times 10^{-3}}{\sigma [kg \, m^{-2}]}$$

where $\sigma$ is the mass per unit surface area of the sail spacecraft. If the sail is pointed at the Sun and $\lambda=1$ (in which case $\sigma$ is a tenth the value of plastic food wrap), then the forces cancel.

4. Conclusion
We should only ever use $f=0$ (no dive) or $f=1$ (deepest dive). Which to do depends on what $\lambda$ and $\Delta v$ we have available, as shown above. This applies for any size initial orbit. In practice the depth of the dive is limited by not hitting the Sun. Furthermore, using more burns, or pointing the sail away from the Sun (to produce non-Keplerian orbits) can achieve larger final velocities (but take longer to achieve).

Published in the American Journal of Physics, 89, 235 (2021)