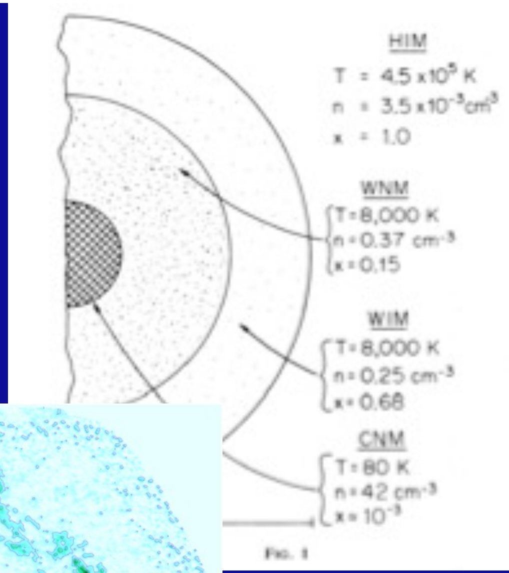
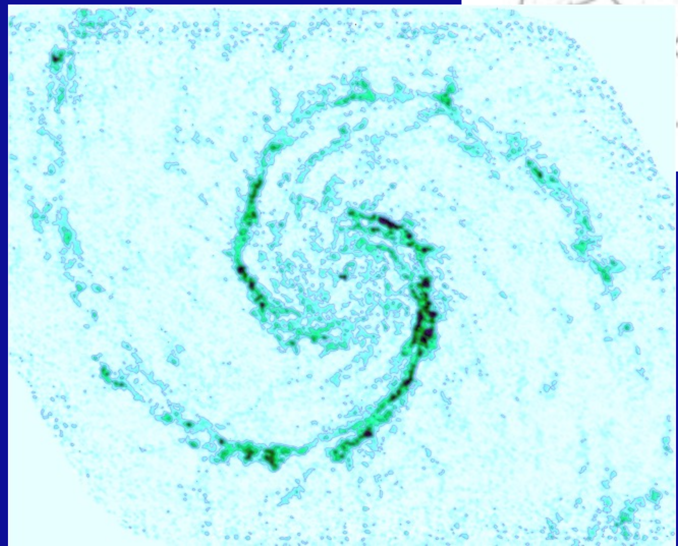
Last week

- Different components of ISM, early models

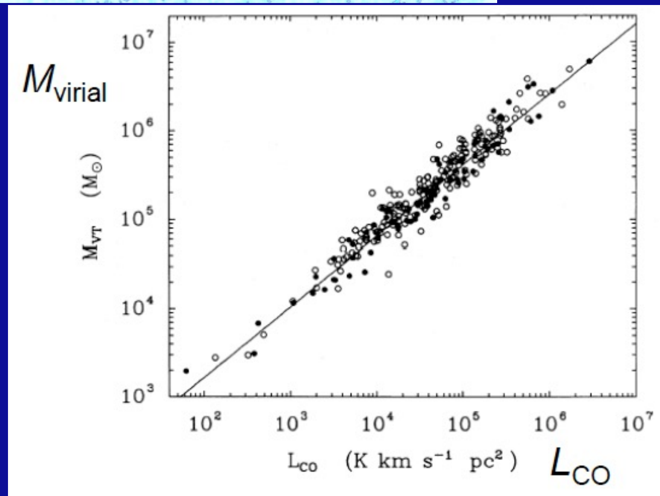


- Basic characteristics of mol. clouds (10-20 K, 2-3 km/s)



- 20% of gas is in molecular form; Most of the mass in massive GMCs

- Important cloud scaling relations Larson relation: σ(v) ∽ $L^{1/2}$



- Cloud fragmentation Clouds are highly structured – filamentary structure

- All stars in the MW born in molecular gas

Topics today

- Virial theorem
- Jeans analysis for gravitational instability
- Magnetic fields
- Cloud formation and turbulence

Virial Analysis

Remember: Virial Equlibrium in Statistical Mechanics

$2 \langle T \rangle + \langle W \rangle = 0$ in closed system (time-averaged quantities)

(Rudolf Clausius 1870, Poincare, Eddington, Chandrasekhar ….)

Here formulated with Newton‘s gravitational potential.

We can perform a more general analysis:

Starting from the Euler equation for an incompressible fluid:

What is the force balance within any structure in hydrostatic equilibrium?
Euler equation including magnetic fields **B** acting on a current **j** and the full convective fluid velocity **v** is:

$$\rho\, D\mathbf{v}/Dt = -\mathrm{grad}(P) - \rho\, \mathrm{grad}(\Phi_g) + 1/c\, \mathbf{j} \times \mathbf{B}$$

$D\mathbf{v}/Dt = (\partial \mathbf{v}/\partial t)_x + (\mathbf{v}\ \mathrm{grad})\mathbf{v}$

$1/2(\partial^2 I/\partial t^2)$ -2T 2U W *M*

($D\mathbf{v}/D\mathbf{t}$ includes the rate of change at fixed spatial position x $(\partial \mathbf{v}/\partial t)_x$ and the change induced by transporting elements to a new location with differing velocity.)

Employing the Poisson equation ($\Delta\Phi_g = 4\pi G\rho$) and requiring mass conservation, one gets after repeated integrations the **VIRIAL THEOREM**

$$1/2\ (\delta^2 I/\delta t^2) = 2T + 2U + W + \mathcal{M}$$

I: Moment of inertia, this decreases when a core is collapsing ($m*r^2$)
T: Kinetic energy U: Thermal energy W: Gravitational energy *M*: Magnetic energy
All terms except W are positive. To keep the cloud stable, the other forces have to match W. Then the left term becomes Zero. No surface terms considered.

Application of the Virial Theorem I

If all forces are too weak to match the gravitational energy, we get

$1/2\ (\delta^2 I/\delta t^2) = W \sim Gm^2/r$

Approximating further $I=mr^2$, the free-fall time is approximately

$t_{ff} \sim \mathrm{sqrt}(r^3/Gm)$

Since the density can be approximated by $\rho=m/r^3$, one can also write

$t_{ff} \sim (G\rho)^{-1/2}$

Or more exactly for a pressure-free 3D homogeneous sphere

$t_{ff} = (3\pi/32G\rho)^{1/2}$

For a giant molecular cloud, this corresponds to

$t_{ff} \sim 7 * 10^6\ \mathrm{yr}\ (m/10^5 M_{sun})^{-1/2}\ (R/25\mathrm{pc})^{3/2}$

For a dense core with $\rho \sim 10^5 \mathrm{cm}^{-3}$ the t_{ff} is approximately 10^5 yr.

However, no globally collapsing GMCs observed → add support!

Two comments:

- Free-fall time for sphere without thermal pressure:
  $d^2r/dt^2 = -G\,M(R)/r^2$ (integrate this equation)

  Free-fall time means that all mass shells reach the center at the same time

- Let us take a free-fall time for the GMCs of 10^7 yr and a total mass of molecular gas in the MW of $2.5\times10^9\ M_{sun}$ then the SFR would be:

  $250\ M_{sun}$/yr compared to the measured SF of $1\ M_{sun}$/yr

We have two possibilities:

Molecular clouds are very long lived or SF is inefficient (feedback)

Argument 1: Large-scale distribution of molecular gas relative to atomic gas and arm-to-interarm contrast: 10^8 yr
Argument 2: Measured lifetimes of clouds: 5-30 Myr: SF is not efficient

Application of the Virial Theorem II

If the cloud complexes are in approximate force equilibrium, the moment of inertia actually does not change significantly and hence $1/2\ (\delta^2 I/\delta t^2)=0$

$$2T + 2U + W + \mathcal{M} = 0$$

This state is called VIRIAL EQUILIBRIUM. What balances gravitation W best?

Thermal Energy: Approximating U by $U \sim 3/2Nk_BT \sim m\mathcal{R}T/\mu$

$$U/|W| \sim m\mathcal{R}T/\mu\ (Gm^2/R)^{-1}$$

$$= 3{*}10^{-3}\ (m/10^5M_{sun})^{-1}\ \ (R/25pc)\ \ (T/15K)$$

--> Clouds cannot be supported by thermal pressure alone!

Magnetic energy: Approximating $\mathcal{M}$ by $M \sim B^2r^3/6$ (cloud approximated as sphere)

$$\mathcal{M}/|W| \sim B^2r^3/6\ (Gm^2/R)^{-1}$$

$$= 0.3\ (B/20\mu G)^2\ (R/25pc)^4\ \ (m/10^5M_{sun})^{-2}$$

--> Magnetic force is important for large-scale cloud stability!

Application of the Virial Theorem III

The last term to consider in $2T + 2U + W + \mathcal{M} = 0$ is the kinetic energy T

$$T/|W| \sim 1/2 m\Delta v^2 (Gm^2/R)^{-1}$$
$$= 0.5\ (\Delta v/4 km/s)\ \ (M/10^5 M_{sun})^{-1}\ \ (R/25pc)$$

Since the shortest form of the virial theorem is $2T = -W$, the above numbers imply that a typical cloud with linewidth of a few km/s is in approximate virial equilibrium.

The other way round, one can derive an approximate relation between the observed line-width and the mass of the cloud:

$$2T = 2 * (1/2 m\Delta v^2) = -W = Gm^2/r$$

→ virial velocity: $v_{vir} = (Gm/r)^{1/2}$

→ or virial mass: $m_{vir} = v^2 r/G$

Application of the Virial Theorem III

The last te … energy T

… /25pc)

Since the sh … e numbers imply that … roximate

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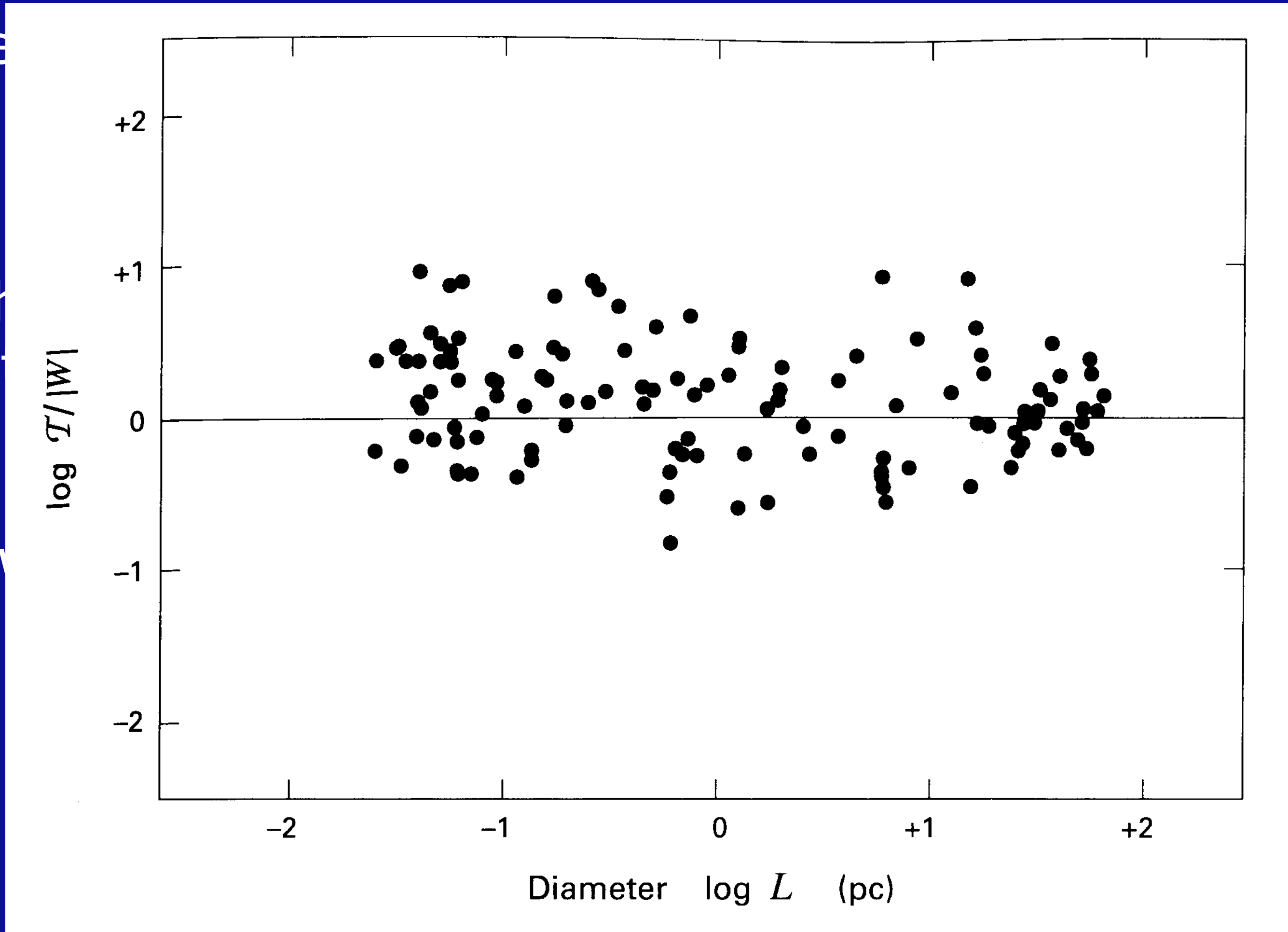

Stability of Molecular Clouds (Summary)

What helps to balance gravity?

Thermal Energy (using $U = \frac{3}{2} N k_B T \sim \frac{M \mathscr{R} T}{\mu}$):

$$\frac{U}{|\mathscr{W}|} \approx \frac{M \mathscr{R} T}{\mu} \left(\frac{G M^2}{R} \right)^{-1} \approx 3 \times 10^{-3} \left(\frac{M}{10^5\,\mathrm{M}_\odot} \right)^{-1} \left(\frac{R}{25\,\mathrm{pc}} \right) \left(\frac{T}{15\,K} \right)$$

Magnetic Energy (using $\mathscr{M} = \frac{|\mathbf{B}|^2 R^3}{6}$):

$$\frac{\mathscr{M}}{|\mathscr{W}|} \approx \frac{|\mathbf{B}|^2 R^3}{6} \left(\frac{G M^2}{R} \right)^{-1} \approx 0.9 \left(\frac{M}{10^5\,\mathrm{M}_\odot} \right)^{-2} \left(\frac{R}{25\,\mathrm{pc}} \right)^4 \left(\frac{B}{20\,\mu G} \right)^2$$

Kinetic Energy (using $\mathscr{T} = \frac{1}{2} M v^2$):

$$\frac{\mathscr{T}}{|\mathscr{W}|} \approx \frac{1}{2} M v^2 \left(\frac{G M^2}{R} \right)^{-1} \approx 0.5 \left(\frac{M}{10^5\,\mathrm{M}_\odot} \right)^{-1} \left(\frac{R}{25\,\mathrm{pc}} \right) \left(\frac{v}{4\,\mathrm{km\,s^{-1}}} \right)^2$$

Stability of Molecular Clouds

THE ENERGY DISSIPATION RATE OF SUPERSONIC, MAGNETOHYDRODYNAMIC TURBULENCE IN MOLECULAR CLOUDS

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ABSTRACT

Molecular clouds have broad line widths, which suggests turbulent supersonic motions in the clouds. These motions are usually invoked to explain why molecular clouds take much longer than a free-fall time to form stars. Classically, it was thought that supersonic hydrodynamical turbulence would dissipate its energy quickly but that the introduction of strong magnetic fields could maintain these motions. A previous paper has shown, however, that isothermal, compressible MHD and hydrodynamical turbulence decay at virtually the same rate, requiring that constant driving occur to maintain the observed turbulence. In this paper, direct numerical computations of uniform, randomly driven turbulence with the ZEUS astrophysical MHD code are used to derive the value of the energy-dissipation coefficient, which is found to be

$$\dot{E}_{\rm kin} \simeq -\eta_v m \tilde{k} v_{\rm rms}^3 ,$$

with $\eta_v = 0.21/\pi$, where $v_{\rm rms}$ is the root-mean-square (rms) velocity in the region, $E_{\rm kin}$ is the total kinetic energy in the region, m is the mass of the region, and $\tilde{k}$ is the driving wavenumber. The ratio τ of the formal decay time $E_{\rm kin}/\dot{E}_{\rm kin}$ of turbulence to the free-fall time of the gas can then be shown to be

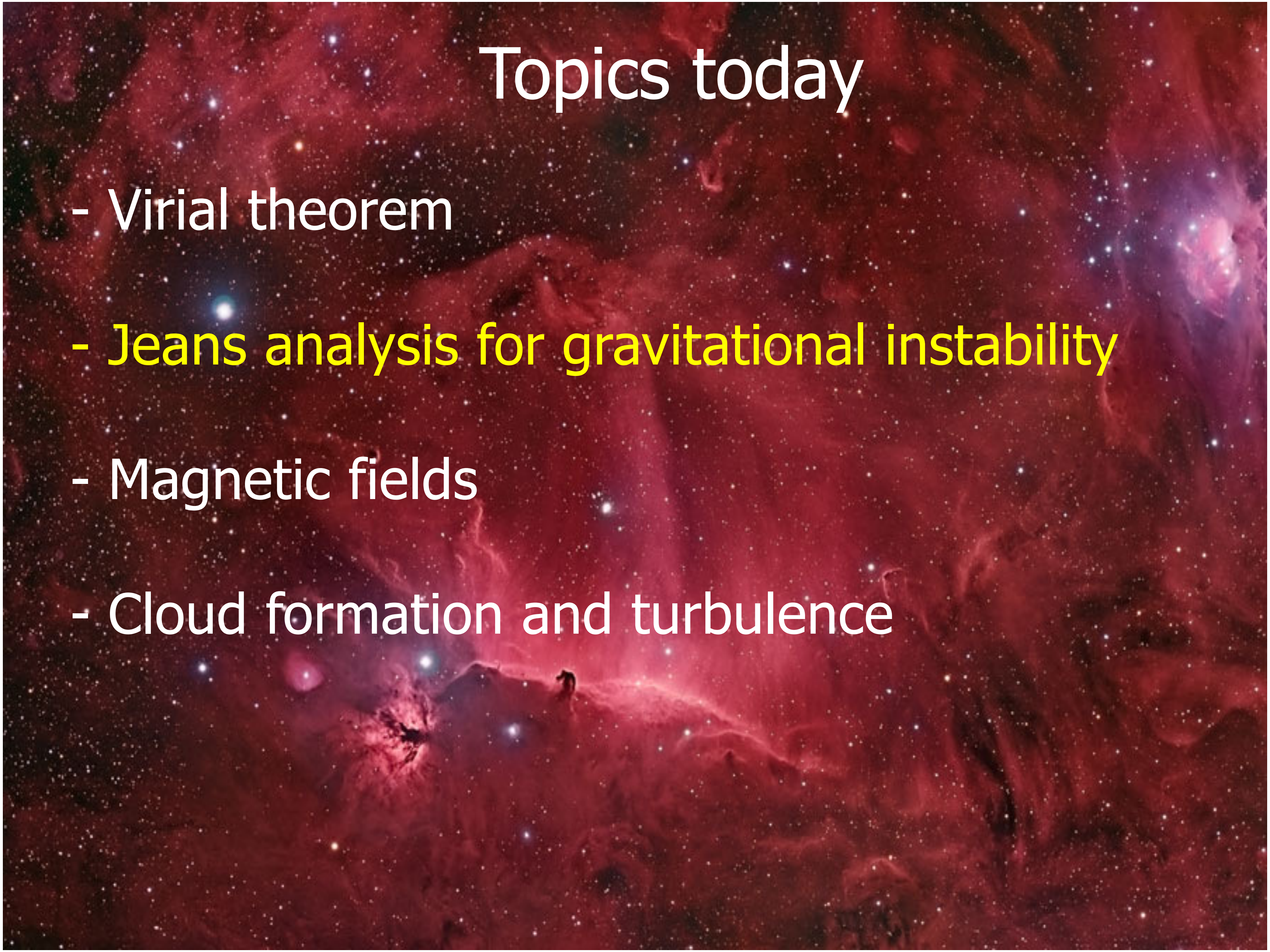
$$\tau(\kappa) = \frac{\kappa}{M_{\rm rms}} \frac{1}{4\pi\eta_v} ,$$

where $M_{\rm rms}$ is the rms Mach number, and κ is the ratio of the driving wavelength to the Jeans wavelength. It is likely that $\kappa < 1$ is required for turbulence to support gas against gravitational collapse, so the decay time will probably always be far less than the free-fall time in molecular clouds, again showing that turbulence there must be constantly and strongly driven. Finally, the typical decay time constant of the turbulence can be shown to be

$$t_0 \simeq 1.0\, \mathscr{L}/v_{\rm rms} ,$$

where $\mathscr{L}$ is the driving wavelength.

Turbulence needs continuous driving!


Topics today
- Virial theorem
- Jeans analysis for gravitational instability
- Magnetic fields
- Cloud formation and turbulence

Jeans analysis I

Start again with Euler equation (without magn. Field):

Equation of motion: $\rho\, D\mathbf{v}/Dt = -\mathrm{grad}(P) - \rho\, \mathrm{grad}(\Phi_g)$

$$\rho\, (\partial \mathbf{v}/\partial t) + (\mathbf{v}\, \mathrm{grad})\mathbf{v} = -\mathrm{grad}(P) - \rho\, \mathrm{grad}(\Phi_g)$$

Continuity equation: $(\partial \rho/\partial t) = -\mathrm{grad}(\rho \mathbf{v})$

Poisson equation: $\Delta \Phi_g = 4\, \pi\, G\, \rho$

Static solution: $\rho = \rho_0 = \mathrm{const}$; $P = P_0 = \mathrm{const}$; $\mathbf{v} = \mathbf{v_0} = \mathrm{const}$; $\Phi_g = \Phi_0 = \mathrm{const}$

Little pertubation: linear stability analysis:

$$\rho = \rho_0 + \rho_1;\ P = P_0 + P_1;\ \mathbf{v} = \mathbf{v_0} + \mathbf{v_1};\ \Phi_g = \Phi_0 + \Phi_1$$

(with $|\rho_1| << \rho_0$ etc.)

Jeans analysis II

Considering only expressions of first order:

$$\partial \mathbf{v_1}/\partial t = -1/\rho_0 \, \mathrm{grad}(P_1) - \mathrm{grad}(\Phi_1) \qquad \text{(Eq. 1)}$$

$$\partial \rho_1/\partial t = -\rho_0 \, \mathrm{grad}(\mathbf{v_1}) \qquad \text{(Eq. 2)}$$

$$\Delta\Phi_1 = 4\pi G\rho_1 \qquad \text{(Eq. 3)}$$

Using furthermore: $P_1 = c_s^2 \rho_1$ and $c_s^2 = kT/(\mu m_H)$

(c_s sound speed; μ mean mass of particle; m_H mass of hydrogen)

Apply grad to Eq. 1: $\mathrm{grad}(\partial \mathbf{v_1}/\partial t) = -\Delta\,(c_s^2\rho_1/\rho_0 + \Phi_1)$

Time derivative of Eq. 2: $\partial^2\rho_1/\partial t^2 = -\rho_0 \, \mathrm{grad}(\partial \mathbf{v_1}/\partial t)$

$$\partial^2\rho_1/\partial t^2 = c_s^2 \Delta\rho_1 + 4\pi G\rho_0\rho_1$$

What type of equation did we obtain?

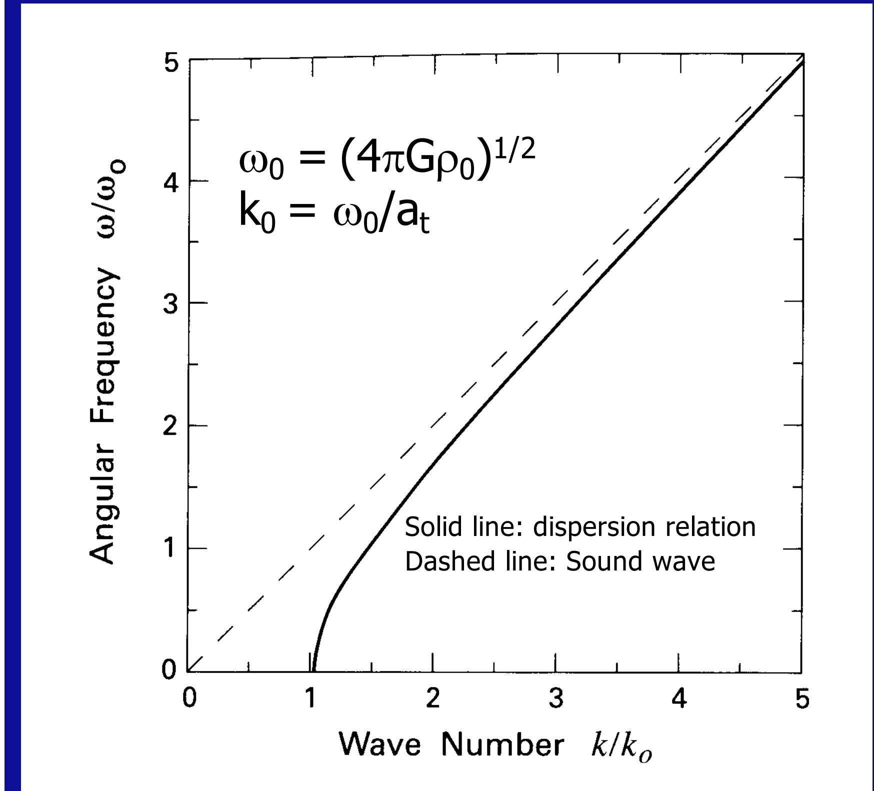
Jeans analysis III

- A traveling wave in an isothermal gas can be described as:

$\rho_1(x,t) = \rho_1^* \exp[i(kx - \omega t)]$ — wave number $k=2\pi/\lambda$ and frequency ω

Then: $\partial^2\rho_1/\partial t^2 = -\omega^2\rho_1$ and $\Delta\rho_1 = -k^2\rho_1$

- This results in *dispersion equation:* $\omega^2 = k^2 c_s^2 - 4\pi G\rho_0$



Large k high-frequency disturbances
→ sound wave $\omega=k\,c_s$
→ isothermal sound speed of background

Low k $(k<=k_0)$ $\omega^2 \to 0$.
→ Jeans-length: $\lambda_J = 2\pi/k_0 = (\pi\, c_s^2/G\rho_0)^{1/2}$

Perturbations larger λ_J have exponentially growing amplitudes → instable

Jeans analysis IV

For $\omega^2 < 0$: time part of wave solution becomes real and with Im(ω) < 0 will density perturbation ρ_1 grow for fixed x: Jeans instability

For $k^2 < 4 \pi G \rho_0 / c_s^2$ Instability or in the Length scale

$$\lambda_J = (\pi c_s^2/G\rho_0)^{1/2} = 0.19\text{pc}\,(T/(10K))^{1/2}\,(n_{H2}/(10^4 cm^{-3})^{-1/2}$$

and Jeans-mass

$$M_J = m_1 c_s^3/(\rho_0^{1/2} G^{3/2}) = 1.0 M_{sun}\,(T/(10K))^{3/2}\,(n_{H2}/(10^4 cm^{-3})^{-1/2}$$

→ Clouds larger λ_J or more massive than M_J may be prone to collapse and fragmentation.

→ Conversely, small or low-mass cloudlets could be stable if there is sufficient external pressure. Otherwise only transient objects.

Jeans analysis V

Examples:

Small HI cloud:

T ~ 100K; n_H ~ 20 cm^{-3}; L ~ 5pc; M ~ 20M_{sun}

→ L_J ~ 13pc

→ Jeans stable

Giant Molecular Cloud (GMC)

T=10K and $n_{H2}=10^3 cm^{-3}$

→ M_J = 3.2 M_{sun}

Orders of magnitude too low → Jeans unstable

→ Additional support necessary, e.g., magnetic field, turbulence …

Jeans fragmentation has an issue:

- Growth rates increase for decreasing wavenumber
- This implies: Fastest growing mode of gravitational instability would be overall (!!!) collapse of the medium and small perturbations do not have the time to collapse before the cloud.

Solutions to the problem:

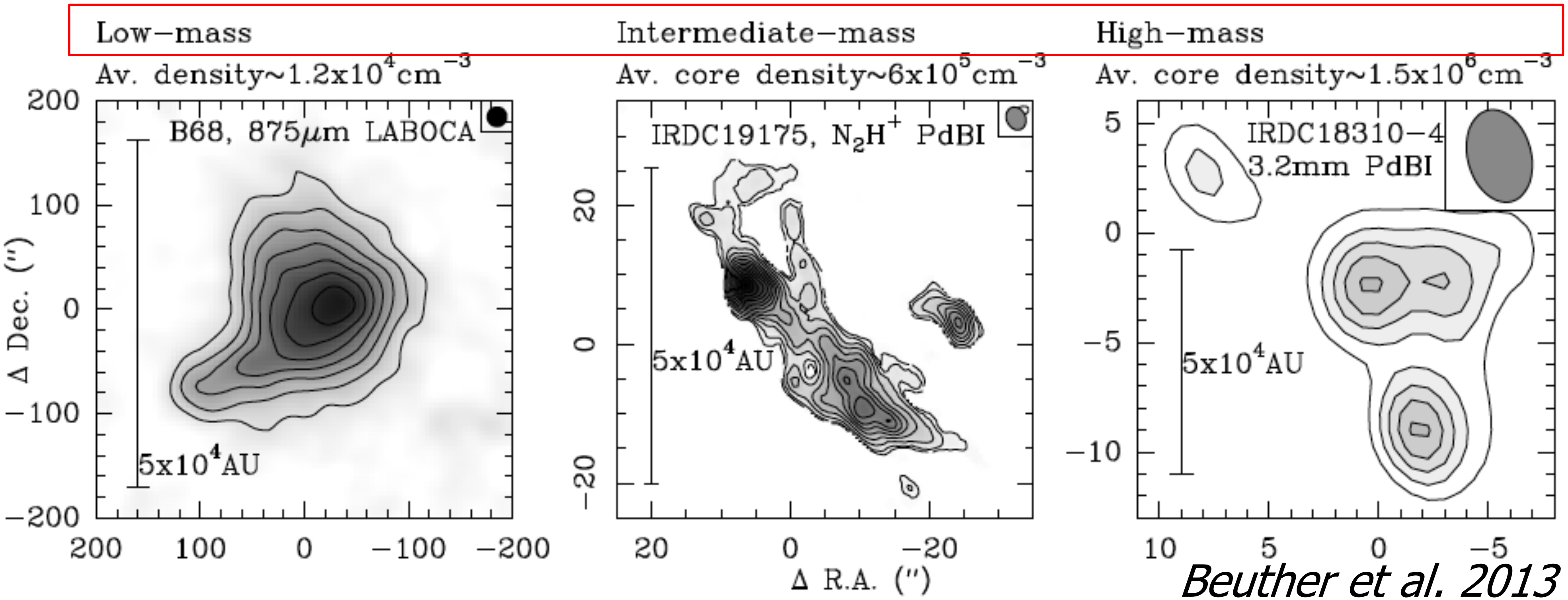
Supersonic turbulence drives the formation of density fluctuations which then can lead to individually collapsing cloud parts.

A few issues to remember:

- Molecular clouds have hierarchical structure
- Linear stability analysis – ISM is turbulent and compressible
- Observations and numerical simulations indicate that fragmentation is a non-linear process
  (strong non-linearities in density, velocity, magnetic fields)
- Radiative transfer not considered (cooling/heating)

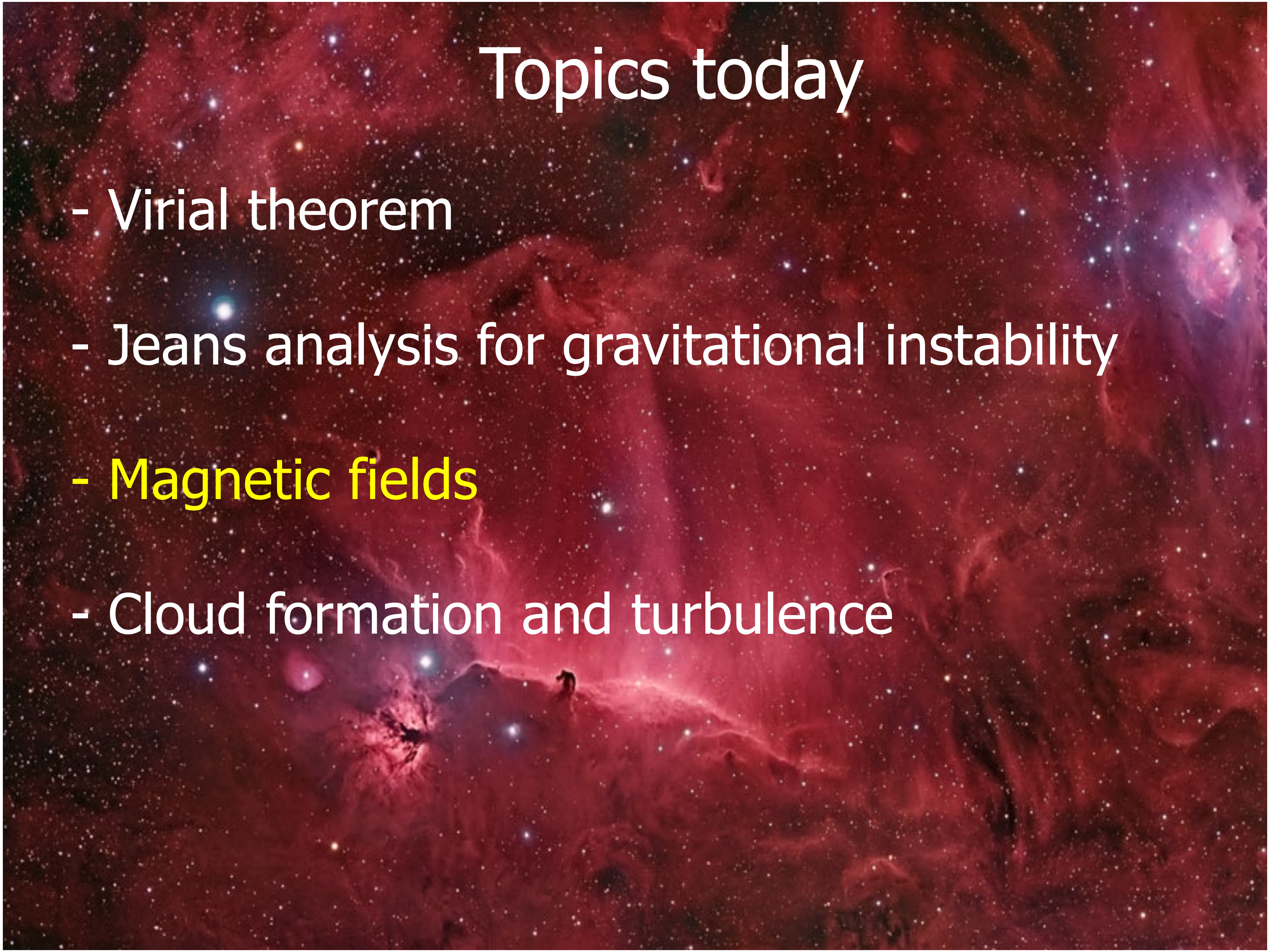
Numerical simulations very important!

Jeans fragmentation in star formation



Beuther et al. 2013

Numerical Simulation of Cloud Evolution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Magnetic fields I

Object	Type	Diagnostic	$\lvert B_{\parallel} \rvert$ [μG]
Ursa Major	Diffuse cloud	HI	10
NGC2024	GMC clump	OH	87
S106	HII region	OH	200
W75N	Maser	OH	3000

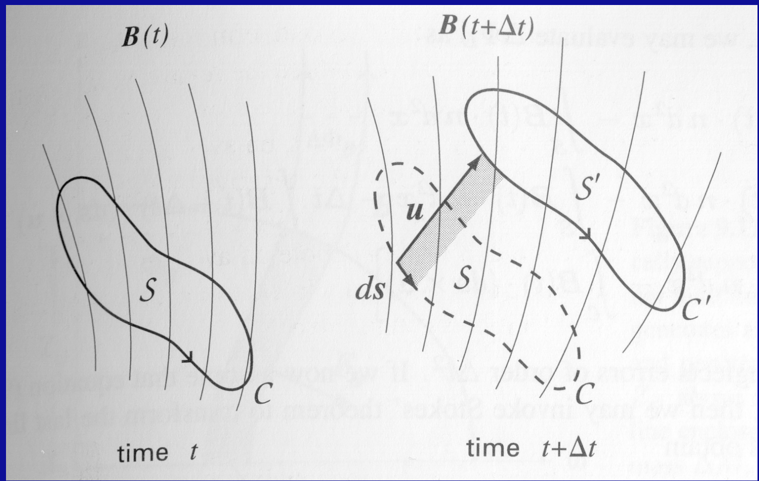
Increasing magnetic field strength with increasing density indicate "field-freezing" between B-field and gas

(B-field couples to ions and electrons, and these via collisons to neutral gas).

Magnetic fields II

This field freezing can be described by ideal MHD:

$$\frac{dB}{dt} = \nabla \times (\mathbf{u} \times B)$$



However, ideal MHD must break down at some point.

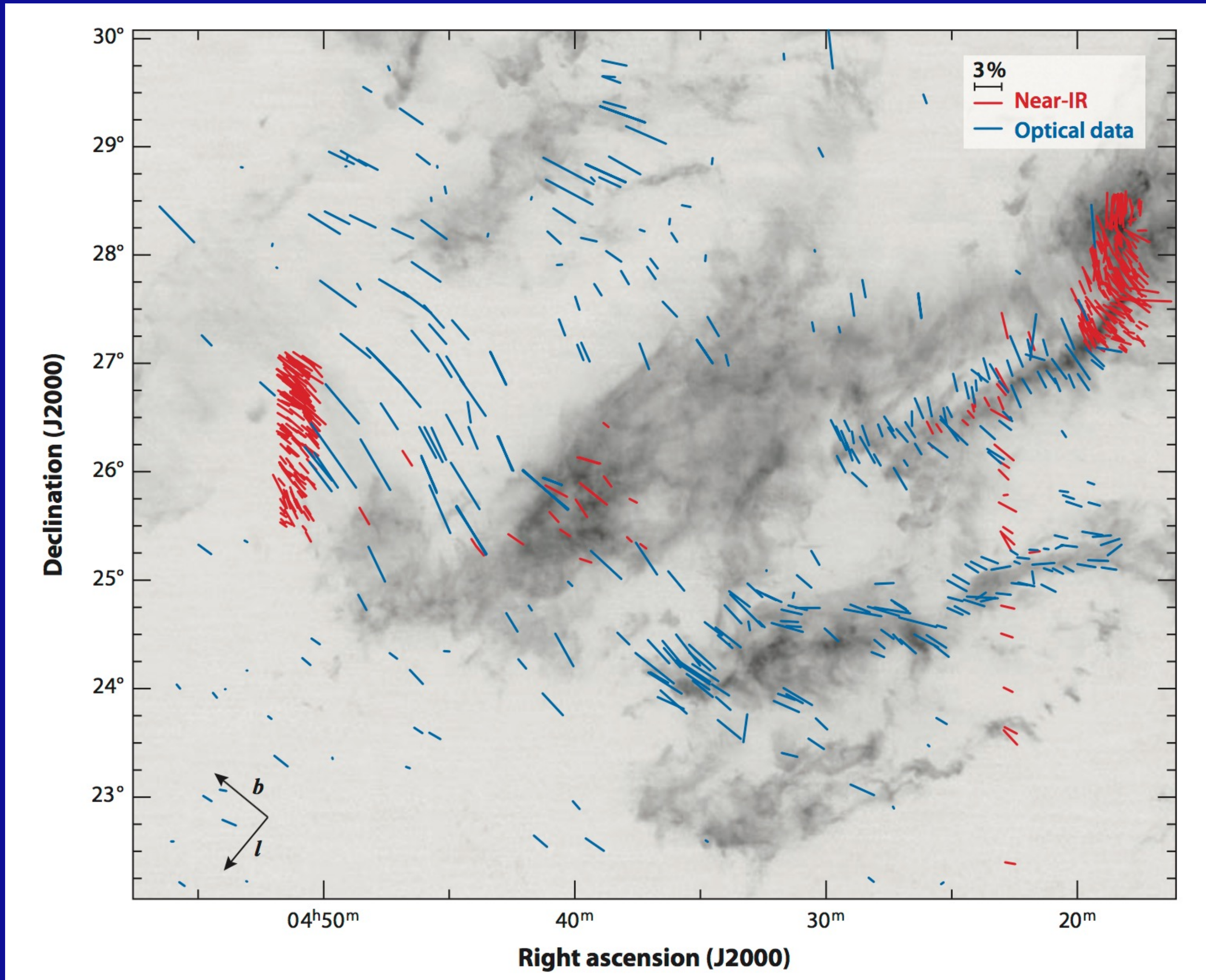
Dense core: $1M_{sun}$, R_0=0.07pc, B_0=30μG *versus* T Tauri star: $R_1=5R_{sun}$

If flux-freezing → magnetic flux $\Phi_M = \pi BR^2$ should remain constant:

→ $B_1=2\times10^7$ G, which exceeds observed values by orders of magnitude

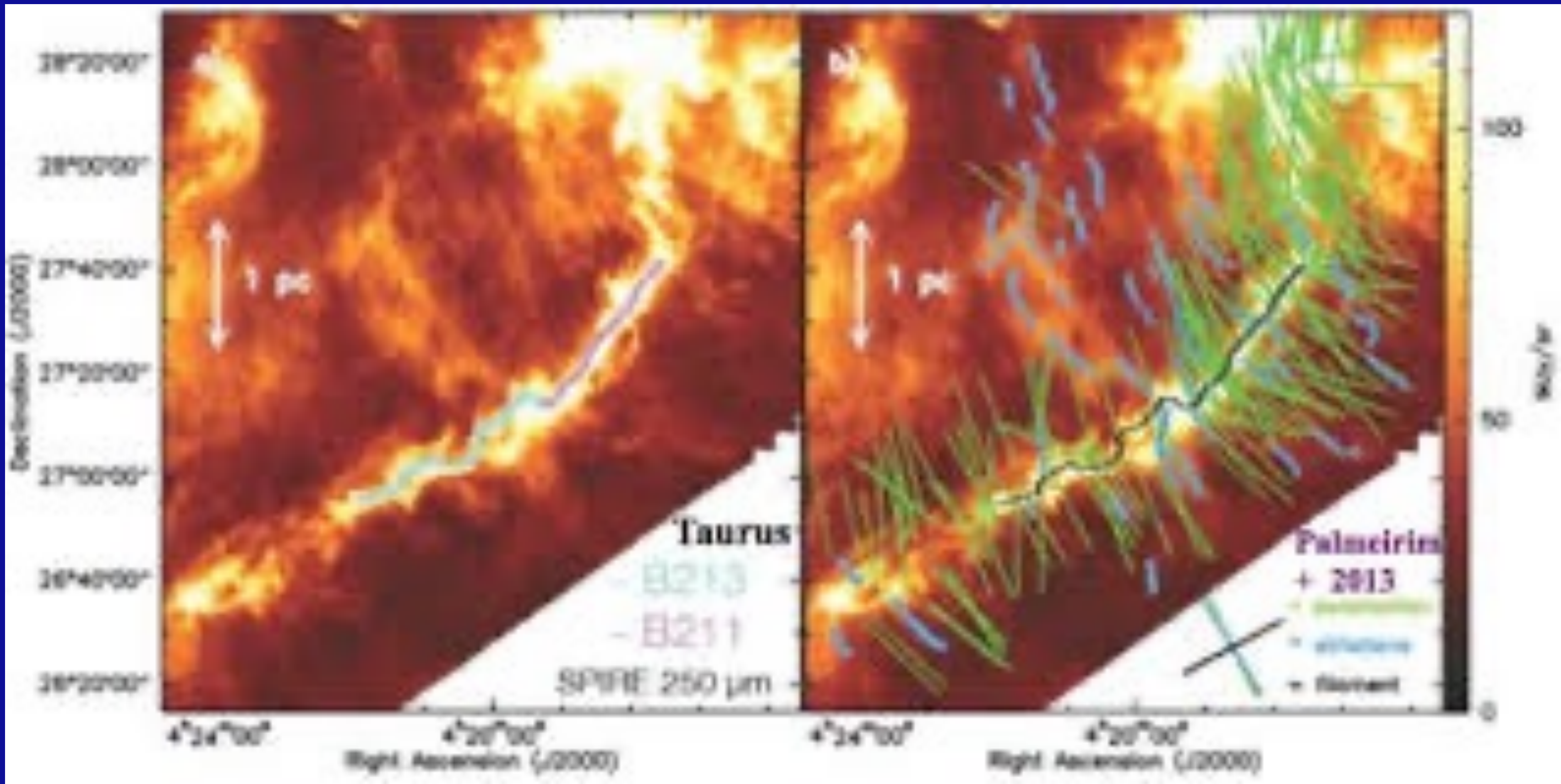
Ambipolar diffusion: neutral and ionized medium decouple, and neutral gas can sweep through during the gravitational collapse.

Magnetic fields morphology in Taurus

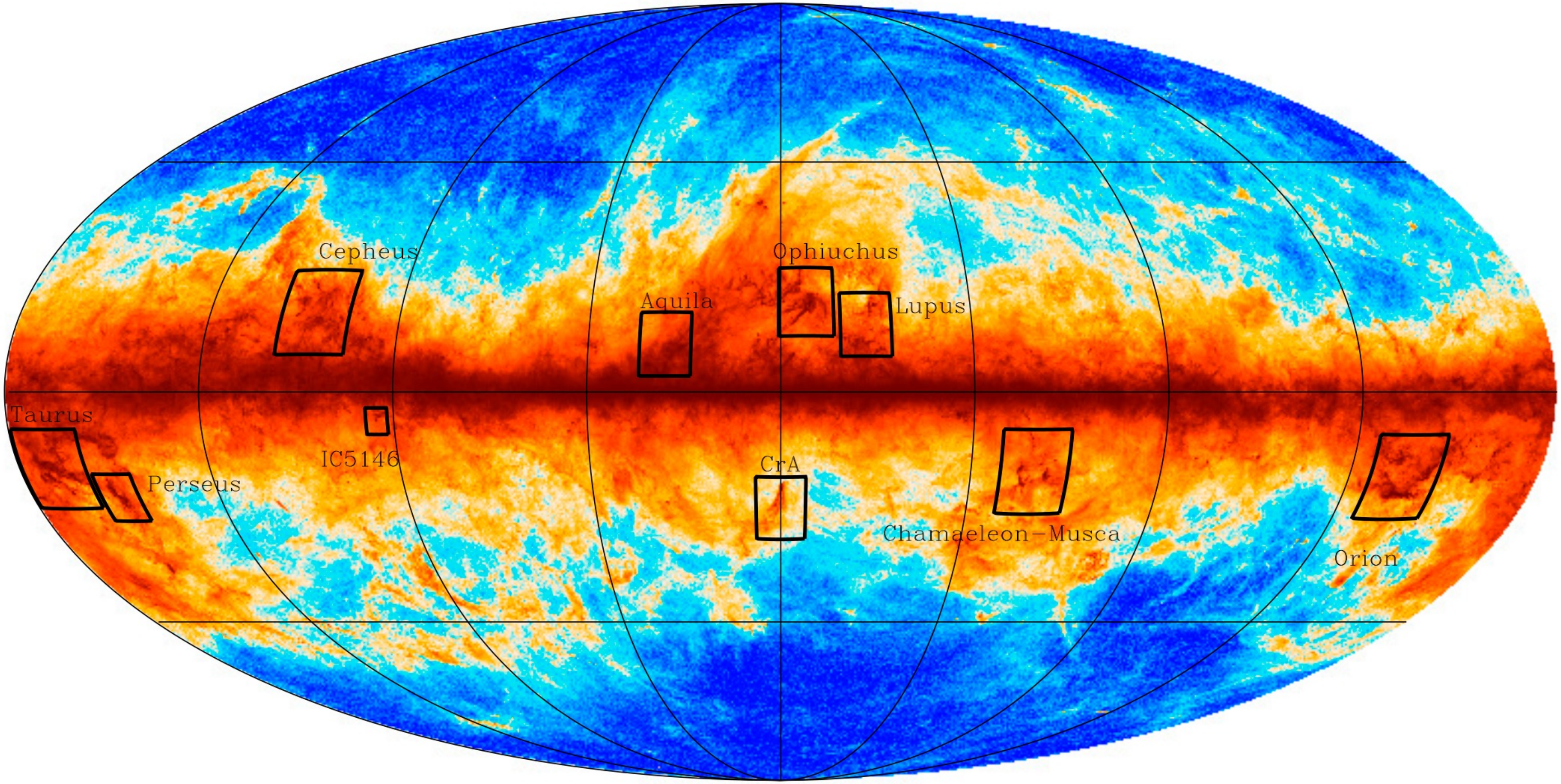


Grey: ^{13}CO; line segments: optical polarization *Chapman et al. 2011*

Magnetic fields morphology in Taurus filament

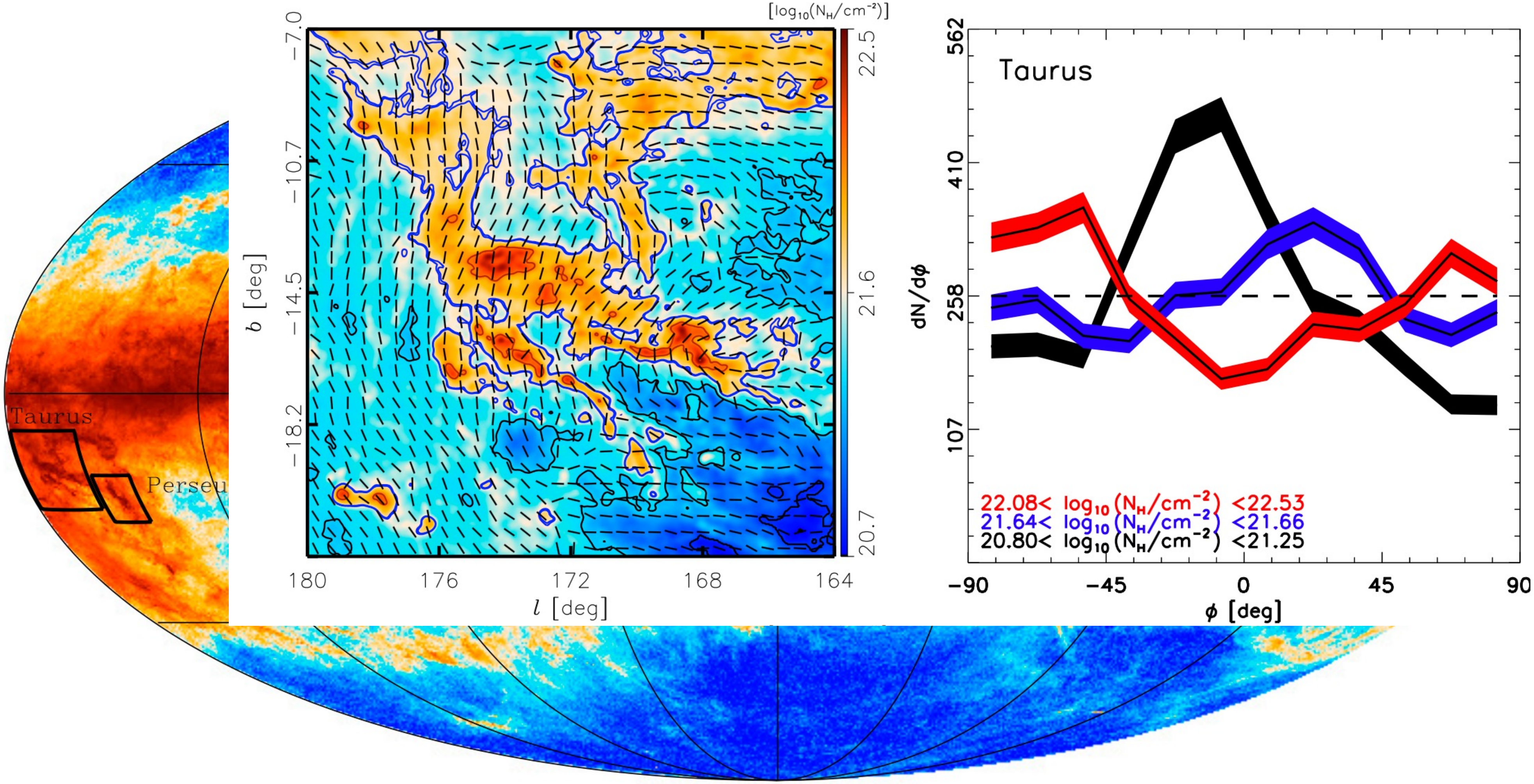


Planck and the magnetic field



Soler et al. 2015

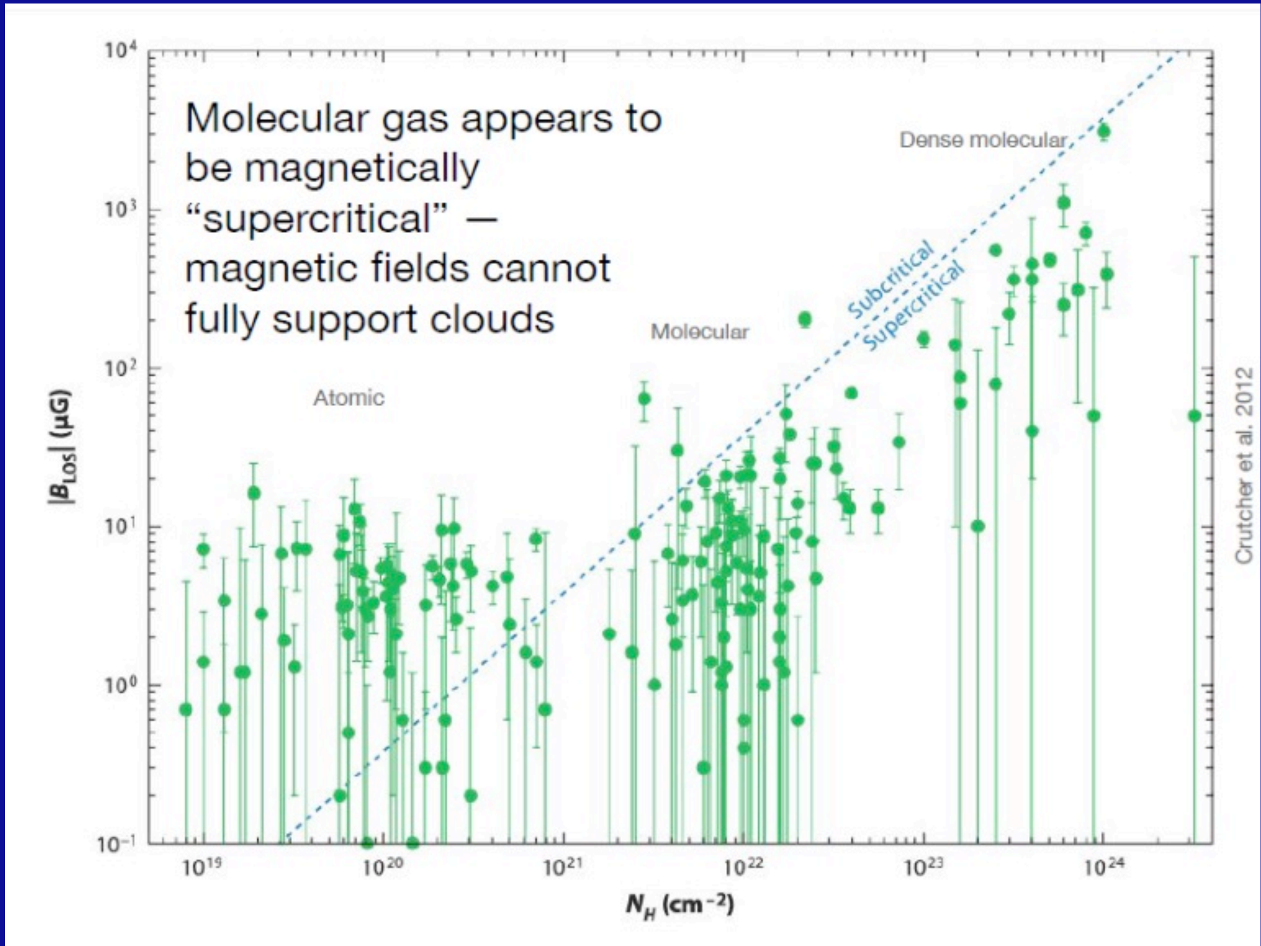
Planck and the magnetic field



Soler et al. 2015

Magnetic fields strength

HI, OH & CN Zeeman



Crutcher (2012)

Jeans-like analysis: $M_{cr} = 1000 M_{sun} (B/(30\mu G)) (R/(2pc)^2$

$M<M_{cr}$ magnetically subcritical; $M>M_{cr}$ magnetically supercritical

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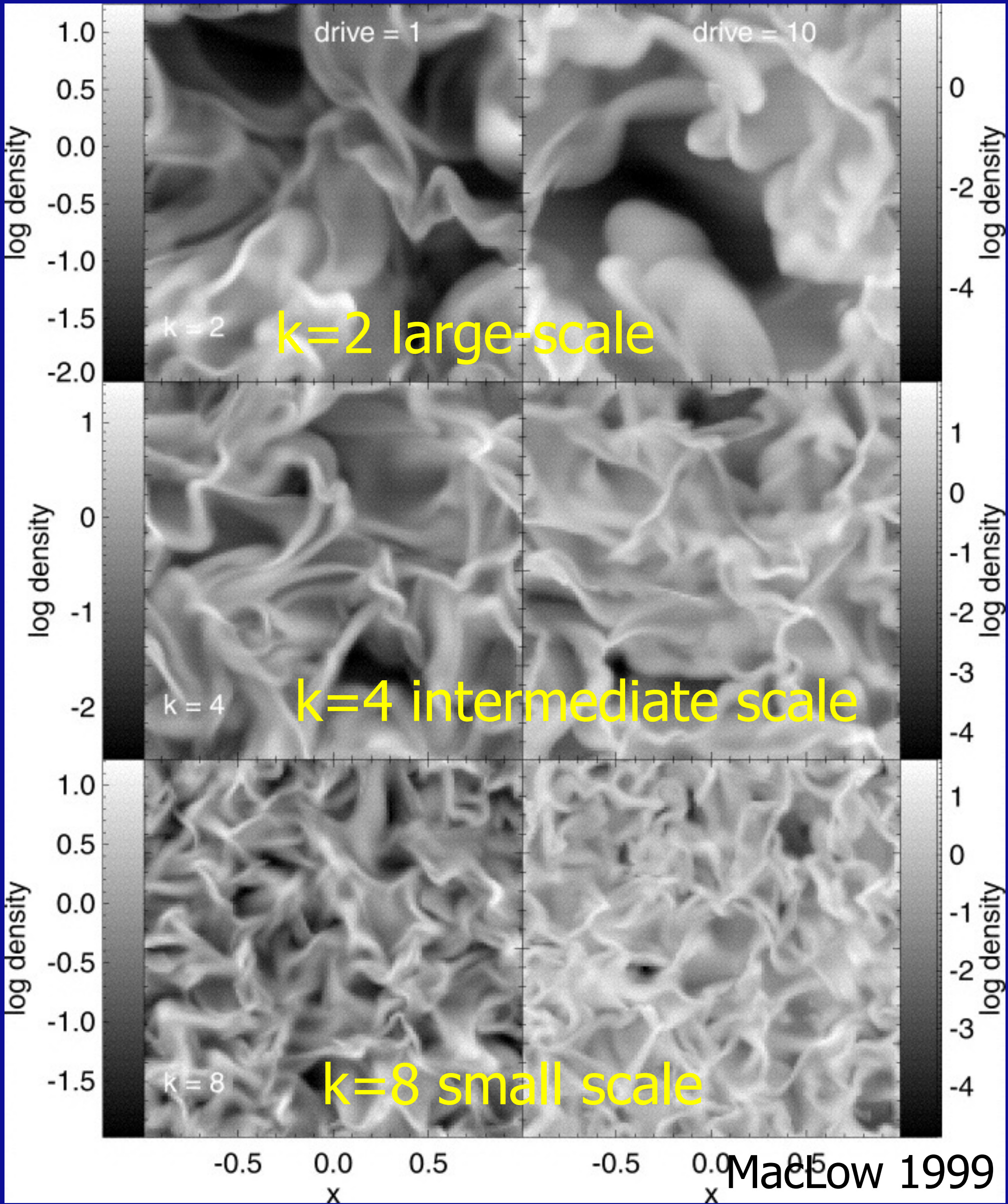
Interstellar Turbulence



- Supersonic → network of shocks

→ Density fluctuations $\delta\rho \propto \mathcal{M}^2$

→ Molecular H_2 can form

- t_{form} = 1.5x10^9yr / (n/1cm^{-3})
  (Hollenbach et al. 1971)

→ either molecular clouds form
slowly in low-density gas or
rapidly in ~10^5yr in n=$10^4$$cm^{-3}$

- Decays on time-scales of order
  the free-fall time-scale

→ Needs continuous driving

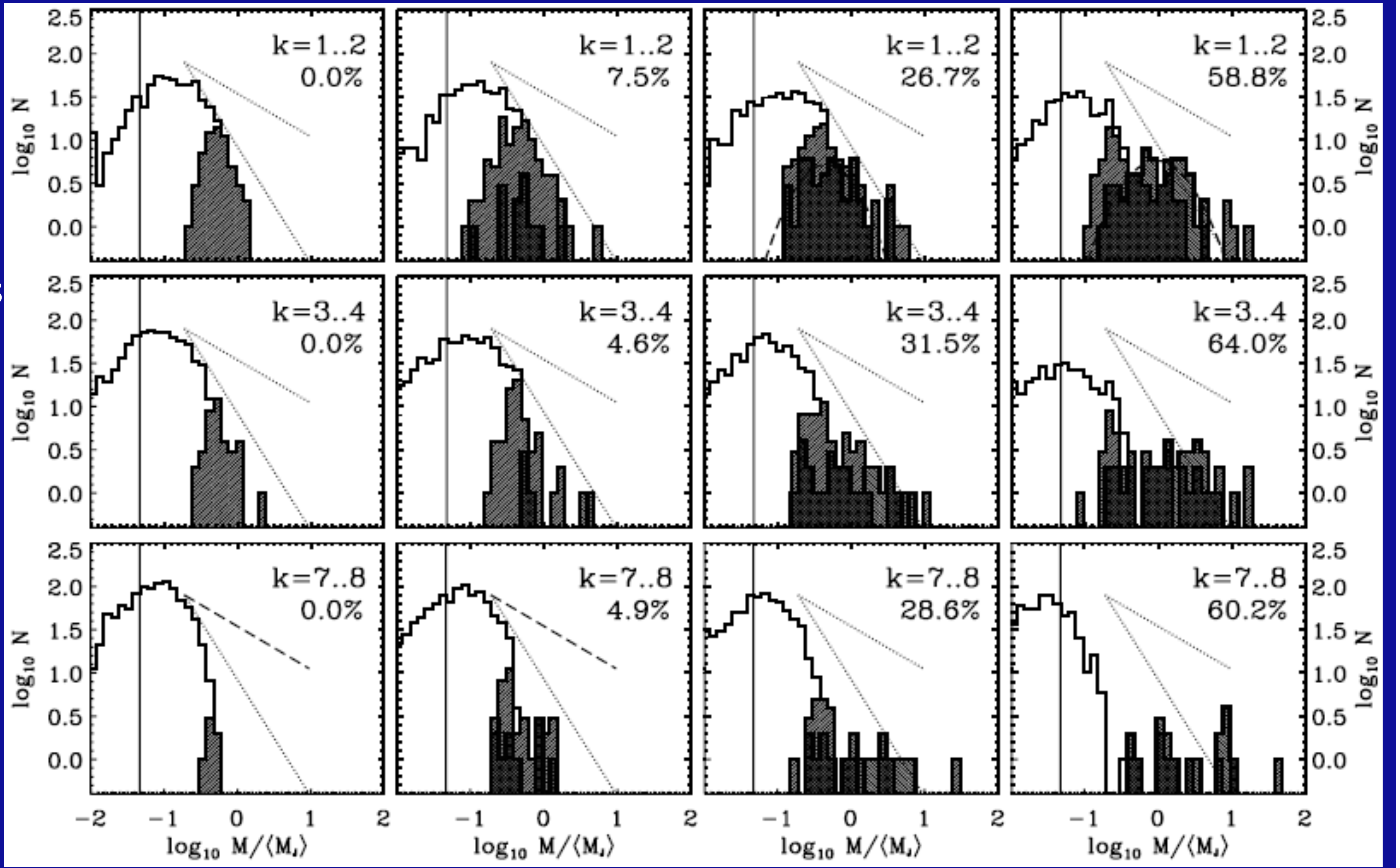
Candidates: Protostellar outflows,
radiation from massive stars,
supernovae explosions

(Gravo)-turbulent fragmentation

Histogram:
Gas clumps
Grey:
Jeans unstable clumps
Dark:
Collapsed core

Slopes: -1.5 & -2.3

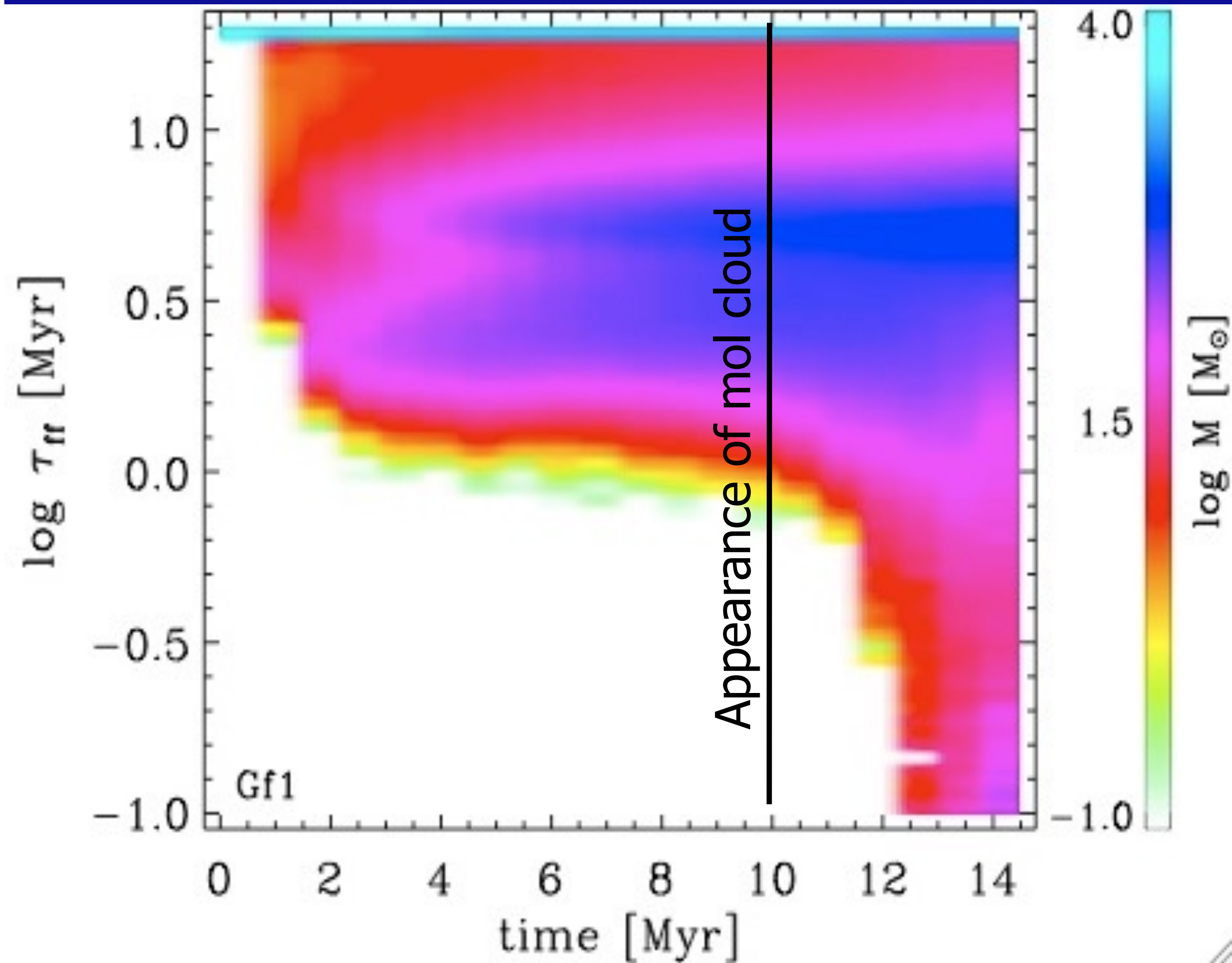
Klessen 2001



2 steps: 1.) Turbulent fragmentation → 2.) Collapse of individual core

- Large-scale driving reproduces shape of IMF.
- Discussion whether largest fragments remain stable or fragment further …

Time scales



Densest regions form stars while the envelope (blue) is not participating.

$$\tau_{ff} = \sqrt{\frac{3\pi}{32G\rho}}$$

→ Densest region have shortest free-fall time.

Heitsch & Hartmann 2008

Summary

- Virial theorem and its application

- Jeans analysis and applications

- Magnetic fields in the interstellar medium

- Turbulence, cloud formation and time scales