Sternentstehung - Star Formation

Winter term 2022/2023

Henrik Beuther, Thomas Henning, & Jonathan Henshaw

18.10 - Introduction & overview (Beuther)
25.10 - Physical processes I (Beuther)
08.11 - Physical processes II (Beuther)
15.11 - Molecular clouds I: the birth places of stars (Henshaw)
22.11 - Molecular clouds II: Jeans analysis (Henshaw)
29.11 - Collapse models I (Beuther)
06.12 - Collapse models II (Henning)
13.12 - Protostellar evolution (Beuther)
20.12 - Pre-main sequence evolution & outflows/jets (Beuther)
10.01 - Accretion disks I (Henning)
17.01 - Accretion disks II (Henning)
24.01 - High-mass star formation, clusters & the IMF (Henshaw)
31.01 - Extragalactic star formation (Henning)
07.02 - Planetarium @ HdA, outlook, questions
13.02 - Examination week, no star formation lecture

Book: Stahler & Palla: The Formation of Stars, Wileys

More information and the current lecture files: https://www2.mpia-hd.mpg.de/homes/beuther/lecture_ws2223.html
beuther@mpia.de, henning@mpia.de, henshaw@mpia.de

Recap from the last lecture

Learning outcomes:

- Molecular gas as a component of the interstellar medium
  - **~20% of the gas in the MW by mass is molecular**
- Why are we interested in molecular gas/clouds?
  - **All stars in all galaxies are born in molecular gas**
- What *even is* a "molecular cloud"?
  - **It turns out they are hard to define due to their dynamic nature — over densities in the ISM where gas is in molecular form and in which gravitational collapse and star formation possible**
- What are the general properties of molecular clouds?
  - **Most of mass is in most massive clouds, clouds have varying surface density, clouds are turbulent, unclear whether or not they are in virial equilibrium**
- The internal structure of molecular clouds
  - **Internal structure is filamentary - complex hierarchical structural network. Cores and stars tend to form in filaments.**

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Molecular clouds II: The fate of molecular clouds

Today’s lecture

Learning outcomes:

- The Virial Theorem
- The stability of molecular clouds
- How long do molecular clouds live?
- Cloud fragmentation

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Learning outcomes:

- **The Virial Theorem**
- The stability of molecular clouds
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The virial theorem

$$\frac{1}{2}\frac{\delta^2 I}{\delta t^2} = 2\mathcal{T} + 2U + \mathcal{W} + \mathcal{M}$$

The virial theorem

Same dimensions as
moment of inertia

$$\frac{1}{2}\frac{\delta^2 \boxed{I}}{\delta t^2} = 2\mathcal{T} + 2U + \mathcal{W} + \mathcal{M}$$

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$$\frac{1}{2}\frac{\delta^2 \boxed{I}}{\delta t^2} = \boxed{2\mathcal{T}} + 2U + \mathcal{W} + \mathcal{M}$$

Kinetic energy

The virial theorem

Same dimensions as
moment of inertia

$$\frac{1}{2}\frac{\delta^2 \boxed{I}}{\delta t^2} = \underbrace{\boxed{2\mathcal{T}}}_{\text{Kinetic energy}} + \underbrace{\boxed{2U}}_{\text{Thermal energy}} + \mathcal{W} + \mathcal{M}$$

The virial theorem

$$\frac{1}{2}\frac{\delta^2 \overset{\text{Same dimensions as moment of inertia}}{\boxed{I}}}{\delta t^2} = \underset{\text{Kinetic energy}}{\boxed{2\mathscr{T}}} + \underset{\text{Thermal energy}}{\boxed{2U}} + \underset{\text{Grav. energy}}{\boxed{\mathscr{W}}} + \mathscr{M}$$

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Warning: This version ignores surface terms, which may be important!

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- $\delta^2 I / \delta t^2$ is the integrated form of the acceleration. For a cloud of fixed shape, it tells us the rate of change of its expansion or contraction
- If it is negative, the terms that are trying to collapse the cloud are larger, and the cloud accelerates inward
- If it is positive, the terms that favour expansion are larger, and the cloud accelerates outward
- If it is zero, the cloud neither contracts nor expands

The virial theorem

Same dimensions as moment of inertia

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How do we get here?

The virial theorem

$$\rho \frac{Du}{Dt} = -\nabla P - \rho \nabla \phi_g + \frac{1}{c}\mathbf{j} \times \mathbf{B}$$

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Full time derivative of fluid velocity

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Rate of change of velocity at fixed spatial location, x

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Rate of change of velocity at fixed spatial location, x

Change induced by transporting to a new location with different velocity

The virial theorem

$$\rho \boxed{\frac{Du}{Dt}} = -\nabla P - \rho \nabla \phi_g + \boxed{\frac{1}{c}\mathbf{j} \times \mathbf{B}}$$

Magnetic force per unit volume acting on current density, j

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Rate of change of velocity at fixed spatial location, x

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Related via Ampère's law $\rightarrow \nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}$

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Rearrange

$$\mathbf{j} = \frac{c}{4\pi}\nabla \times \mathbf{B}$$

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Substitute

$$\frac{1}{c}\mathbf{j} \times \mathbf{B} = \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

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Use

$$(\mathbf{A} \cdot \nabla)\mathbf{A} = \frac{1}{2}\nabla |\mathbf{A}|^2 + (\nabla \times A) \times A$$

$$\frac{1}{c}\mathbf{j} \times \mathbf{B} = \frac{1}{4\pi}(\mathbf{B} \cdot \nabla)\mathbf{B} - \frac{1}{8\pi}\nabla |\mathbf{B}|^2$$

The virial theorem

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Substitute back in…

The virial theorem

$$\left(\frac{\delta \mathbf{u}}{\delta t}\right)_x + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla P - \rho \nabla \phi_g + \frac{1}{4\pi}(\mathbf{B} \cdot \nabla)\mathbf{B} - \frac{1}{8\pi}\nabla |\mathbf{B}|^2$$

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Tension associated with
curved magnetic fields

The virial theorem

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Tension associated with curved magnetic fields

Gradient of magnetic pressure

The virial theorem

This is the local behaviour of the fluid.

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To derive a relation between global properties of the gaseous body, we need to integrate over the volume. Employing Poisson's equation ($\nabla^2 \phi_g = 4\pi G \rho$) and requiring mass conservation, after repeated integration by parts one arrives at the virial theorem:

$$\frac{1}{2}\frac{\delta^2 I}{\delta t^2} = 2\mathcal{T} + 2U + \mathcal{W} + \mathcal{M}$$

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For a detailed derivation see Appendix D in Stahler and Palla

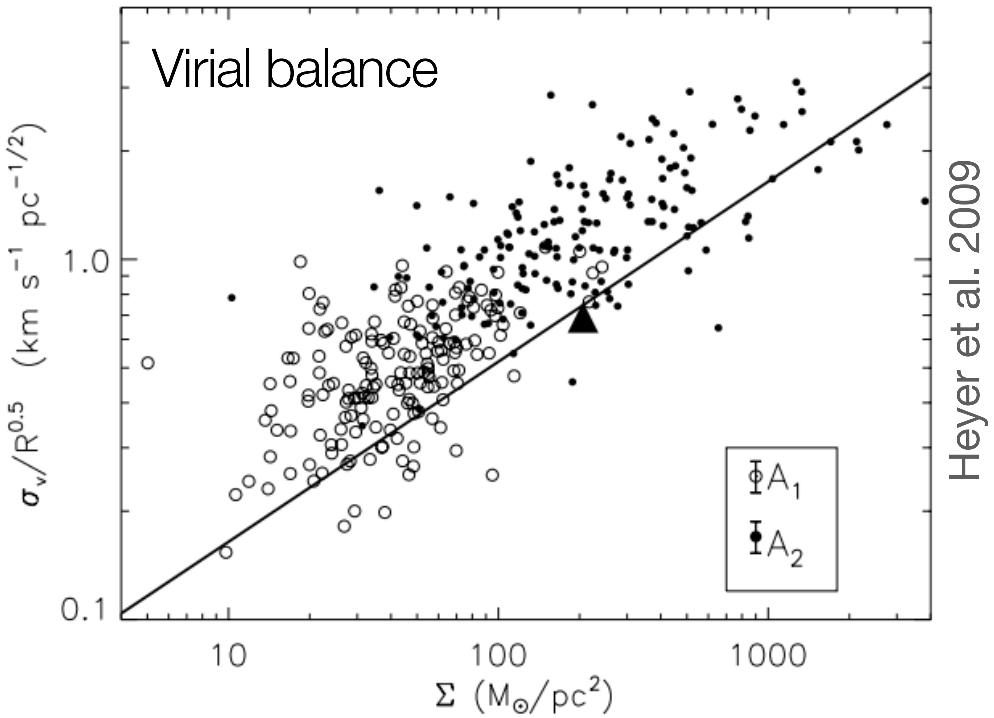
$$\frac{1}{2}\frac{\delta^2 I}{\delta t^2} = 2\mathcal{T} + 2U + \mathcal{W} + \mathcal{M}$$

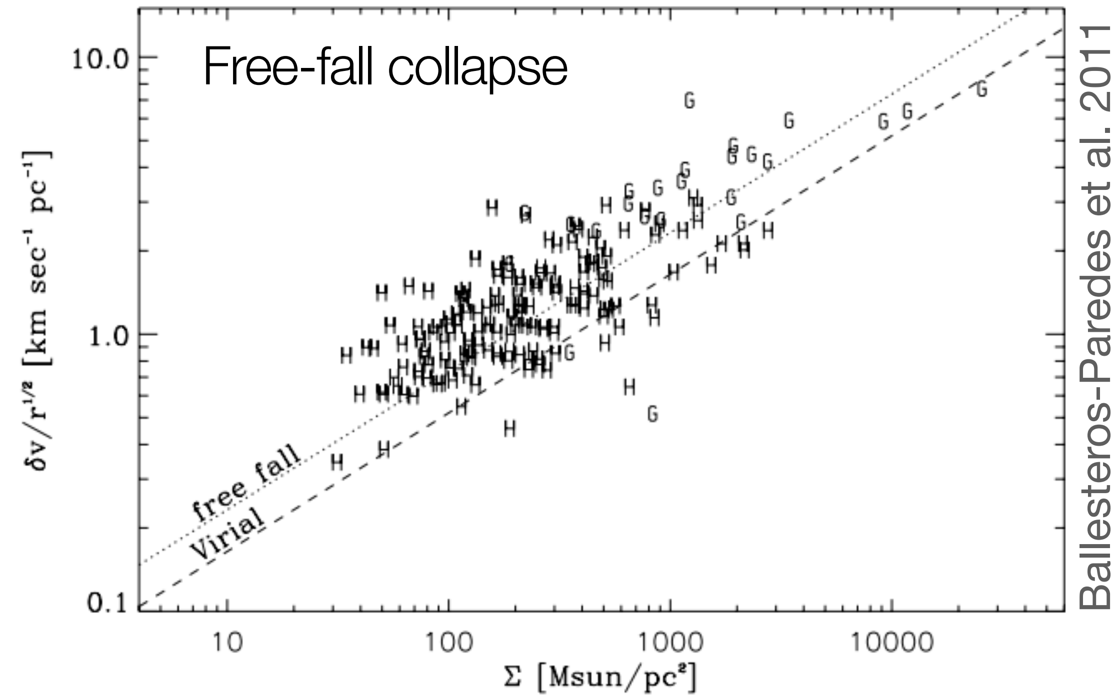
Today’s lecture

Learning outcomes:

- The Virial Theorem
- **The stability of molecular clouds**
- How long do molecular clouds live?
- Cloud fragmentation

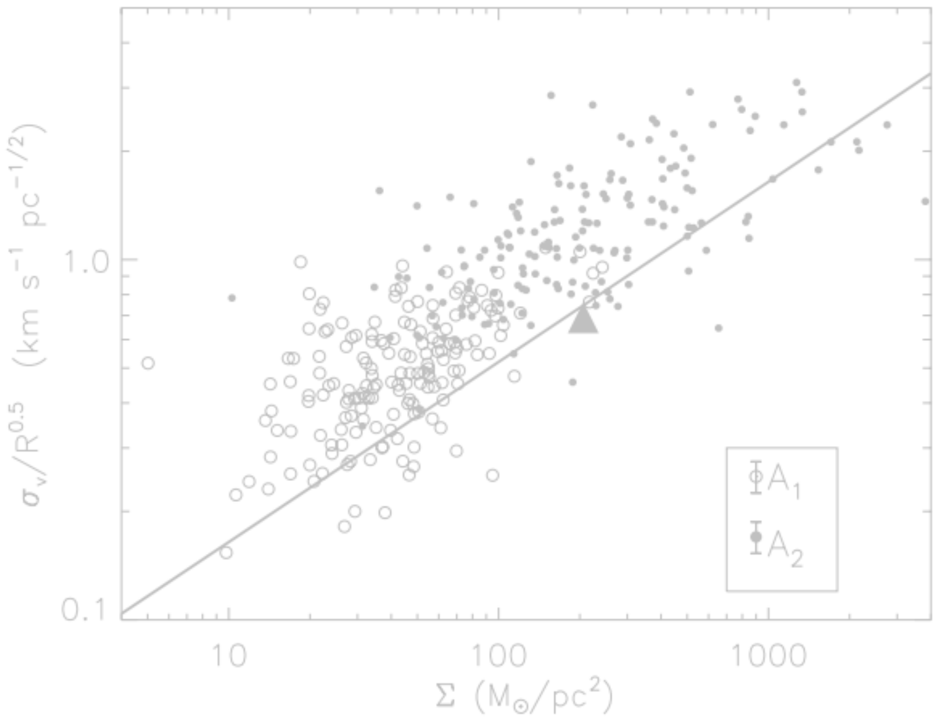
The stability of molecular clouds





What is the origin of the velocity dispersion in molecular clouds?

The stability of molecular clouds



Lets apply the virial theorem to define a characteristic timescale for gravitational collapse

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The stability of molecular clouds

Lets apply the virial theorem to define a characteristic timescale for gravitational collapse

If all forces are too weak to match the gravitational energy, we would have free-fall collapse

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Units of energy
$kg^{-1}m^3s^{-2} * kg^2 * m^{-1}$

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Use dimensional analysis with

$I \approx MR^2$

$$\frac{MR^2}{t^2} \sim \frac{GM^2}{R}$$

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Use dimensional analysis with

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$$\frac{MR^2}{t^2} \sim \frac{GM^2}{R}$$

Solve for time

$$t = \left(\frac{R^3}{GM}\right)^{1/2}$$

The stability of molecular clouds

Lets apply the virial theorem to define a characteristic timescale for gravitational collapse

$$t_{\rm ff} \approx (G\rho)^{-1/2}$$

More precisely we can define the free-fall time…

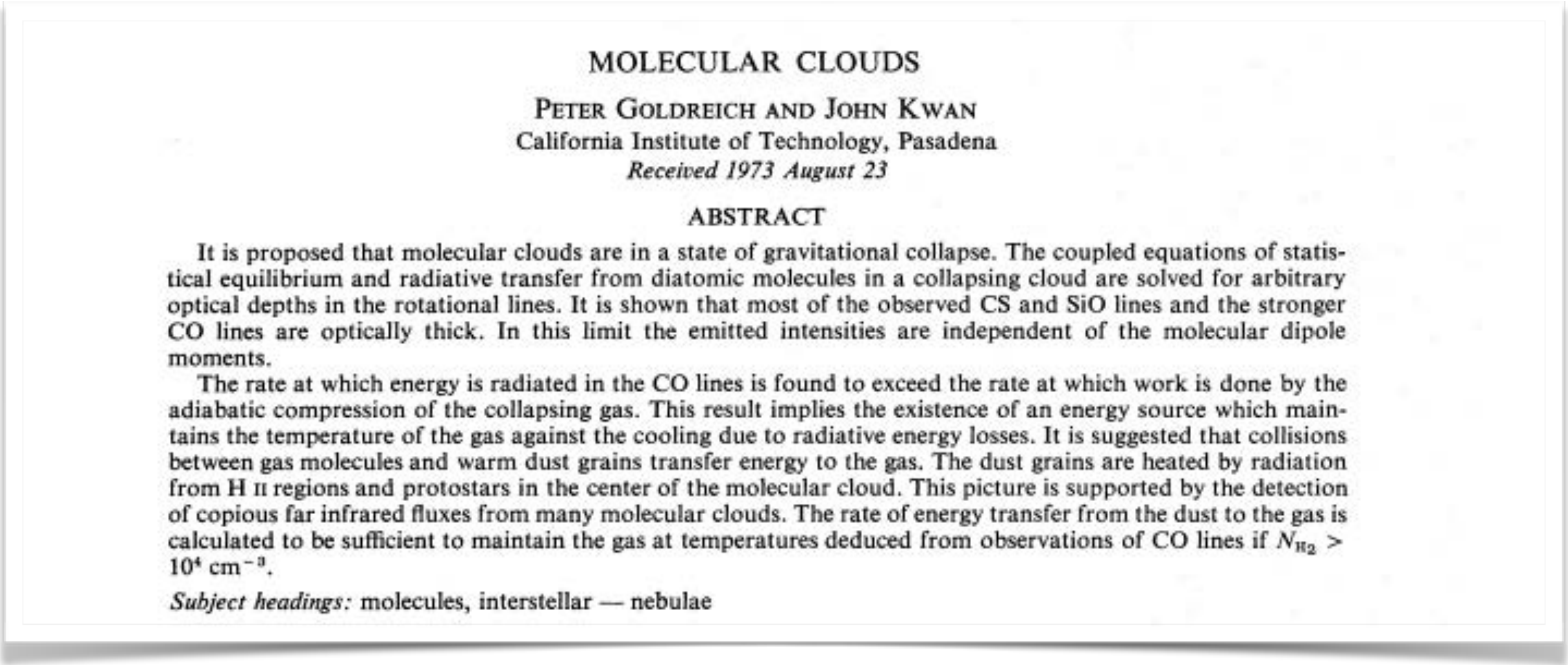
$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

For a Giant Molecular Cloud…

$$t_{\rm ff} = 5 \times 10^7 \left(\frac{M}{10^5\,{\rm M}_\odot}\right)^{-1/2} \left(\frac{R}{100\,{\rm pc}}\right)^{3/2} {\rm yr}$$

The stability of molecular clouds

The idea that molecular clouds are in a state of collapse has been around for some time!



MOLECULAR CLOUDS

PETER GOLDREICH AND JOHN KWAN
California Institute of Technology, Pasadena
Received 1973 August 23

ABSTRACT

It is proposed that molecular clouds are in a state of gravitational collapse. The coupled equations of statistical equilibrium and radiative transfer from diatomic molecules in a collapsing cloud are solved for arbitrary optical depths in the rotational lines. It is shown that most of the observed CS and SiO lines and the stronger CO lines are optically thick. In this limit the emitted intensities are independent of the molecular dipole moments.

The rate at which energy is radiated in the CO lines is found to exceed the rate at which work is done by the adiabatic compression of the collapsing gas. This result implies the existence of an energy source which maintains the temperature of the gas against the cooling due to radiative energy losses. It is suggested that collisions between gas molecules and warm dust grains transfer energy to the gas. The dust grains are heated by radiation from H II regions and protostars in the center of the molecular cloud. This picture is supported by the detection of copious far infrared fluxes from many molecular clouds. The rate of energy transfer from the dust to the gas is calculated to be sufficient to maintain the gas at temperatures deduced from observations of CO lines if $N_{H_2} > 10^4$ cm^{-3}.

Subject headings: molecules, interstellar — nebulae

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From last lecture: mass of molecular gas in the Galaxy is $\sim 2.5 \times 10^9\,\mathrm{M}_\odot$. If we assume this mass is within GMCs collapsing on a free-fall timescale of $5 \times 10^7\,\mathrm{yr}$, and all of the mass is converted to stars (i.e. is highly efficient), this implies a star formation rate…

$$\approx 50\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$$

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Compare this to current estimates of the observed star formation rate….

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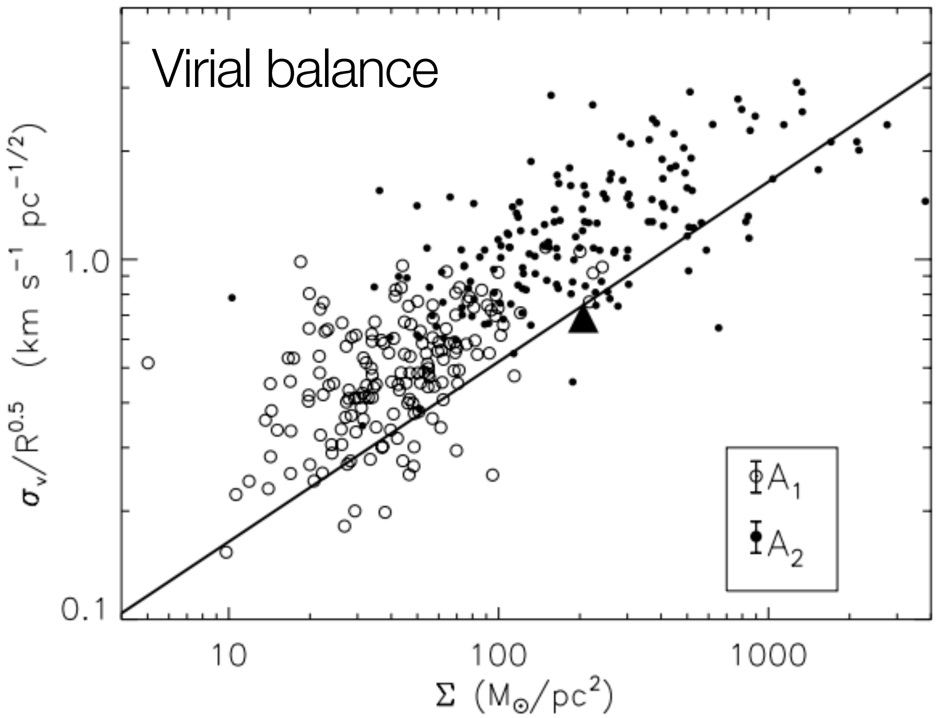
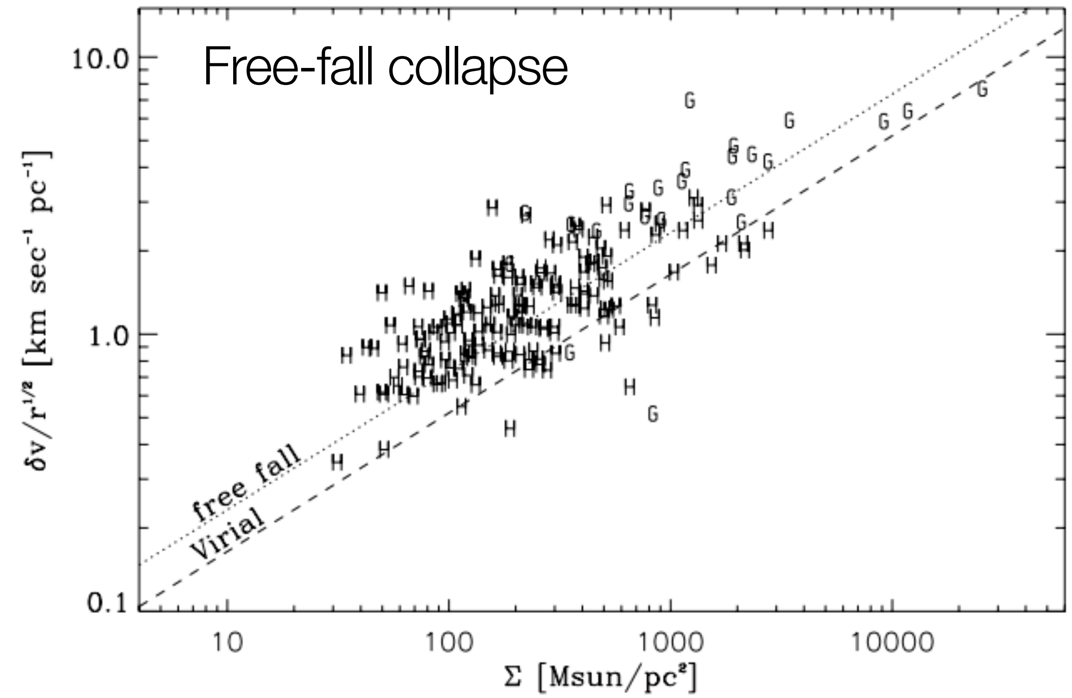
$$\approx 1\,\mathrm{M}_\odot\,\mathrm{yr}^{-1} \longrightarrow$$

If clouds are collapsing then only a tiny fraction of the cloud must be converted into stars

The stability of molecular clouds

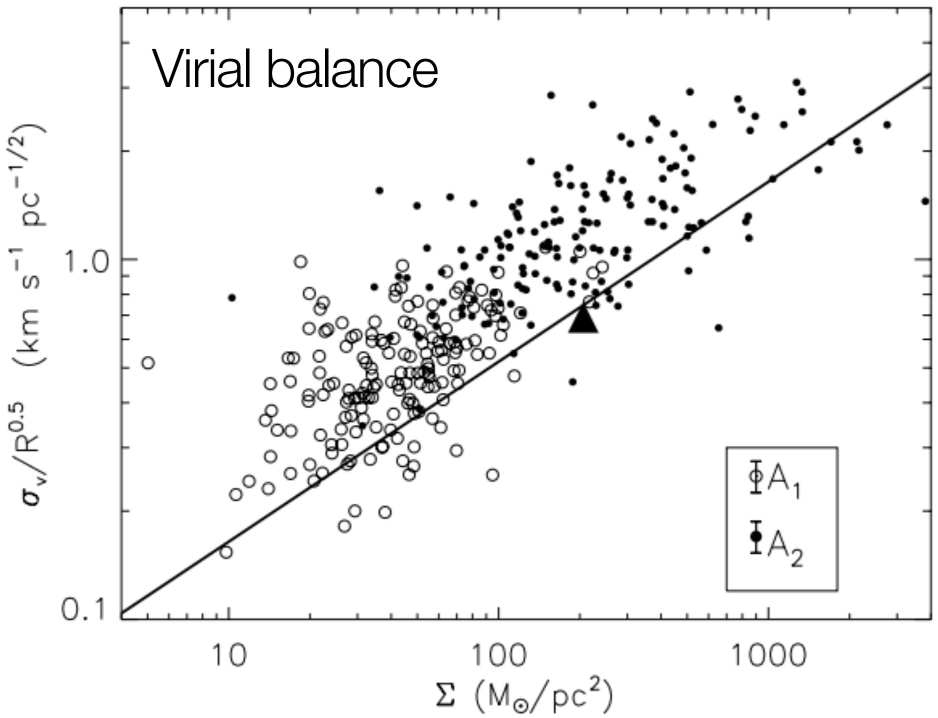


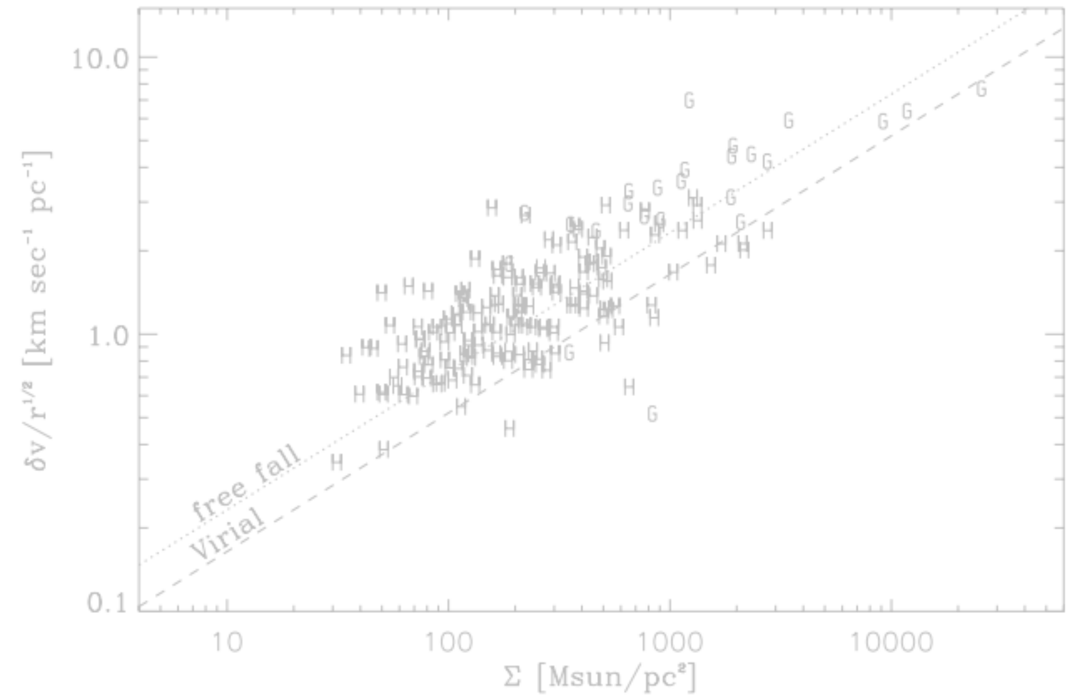
Heyer et al. 2009



Ballesteros-Paredes et al. 2011

The stability of molecular clouds





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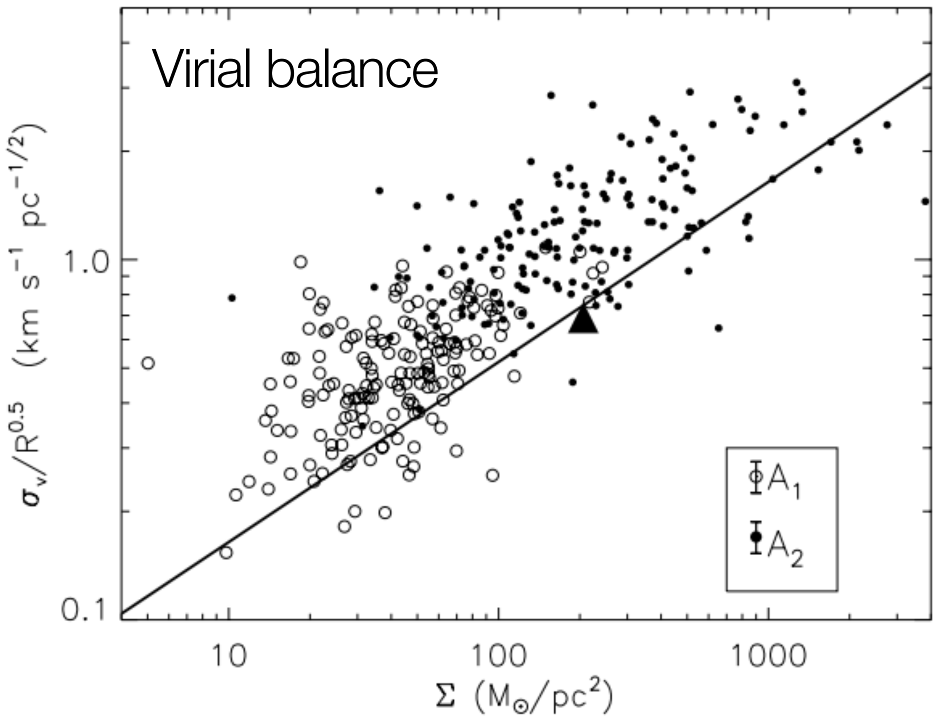
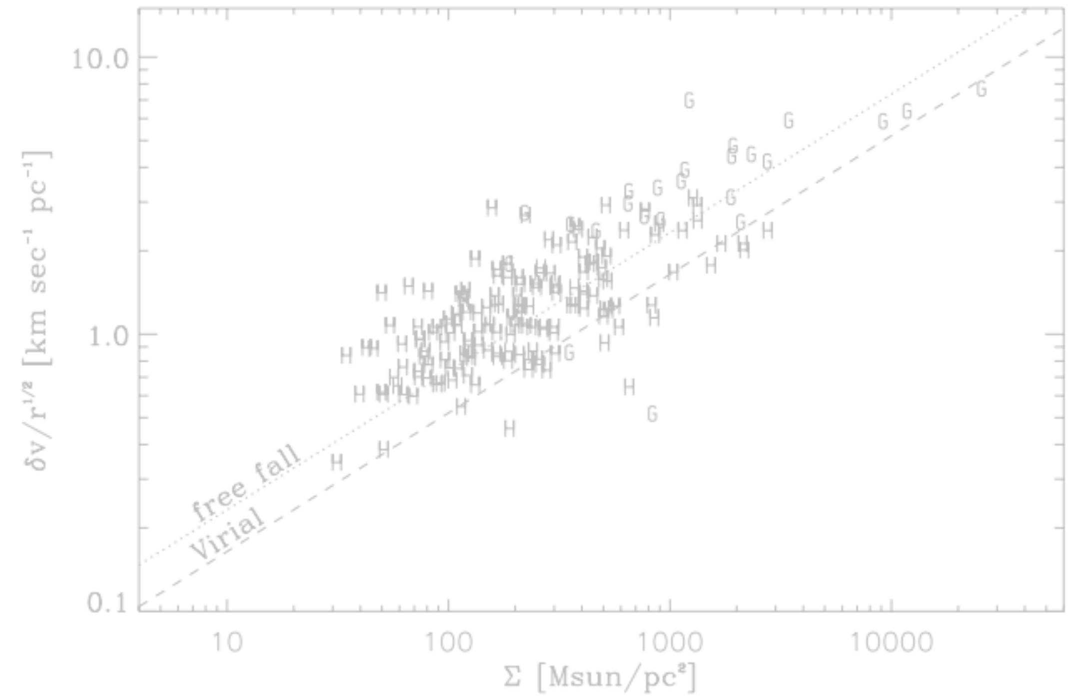
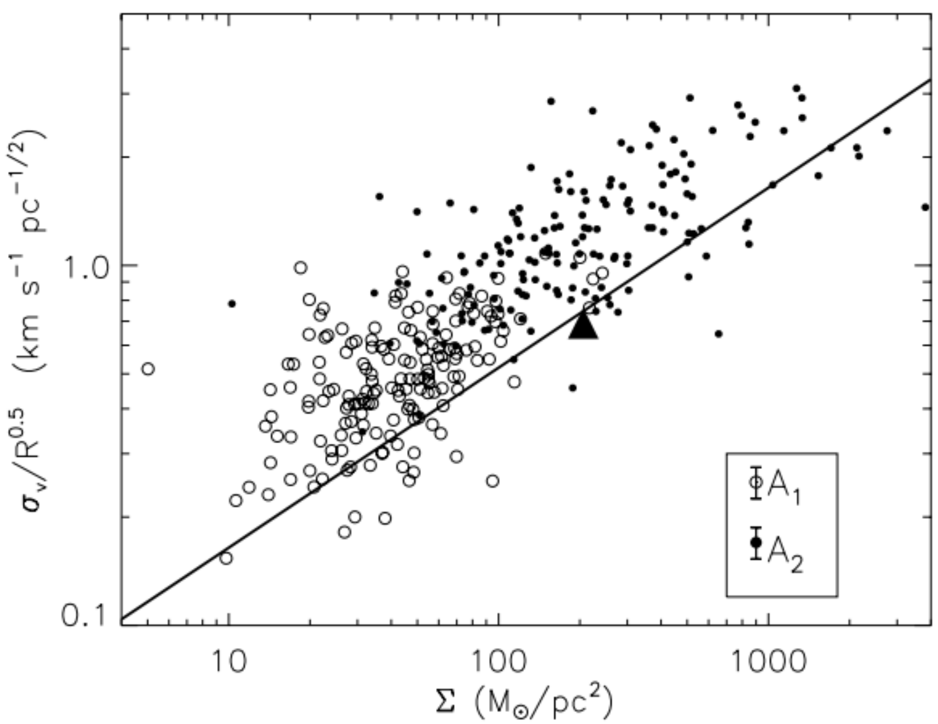
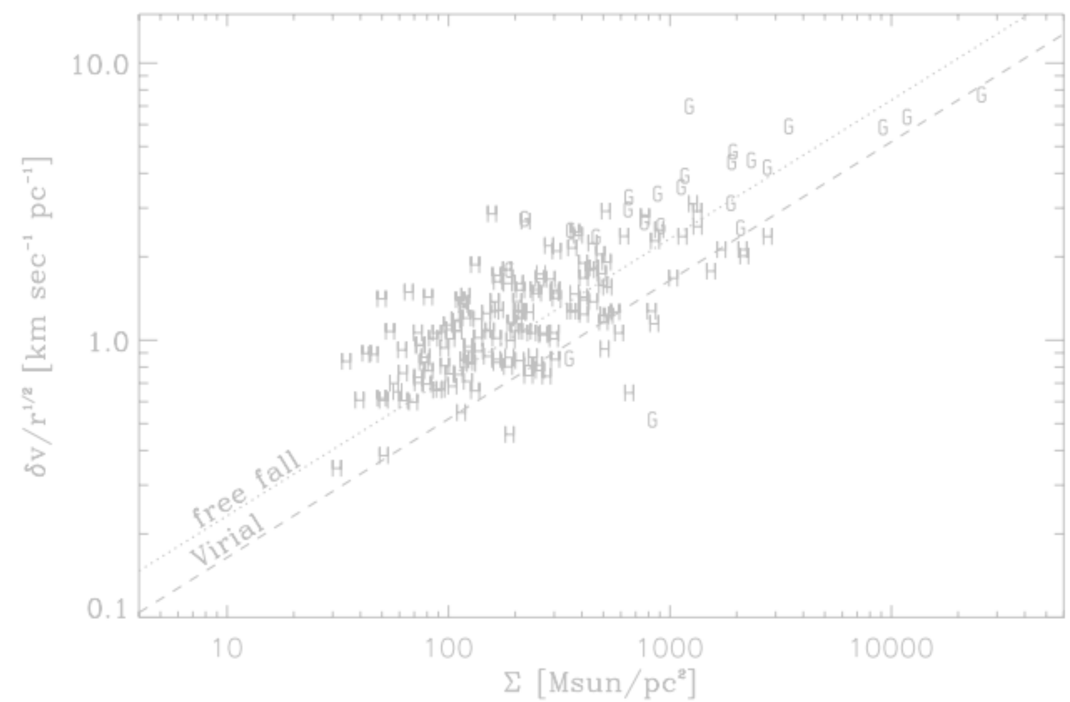




The stability of molecular clouds





What helps to balance gravity?

Thermal Energy (using $U = \frac{3}{2} N k_B T \sim \frac{M\mathscr{R}T}{\mu}$):

$$\frac{U}{|\mathscr{W}|} \approx \frac{M\mathscr{R}T}{\mu}\left(\frac{GM^2}{R}\right)^{-1} \approx 3\times 10^{-3}\left(\frac{M}{10^5\,\mathrm{M}_\odot}\right)^{-1}\left(\frac{R}{25\,\mathrm{pc}}\right)\left(\frac{T}{15\,K}\right)$$

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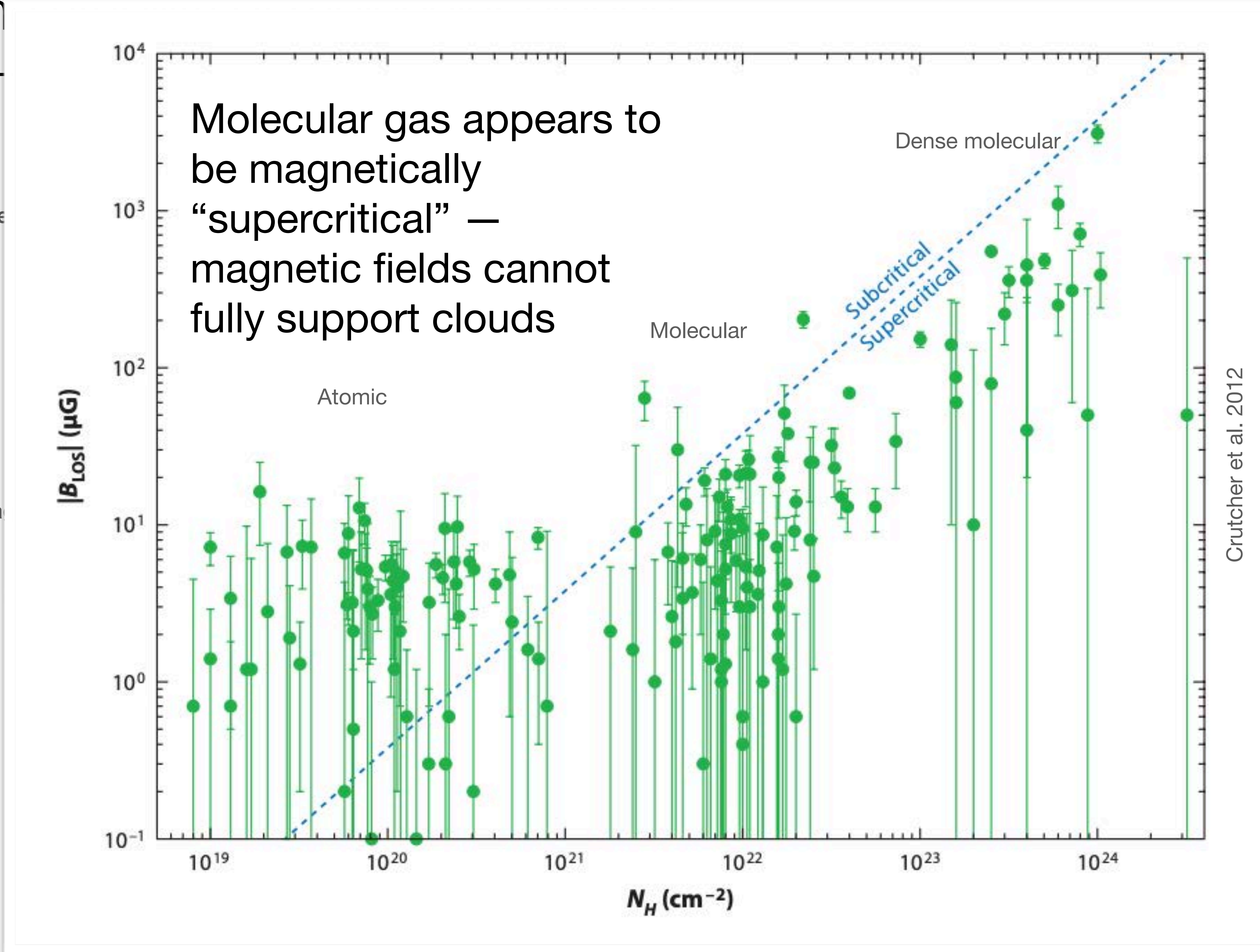

Molecular gas appears to be magnetically "supercritical" — magnetic fields cannot fully support clouds
Dense molecular
Subcritical
Supercritical
Molecular
Atomic
$|B_{LOS}|$ (μG)
N_H (cm^{-2})
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{19}
10^{20}
10^{21}
10^{22}
10^{23}
10^{24}
Crutcher et al. 2012

The stability of molecular clouds

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The stability of molecular clouds

THE ENERGY DISSIPATION RATE OF SUPERSONIC, MAGNETOHYDRODYNAMIC TURBULENCE IN MOLECULAR CLOUDS

MORDECAI-MARK MAC LOW[1]
Max-Planck-Institut für Astronomie, Königstuhl 17, D-69117 Heidelberg, Germany
Received 1998 September 16; accepted 1999 June 1

ABSTRACT

Molecular clouds have broad line widths, which suggests turbulent supersonic motions in the clouds. These motions are usually invoked to explain why molecular clouds take much longer than a free-fall time to form stars. Classically, it was thought that supersonic hydrodynamical turbulence would dissipate its energy quickly but that the introduction of strong magnetic fields could maintain these motions. A previous paper has shown, however, that isothermal, compressible MHD and hydrodynamical turbulence decay at virtually the same rate, requiring that constant driving occur to maintain the observed turbulence. In this paper, direct numerical computations of uniform, randomly driven turbulence with the ZEUS astrophysical MHD code are used to derive the value of the energy-dissipation coefficient, which is found to be

$$\dot{E}_{\rm kin} \simeq -\eta_v m \tilde{k} v_{\rm rms}^3 ,$$

with $\eta_v = 0.21/\pi$, where $v_{\rm rms}$ is the root-mean-square (rms) velocity in the region, $E_{\rm kin}$ is the total kinetic energy in the region, m is the mass of the region, and $\tilde{k}$ is the driving wavenumber. The ratio τ of the formal decay time $E_{\rm kin}/\dot{E}_{\rm kin}$ of turbulence to the free-fall time of the gas can then be shown to be

$$\tau(\kappa) = \frac{\kappa}{M_{\rm rms}} \frac{1}{4\pi\eta_v} ,$$

where $M_{\rm rms}$ is the rms Mach number, and κ is the ratio of the driving wavelength to the Jeans wavelength. It is likely that $\kappa < 1$ is required for turbulence to support gas against gravitational collapse, so the decay time will probably always be far less than the free-fall time in molecular clouds, again showing that turbulence there must be constantly and strongly driven. Finally, the typical decay time constant of the turbulence can be shown to be

$$t_0 \simeq 1.0\, \mathscr{L}/v_{\rm rms} ,$$

where $\mathscr{L}$ is the driving wavelength.

The stability of molecular clouds

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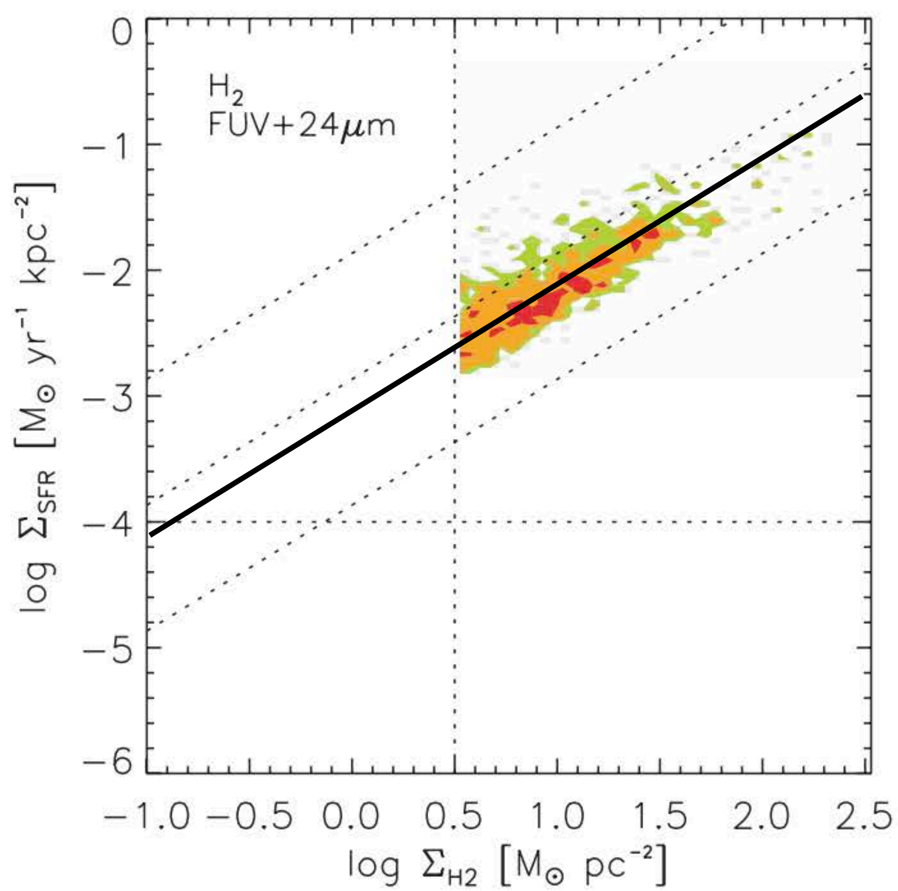
So what is going on?



Bigiel et al. 2008

Recall from last lecture: The linear (i.e. index = 1) relationship here implies a constant “depletion time” - the time it would take to exhaust all molecular gas at the current star formation rate…

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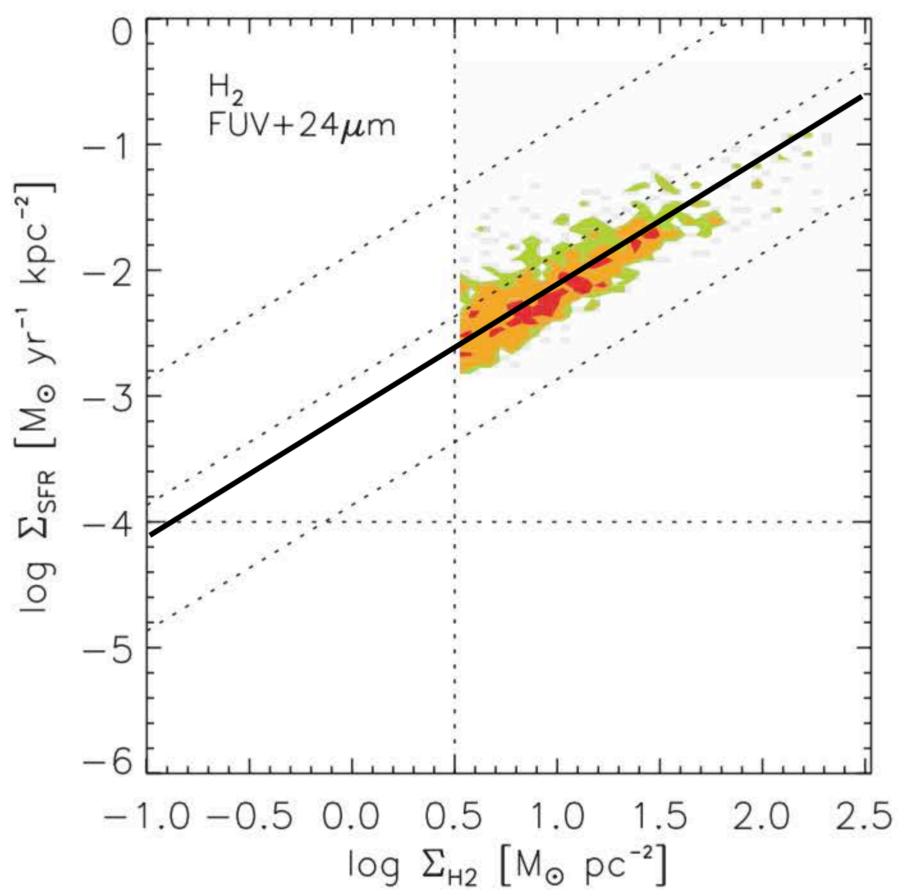

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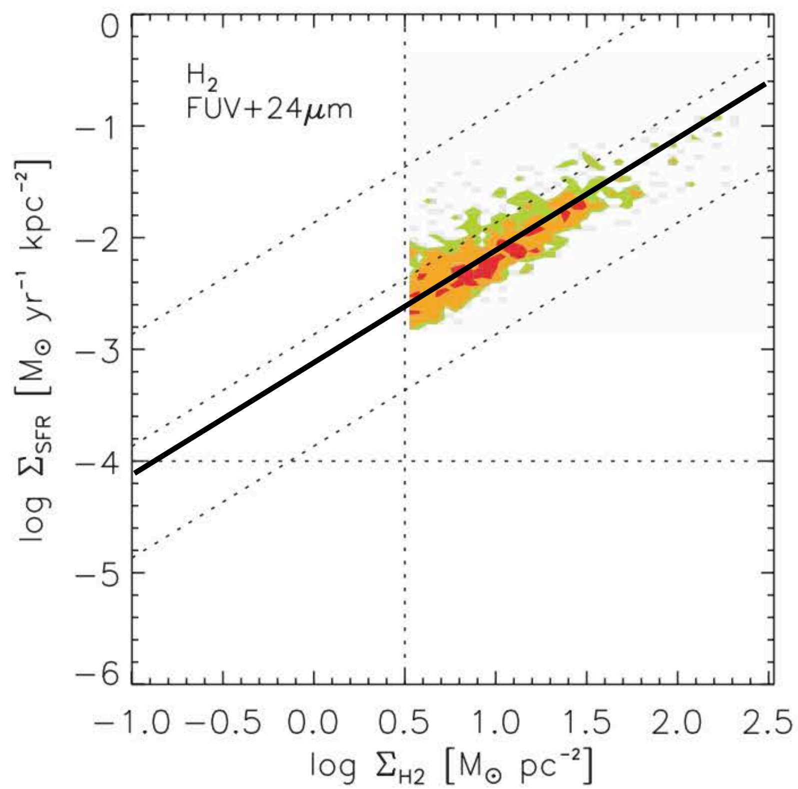
- More generally we can define: $\epsilon_{\mathrm{sf}} = t_{\mathrm{sf}}/t_{\mathrm{dep}}$, where t_{sf} is the time it takes to convert gas into stars and ϵ_{sf} is the fraction of mass converted into stars on that timescale, such that $\epsilon_{\mathrm{sf}} = \dfrac{\mathrm{SFR}}{M/t_{\mathrm{sf}}}$

The stability of molecular clouds



Bigiel et al. 2008

$$\epsilon_{\rm sf} = t_{\rm sf}/t_{\rm dep}$$

Slow (takes many free-fall times) and efficient (large fraction of GMC mass is converted into stars) star formation: Requires some mechanism(s) to provide long-term support for clouds against gravitational collapse (turbulence, magnetic fields, differential rotation).

Fast (occurs on a few free-fall times) and inefficient (only a small fraction of GMC mass is converted into stars) star formation: Requires strong stellar feedback from massive stars to keep the efficiency low.

How long do molecular clouds live?

Today’s lecture

Learning outcomes:

- The Virial Theorem
- The stability of molecular clouds
- **How long do molecular clouds live?**
- Cloud fragmentation

How long do molecular clouds live?

From Heyer & Dame 2015

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"No, *you* don't know what *you're* talking about!"

—Phil Solomon and Pat Thaddeus debating cloud lifetimes at a meeting, circa 1980

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- Large-scale distribution of H_2 with respect to atomic and ionised gas components
- Arm to interarm contrast in spiral galaxies
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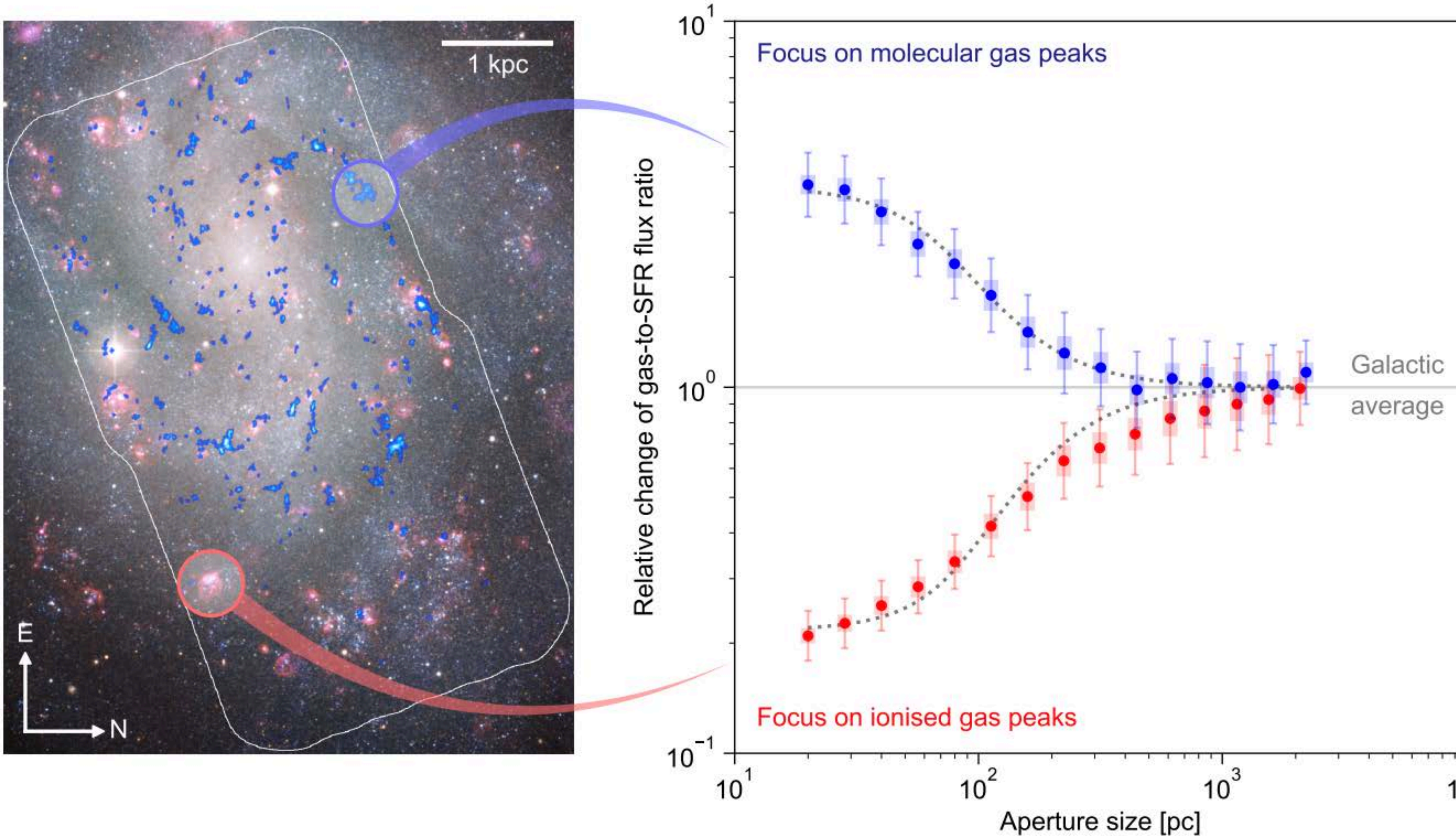
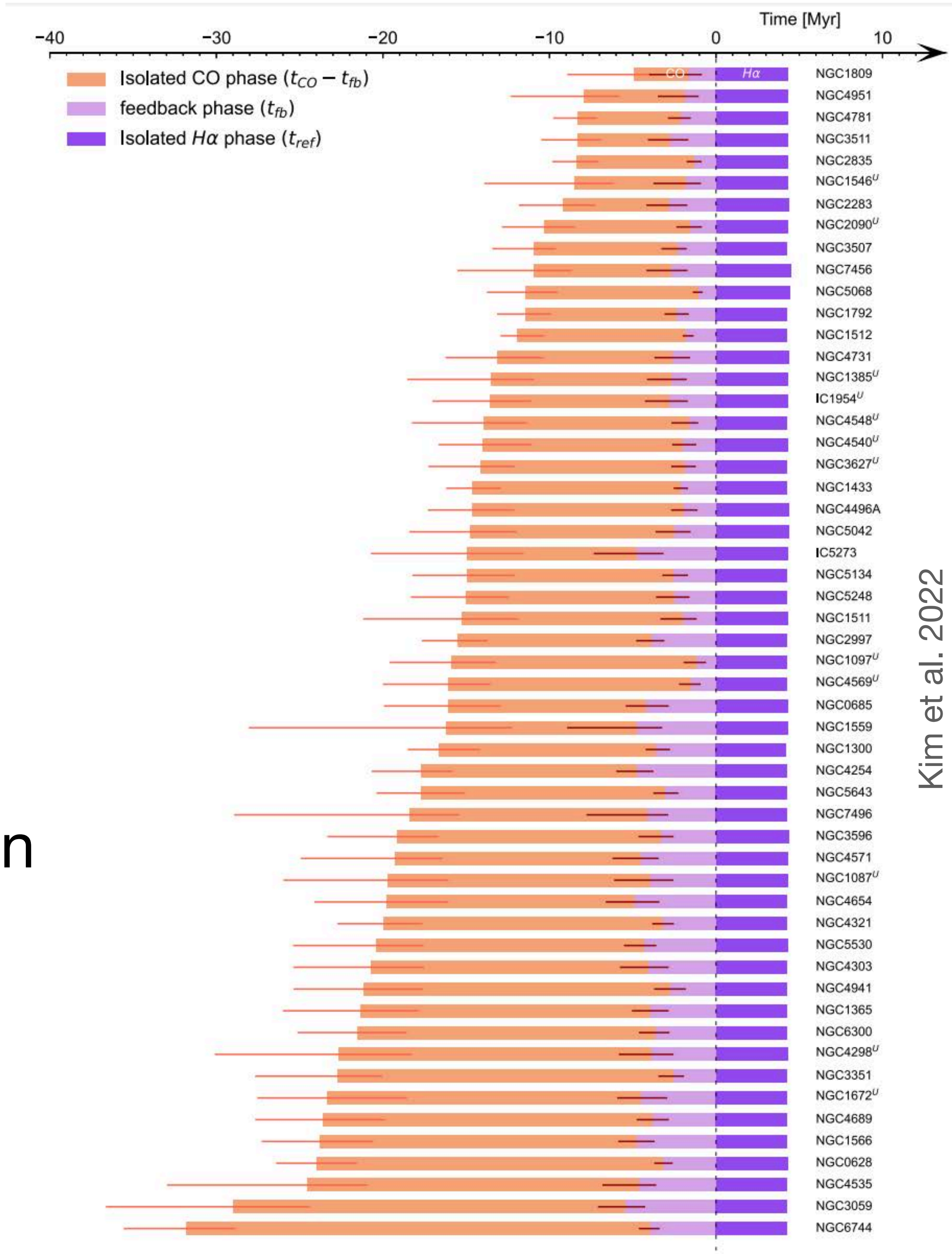


Kruijssen et al. 2019

How long do molecular clouds live?

- Clouds live for 5-30Myr
- Characterised by a long inert phase
- Once high-mass stars form - clouds are quickly dispersed
- Lifetimes are much shorter than the depletion time —GMCs achieve low integrated SFE



Kim et al. 2022

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- **Cloud fragmentation**

Cloud Fragmentation

Important point related to time/efficiency problem from last time:
Clouds are highly structured!

Cloud Fragmentation

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Clouds are highly structured!

Question then becomes: How do dense cores condense out of the lower density regions of GMCs?

"Conventional" picture is one of *local* gravitational instabilities developing in a *globally stable* molecular cloud via the so-called ***Jeans Instability***

Cloud Fragmentation

Consider a uniform isothermal gas in hydrostatic equilibrium with gravity balanced by pressure gradient

1 $$\frac{\delta \mathbf{u}}{\delta t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\nabla P}{\rho} - \nabla \phi_g$$ Equation of motion (no magnetic field)

2 $$\frac{\delta \rho}{\delta t} + \nabla(\rho \mathbf{u}) = 0$$ Continuity equation

3 $$P = nk_B T = \rho c_s^2$$ Equation of state

4 $$\nabla^2 \phi_g = 4\pi G \rho$$ Poisson's equation

Cloud Fragmentation

The unperturbed fluid motion is steady, $\delta/\delta t = 0$, with uniform density and pressure P_0 and ρ_0, and zero velocity $\mathbf{u}_0 = 0$. Now consider small perturbations to that fluid.

$$P = P_0 + P_1$$

$$\rho = \rho_0 + \rho_1$$

$$\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1 = \mathbf{u}_1$$

$$\phi_g = \phi_{g,0} + \phi_{g,1}$$

Where subscript 0 properties are constant (uniform, steady state), and subscript 1 properties reflect small perturbations. Our equations then become…

Cloud Fragmentation

With small perturbations…

5 $$\frac{\delta \mathbf{u}_1}{\delta t} + (\mathbf{u}_1 \cdot \nabla)\mathbf{u}_1 = -\frac{\nabla(P_0 + P_1)}{(\rho_0 + \rho_1)} - \nabla(\phi_{g,0} + \phi_{g,1})$$

6 $$\frac{\delta(\rho_0 + \rho_1)}{\delta t} + \nabla((\rho_0 + \rho_1)\mathbf{u}_1) = 0$$

7 $$(P_0 + P_1) = (\rho_0 + \rho_1)c_s^2$$

8 $$\nabla^2(\phi_{g,0} + \phi_{g,1}) = 4\pi G(\rho_0 + \rho_1)$$

We can simplify these equations by linearising (expanding them to first order and throwing away all products of small terms — subscript 1)…

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"Jeans Swindle" - we will ignore any gradient in the background potential

We can simplify these equations by linearising (expanding them to first order and throwing away all products of small terms — subscript 1)…

With small perturbations…linearised equations…

5 $$\frac{\delta \mathbf{u}_1}{\delta t} = -\frac{\nabla P_1}{\rho_0} - \nabla \phi_{g,1}$$

6 $$\frac{\delta \rho_1}{\delta t} + \rho_0 \nabla \mathbf{u}_1 = 0$$

7 $$P_1 = \rho_1 c_s^2$$

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Cloud Fragmentation

We are going to…

5 $$\frac{\delta \mathbf{u}_1}{\delta t} = -\frac{\nabla P_1}{\rho_0} - \nabla \phi_{g,1}$$

Take the divergence of this

6 $$\frac{\delta \rho_1}{\delta t} + \rho_0 \nabla \mathbf{u}_1 = 0$$

Take the partial derivative of this wrt time

7 $$P_1 = \rho_1 c_s^2$$

8 $$\nabla^2 \phi_{g,1} = 4\pi G \rho_1$$

Substitute for these

Cloud Fragmentation

5 $\Rightarrow$ $$\nabla \cdot \left(\frac{\delta \mathbf{u}_1}{\delta t} \right) = - \frac{\nabla^2 P_1}{\rho_0} - \nabla^2 \phi_{g,1}$$

6 $\Rightarrow$ $$\frac{\delta^2 \rho_1}{\delta t^2} + \rho_0 \frac{\delta}{\delta t} (\nabla \mathbf{u}_1) = 0$$

Here we note that, $\delta(\nabla \mathbf{u}_1)/\delta t = \nabla \cdot (\delta \mathbf{u}_1 / \delta t)$, so substitute…

$$\frac{\delta^2 \rho_1}{\delta t^2} - \nabla^2 P_1 - \rho_0 \nabla^2 \phi_{g,1} = 0$$

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$$\Downarrow$$

$$\frac{\delta^2 \rho_1}{\delta t^2} = c_s^2 \nabla^2 \rho_1 + 4\pi G \rho_0 \rho_1$$

Cloud Fragmentation

Jeans' classic analysis considered self-gravitating waves propagating through a uniform, isothermal gas. He considered a plane-travelling wave perturbation of the form…

$$\rho(x, t) = \rho_0 + \delta\rho \exp[i(kx - \omega t)]$$

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We find…

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0$$

This is a dispersion relation that describes the propagation of the waves.

Cloud Fragmentation

Here we notice something interesting…

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0$$

If k is large $\omega^2 > 0$, the disturbance behaves like a sound wave. However, if k is small we can have $\omega^2 < 0$, then ω becomes imaginary…This leads to the perturbation growing exponentially…

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Lets put in some numbers…

Cloud Fragmentation

For typical conditions in the interior of molecular clouds, the Jeans Length and Jeans Mass are:

$$\lambda_J = \sqrt{\frac{\pi c_s^2}{G\rho_0}} = 0.19 \left(\frac{T}{10\,\mathrm{K}}\right)^{3/2} \left(\frac{n_{\mathrm{H}_2}}{10^4\,\mathrm{cm}^{-3}}\right)^{-1/2} \mathrm{pc}$$

$$M_J = \frac{\pi^{5/2}}{6}\frac{c_s^3}{\rho_0^{1/2}G^{3/2}} = 1.0\left(\frac{T}{10\,\mathrm{K}}\right)^{3/2} \left(\frac{n_{\mathrm{H}_2}}{10^4\,\mathrm{cm}^{-3}}\right)^{-1/2} \mathrm{M}_\odot$$

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As a gravitationally unstable region begins to collapse, density increases — both the Jeans mass and length also decrease (the temperature remains roughly constant because cooling is efficient).

The cloud fragments into lower and lower mass pieces, each collapsing on its own free-fall time — Jeans fragmentation

Cloud Fragmentation

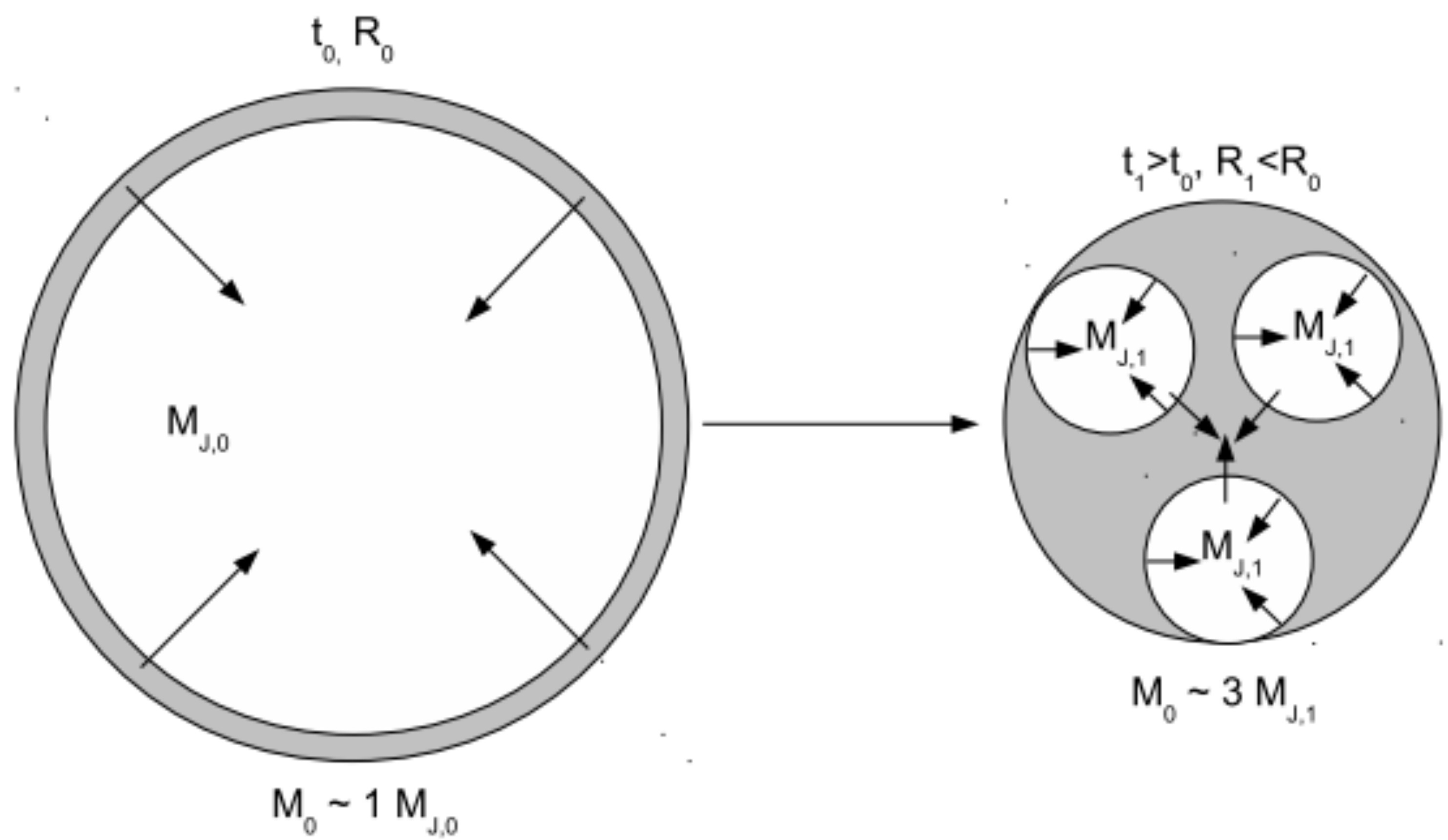
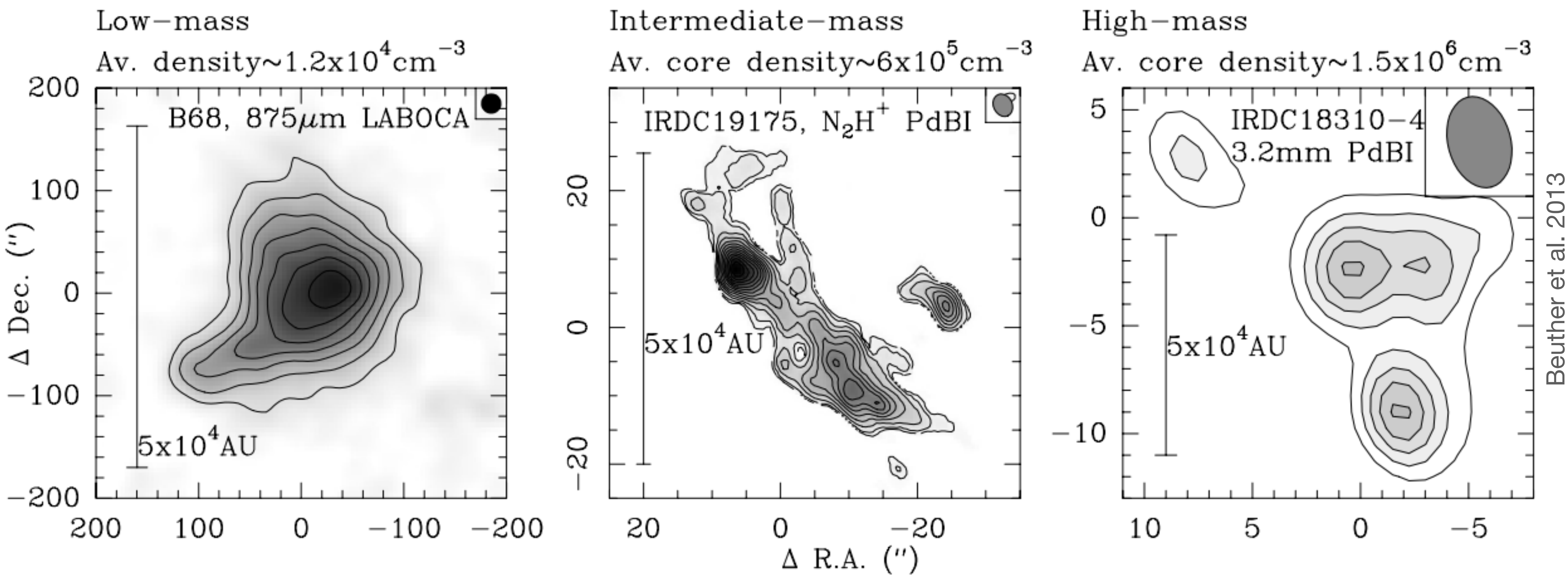


Figure 1. Schematic representation of the first step of fragmentation. A cloud of mass M slightly greater than its initial Jeans mass $M_{J,0}$ begins to contract gravitationally. As it contracts, its mean density increases, and thus its Jeans mass decreases, so that at later times the number of Jeans masses it contains is larger, and fragments again into objects with masses of the order of the instantaneous Jeans mass. The fragments begin to contract themselves, while participating of the large-scale contraction flow as well.

Vazquez-Semadeni et al. 2019

Cloud Fragmentation



Jeans stable

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Dynamical fragmentation — a problem

We can rewrite the dispersion relation in terms of a dimensionless growth rate, $\Omega = i\omega t_{\rm grav}$, where $t_{\rm grav} = (4\pi G\rho_0)^{-1/2}$, and a dimensionless wavenumber, $\nu = k/k_J$, such that

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This indicates that the growth rate increases for decreasing wavenumber, and has a maximum when $\nu = k/k_J = 0$. In other words: the fastest growing mode of the gravitational instability is the one which corresponds to the overall collapse of the medium, and small perturbations do not have time to collapse before the whole cloud, i.e.

Fragmentation very difficult

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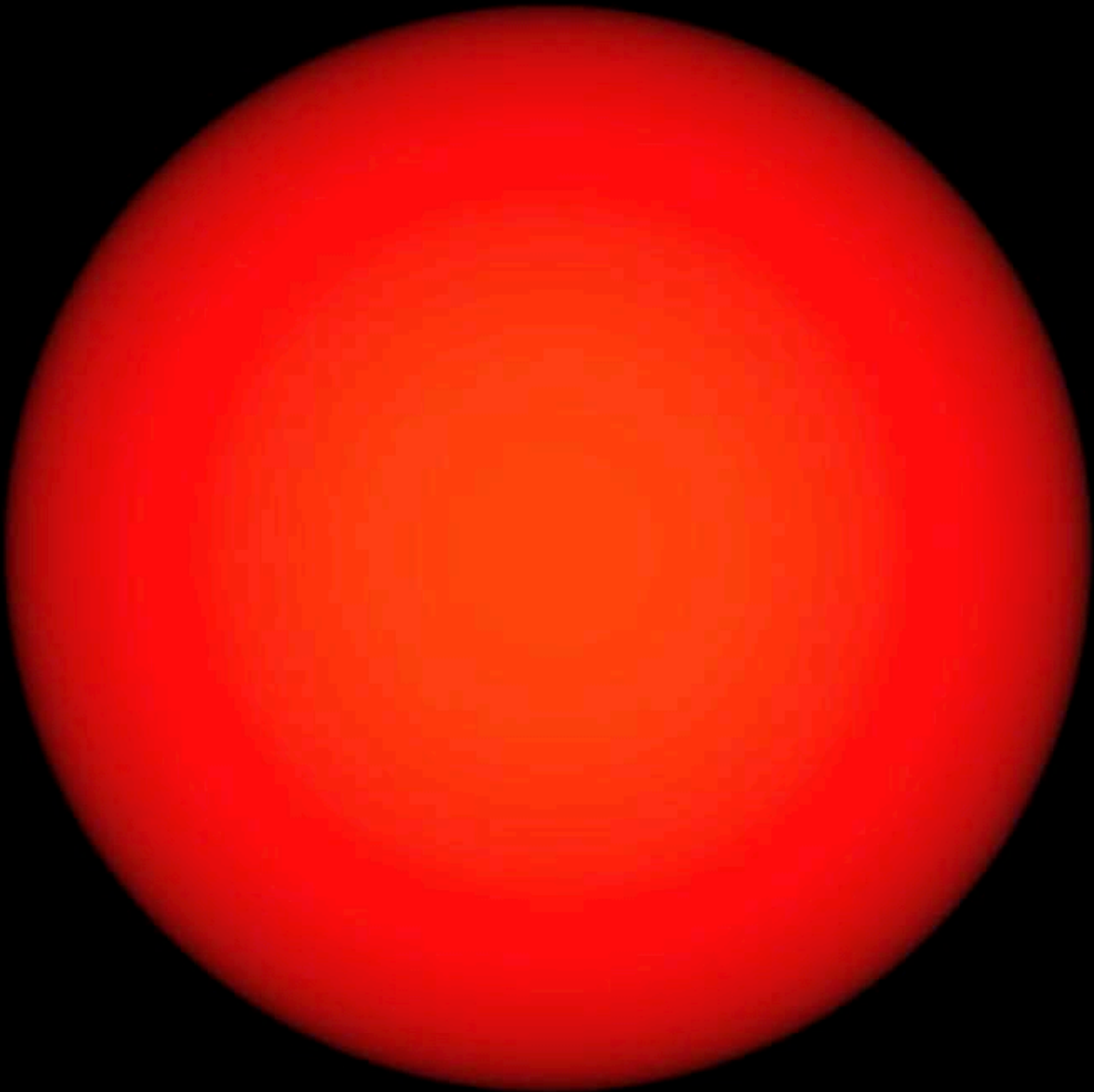
Fragmentation very difficult

Solution: supersonic turbulence drives the formation of density fluctuations — clouds are not homogeneous from the get-go!

UK Astrophysical
Fluids Facility



Matthew Bate

UNIVERSITY OF EXETER

Federrath 2015

Summary

Learning outcomes:

- The Virial Theorem
  - Describes the balance of forces — can be used to assess the stability of molecular clouds
- The stability of molecular clouds
  - Without support, and with high-efficiency gas—> stars, SFR would be much too high. SF must either take a long time or be highly inefficient
- How long do molecular clouds live?
  - 5-30 Myr on average. Implication is that star formation is inefficient. Only a small fraction of a molecular cloud ever gets converted into stars. Once high-mass stars form — molecular clouds are rapidly dispersed
- Cloud fragmentation
  - Jeans' classical analysis predicts a critical length-scale and mass-scale above which gravitational collapse sets in. Fragmentation requires initial seeding of density fluctuations

Sternentstehung - Star Formation

Winter term 2022/2023
Henrik Beuther, Thomas Henning, & Jonathan Henshaw

18.10 - Introduction & overview (Beuther)
25.10 - Physical processes I (Beuther)
08.11 - Physical processes II (Beuther)
15.11 - Molecular clouds I: the birth places of stars (Henshaw)
22.11 - Molecular clouds II: Jeans analysis (Henshaw)
29.11 - Collapse models I (Beuther)
06.12 - Collapse models II (Henning)
13.12 - Protostellar evolution (Beuther)
20.12 - Pre-main sequence evolution & outflows/jets (Beuther)
10.01 - Accretion disks I (Henning)
17.01 - Accretion disks II (Henning)
24.01 - High-mass star formation, clusters & the IMF (Henshaw)
31.01 - Extragalactic star formation (Henning)
07.02 - Planetarium @ HdA, outlook, questions
13.02 - Examination week, no star formation lecture

Book: Stahler & Palla: The Formation of Stars, Wileys

More information and the current lecture files: https://www2.mpia-hd.mpg.de/homes/beuther/lecture_ws2223.html
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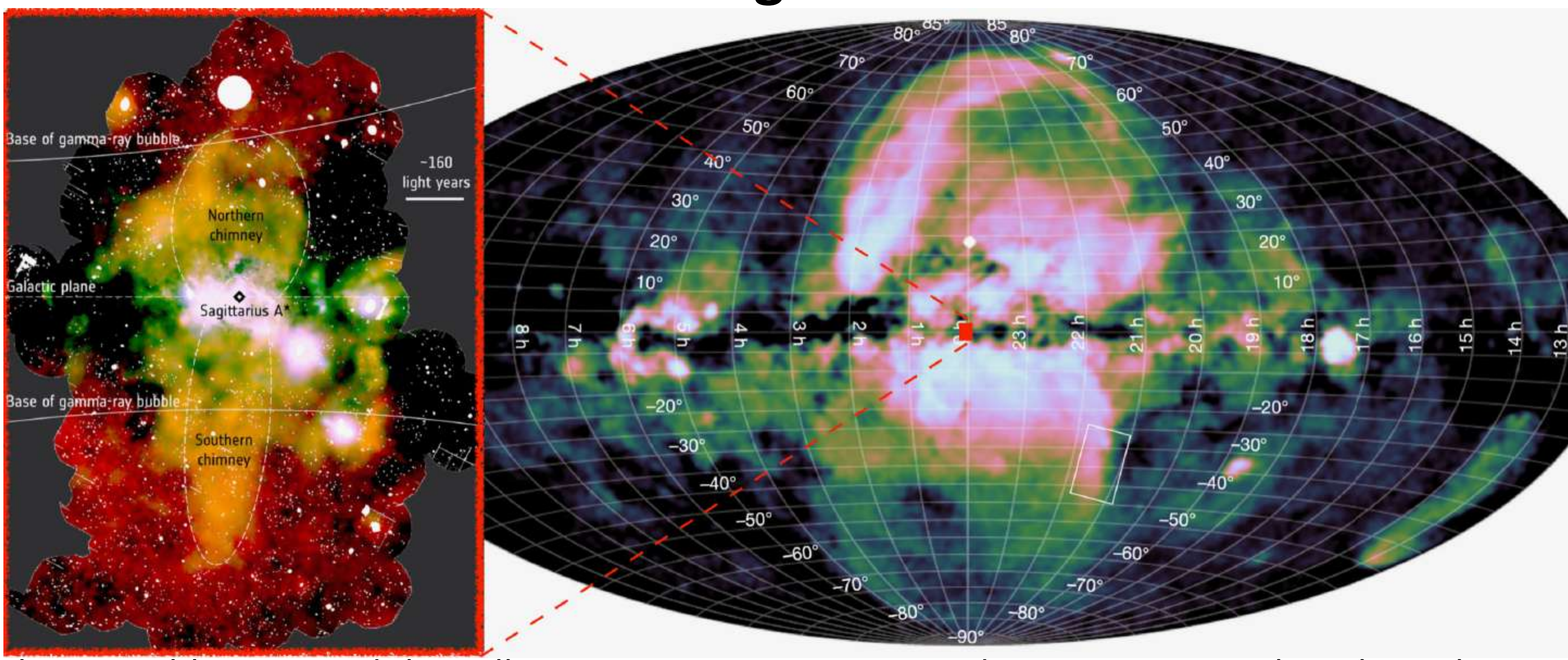
Heidelberg Joint Astronomical Colloquium

Winter Semester 2022 — Tuesday November 22nd, 16:00

Main Lecture Theatre, Philosophenweg 12

Gabriele Ponti (Osservatorio Astronomico di Brera)

Our X-ray view of the Milky Way center, its outflow, and the Circumgalactic Medium



Those unable to attend the colloquium in person are invited to participate online through Zoom.
More information is given on HePhySTO: https://www.physik.uni-heidelberg.de/hephysto/