Sternentstehung - Star Formation

Winter term 2024/2025

Henrik Beuther, Thomas Henning & Caroline Gieser

15.10 Today: Introduction & Overview (Beuther)
22.10 Physical processes I (Beuther)
29.10 --
05.11 Physcial processes II (Beuther)
12.11 Molecular clouds as birth places of stars (Beuther)
19.11 Molecular clouds (cont.), Jeans Analysis (Henning)
26.11 Collapse models I (Beuther)
03.12 Collapse models II (Beuther)
10.12 Protostellar evolution (Gieser)
17.12 Pre-main sequence evolution & outflows/jets (Henning)
07.01 Accretion disks I (Henning)
14.01 Accretion disks II (Henning)
21.01 High-mass star formation, clusters and the IMF (Gieser)
28.01 Extragalactic star formation (Henning)
04.02 Planetarium@HdA, outlook, questions
11.02 Examination week, no star formation lecture (Beuther, Gieser, Henning)

Book: Stahler & Palla: The Formation of Stars, Wileys

More Information and the current lecture files: http://www.mpia.de/homes/beuther/lecture_ws2425.html

beuther@mpia.de, henning@mpia.de , gieser@mpia.de

Last week

- Virial theorem and applications to cloud (in)stability

- Jeans analysis and applications to fragmentation

- Magnetic fields

- Cloud formation and lifetimes

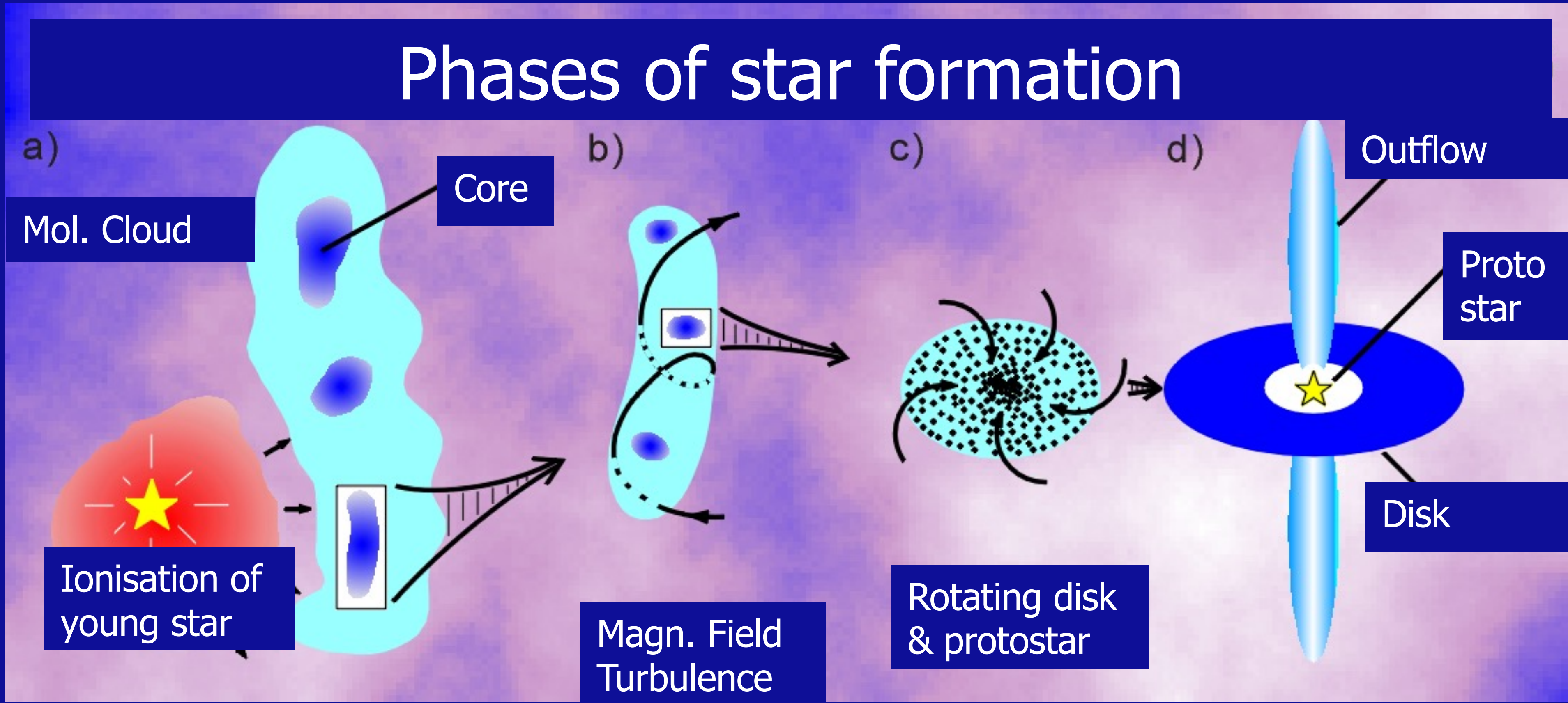
Topics today

- Isothermal sphere, hydrostatic equilibrium,
  grav. stability, Bonnor-Ebert spheres

- Rotational support

- Magnetic support and ambipolar diffusion

Star formation paradigm



https://www.mpifr-bonn.mpg.de/473576/starform

Isothermal Sphere I

Three equations governing the equilibrium are:

Hydrostatic equilibrium

$$-\frac{1}{\rho}\nabla P - \nabla\Phi_g = 0 \quad (1)$$

Ideal isothermal gas

$$P = \rho a_t^2 \quad (2)$$

where the Φ_g obeys Poisson equation

$$\nabla^2\Phi_g = 4\pi G\rho \quad (3)$$

Substituting equation 2 in 1 and after integration

$$ln\rho + \Phi_g/a^2 = const. \quad (4)$$

In the spherical case, this is

$$\rho(r) = \rho_c exp(-\Phi_g/a^2) \quad (5)$$

P: Pressure
ρ: density
Φ_g: grav. Potential
a_t: sound speed

Isothermal Sphere II

With ρ_c the density at the center and $\Phi_g(r=0)=0$, the Poisson eq. becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi_g}{dr}\right) = 4\pi G\rho \quad (1)$$

$$= 4\pi G\rho_c exp(-\Phi_g/a^2) \quad (2)$$

Often, this equations is used in dimensionless form with the dimensionless potential:

$$\phi = \Phi_g/a^2$$

and the dimensionless length ξ

$$\xi = \sqrt{\frac{4\pi G\rho_c}{a^2}}r$$

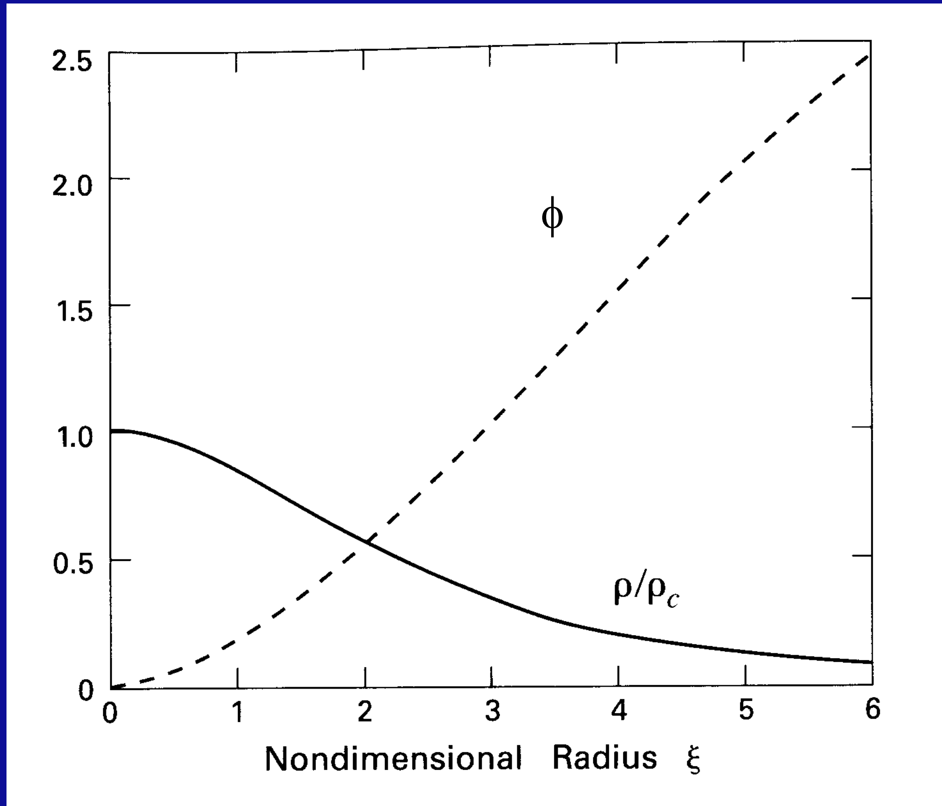
Then the Poisson eq. turns into the Lane-Emden eq.

$$\frac{1}{\xi^2}\frac{1}{d\xi}\left(\xi^2\frac{d\phi}{d\xi}\right) = exp(-\phi) \quad (3)$$

Boundary conditions:
$\phi(0) = 0$
$\phi'(0) = 0$
Gravitational potential and force are 0 at the center.

→ Numerical integration: gravitational potential versus radius … then density

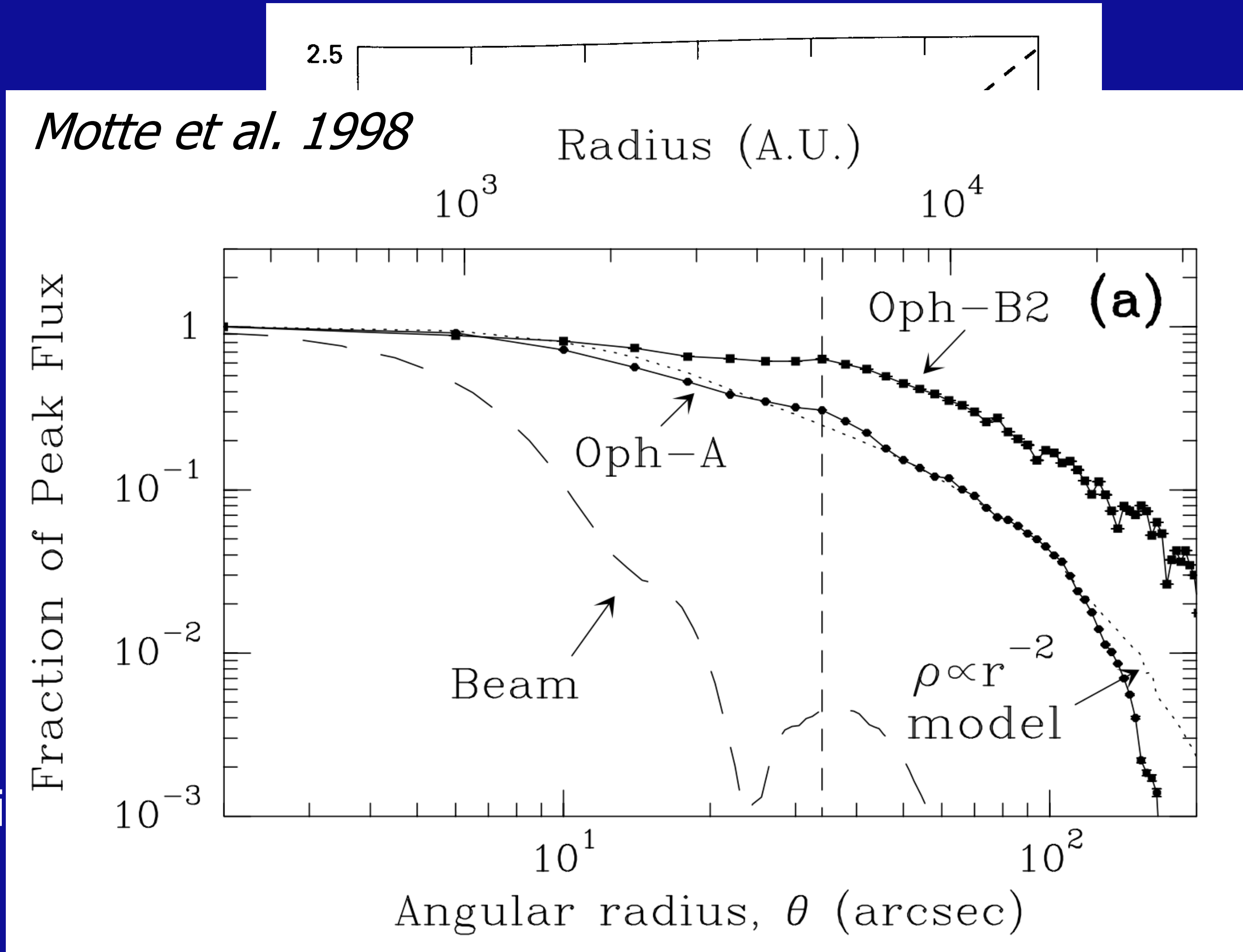
Isothermal Sphere III



- Density and pressure ($P=\rho a^2$) drop monotonically away from the center.
  → important to offset inward pull from gravity for grav. collapse.
- After numerical integration of the Lane-Emden equation
  → density ρ/ρ_c approaches asymtotically $2/\xi^2$.
- Hence the dimensional density profile of the isothermal sphere is:

$$\rho(r) = a^2/(2\pi G r^2) \sim 1/r^2.$$

Isothermal Sphere III



- Densi … the center.
… collapse.
- After …

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Isothermal Sphere IV

The dimensional mass is

$$M = 4\pi \int_0^{r_0} \rho r^2 dr \quad (1)$$

$$= 4\pi\rho_c \left(\frac{a_t^2}{4\pi G\rho_c}\right)^{3/2} \int_0^{\xi_0} e^{-\phi}\xi^2 d\xi \quad (2)$$

Using the Lane-Emden eq. and the boundary condition $\phi'(0) = 0$

$$\rightarrow M = 4\pi\rho_c \left(\frac{a_t^2}{4\pi G\rho_c}\right)^{3/2} \left(\xi^2\frac{d\phi}{d\xi}\right)_{\xi_0} \quad (3)$$

Defining furthermore a dimensionless mass m

$$m = \frac{P_0^{1/2} G^{3/2} M}{a_t^4}, \text{ with } \mathrm{P}_0 = \rho_0 \mathrm{a}_\mathrm{t}^2 \quad (4)$$

the dimensionless mass equals

$$m = \left(4\pi\frac{\rho_c}{\rho_0}\right)^{-1/2} \left(\xi^2\frac{d\phi}{d\xi}\right)_{\xi_0} \quad (5)$$

Since ξ_0 is known for each ρ_c/ρ_0, and $\left(\xi^2\frac{d\phi}{d\xi}\right)_{\xi_0}$ can be read from the previous figure, one can evaluate m.

With:
$r = \sqrt{(a_t^2/(4\pi G\rho_c))}*\xi$
$\rho = \rho_c \exp(-\phi)$
Subscript 0 at cloud edge

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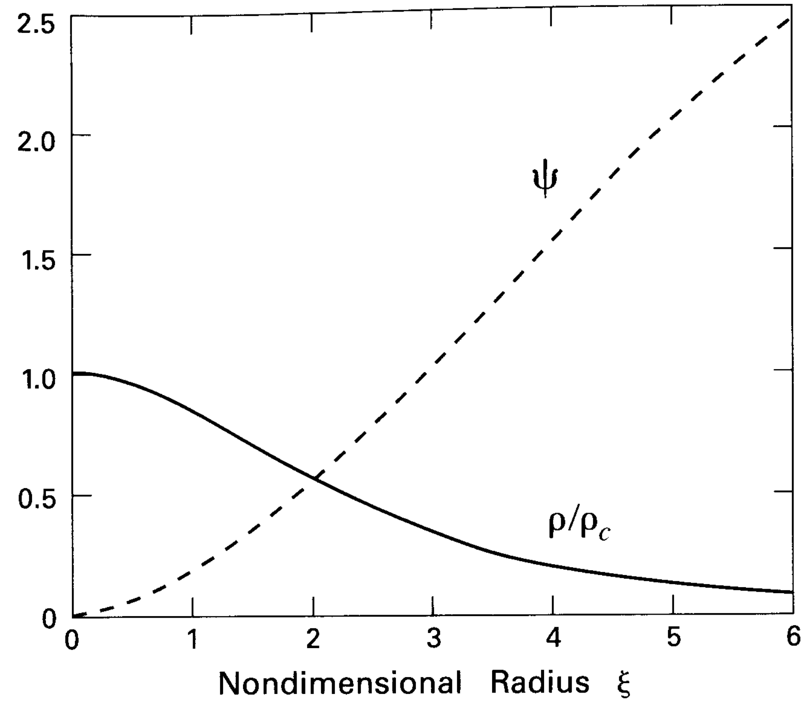
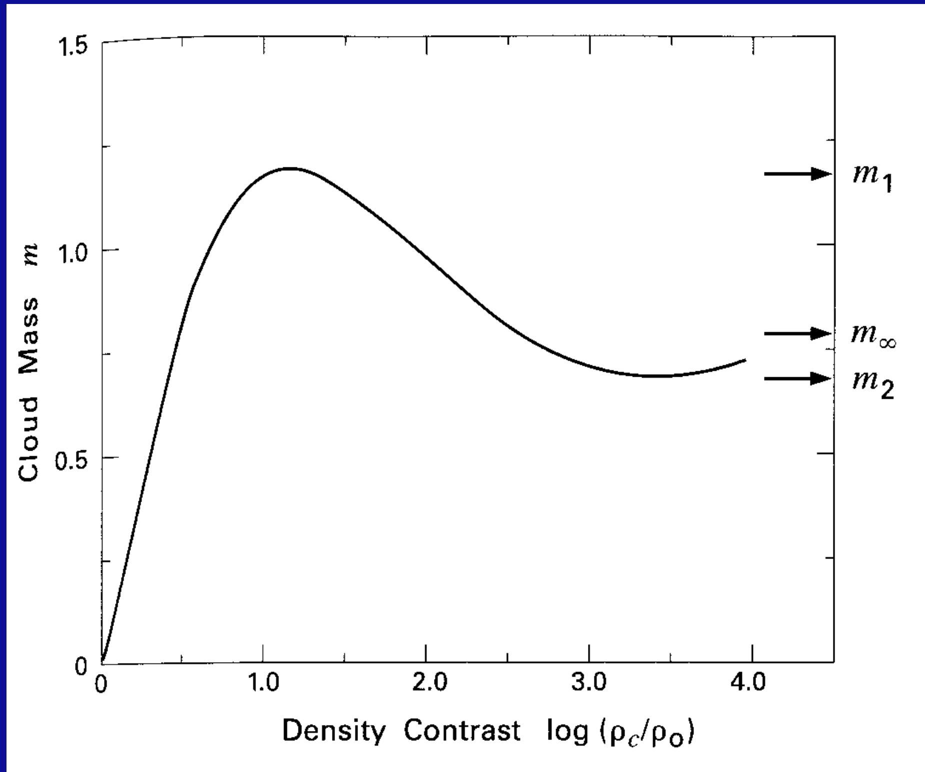


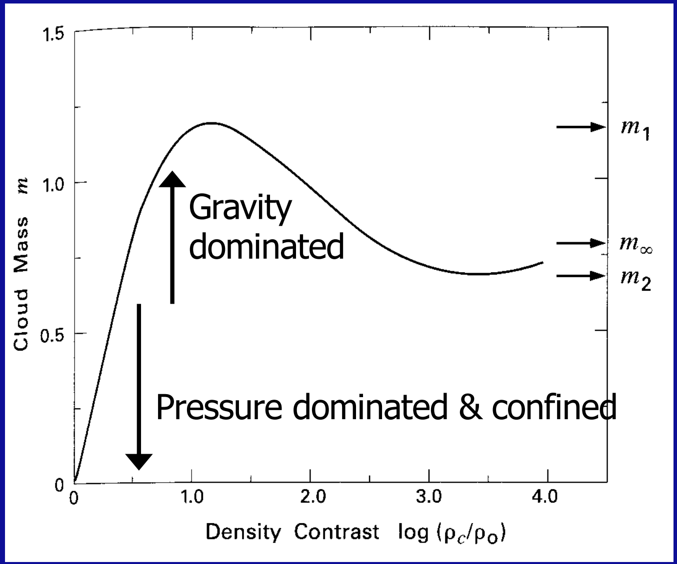
Isothermal Sphere V



The beginning is for a radius ξ_0=0, hence ρ_c/ρ_0=1 and m=0.

For increasing ρ_c/ρ_0, m (and Φ) increases until ρ_c/ρ_0=14.1, corresponding to the dimensionless radius ξ_0=6.5.

Gravitational stability



- Low density-contrast cloud: Increasing outer pressure P_0 → rise of m & ρ_c/ρ_0.
- With internal pressure $P=\rho a_t^2$ and $\rho \sim 1/r^2$ decreasing outward → inner P rises more strongly than P_0 → cloud remains stable.

- High-density contrast: following the Boyle-Mariotte law for an ideal gas:

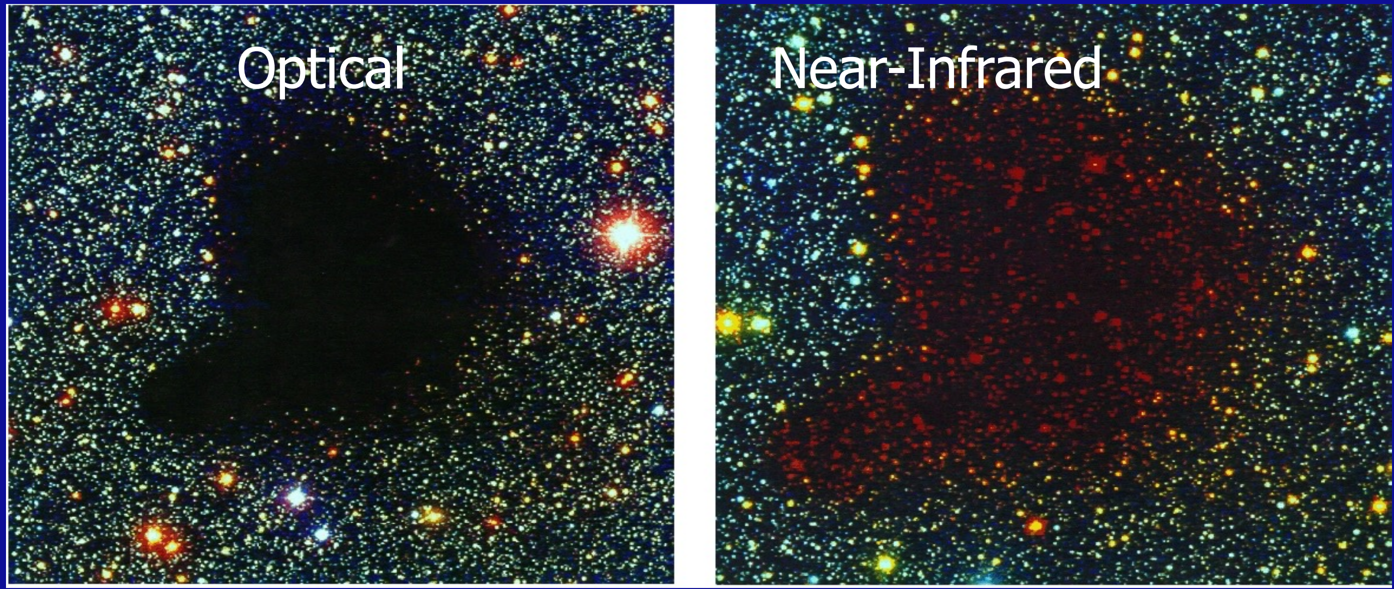
$$PV = \text{const} \rightarrow P*4/3\pi r^3 = \text{const}$$

→ core shrinks with increasing outer pressure P_0.

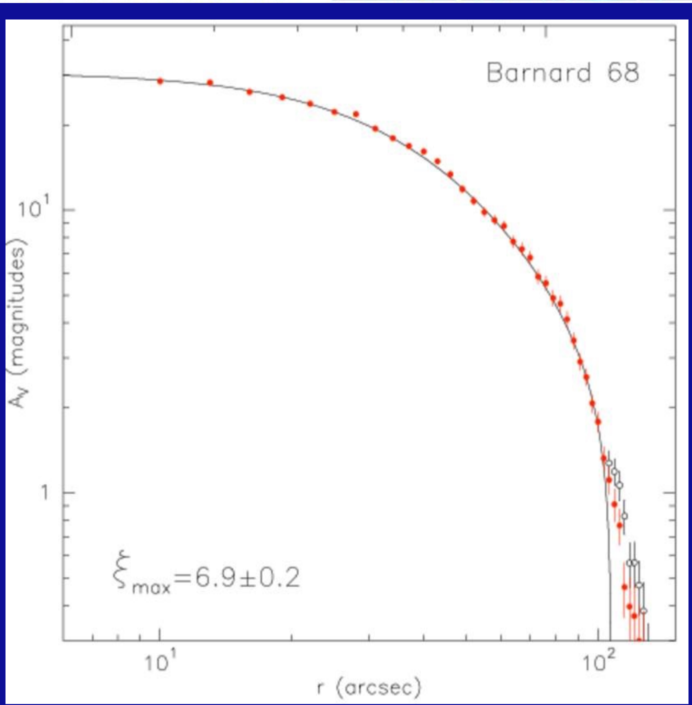
- All clouds with $\rho_c/\rho_0 > 14.1$ ($\xi_0=6.5$) are gravitationally unstable, the critical mass is the Bonnor-Ebert mass (eq. 4, 2 slides ago, Ebert 1955, Bonnor 1956)

$$M_{BE} = (m_1 a_t^4)/(P_0^{1/2} G^{3/2})$$

Gravitational stability: The case of B68



ξ_0=6.9 is only marginally about the critical value 6.5 → gravitational stable or at the verge of collapse



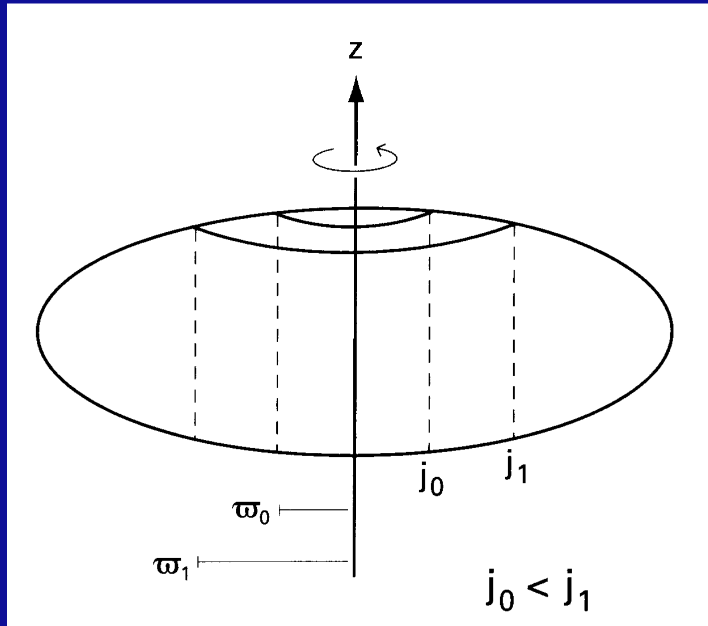
Topics today

- Isothermal sphere, hydrostatic equilibrium, grav. stability, Bonnor-Ebert spheres

- Rotational support

- Magnetic support and ambipolar diffusion

Basic rotational configurations I



Adding a centrifugal potential Φ_{cen}, the hydrodynamic equation reads

$$-1/\rho \ \text{grad}(P) - \text{grad}(\Phi_g) - \text{grad}(\Phi_{cen}) = 0$$

With Φ_{cen} defined as

$$\Phi_{cen} = - \int (j^2/\omega^3) \, d\omega$$

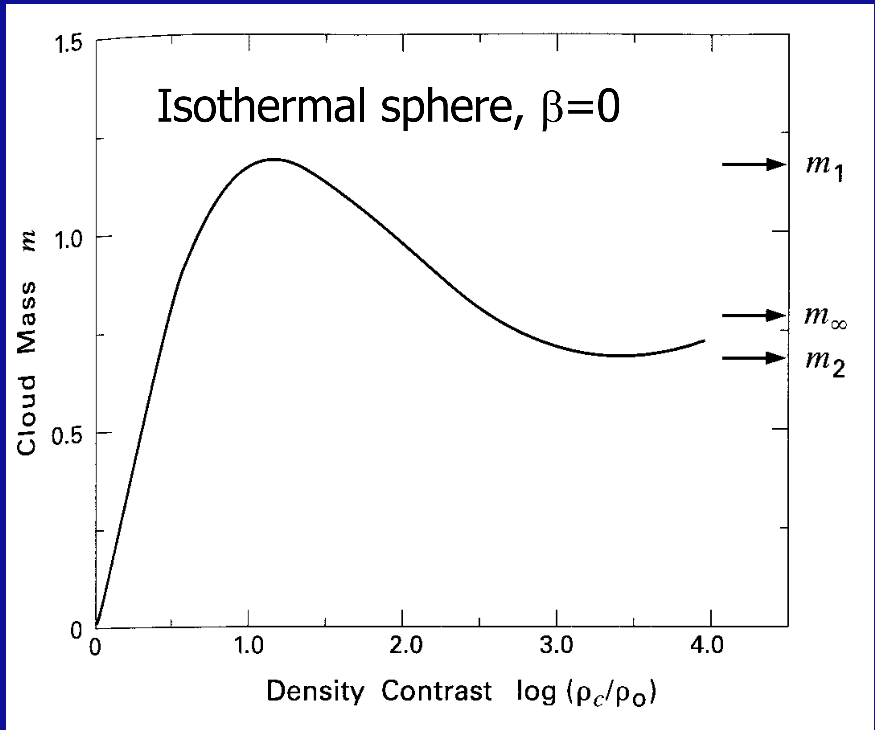
j: specific angular momentum

ω: cylindrical radius

and $j=\omega u$ with u the velocity around the rotation axis

Rotation flattens cores and may be additional support against collapse.

Basic rotational configurations II



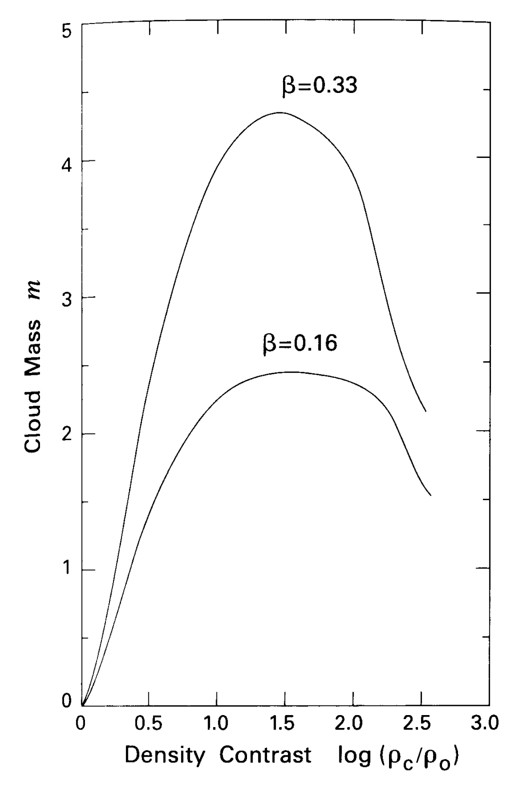
Rotational models: in addition to the density contrast $\rho_c/\rho_0 \rightarrow \beta$

β defined as ratio of rotational to gravitational energy:

$$\beta = T_{rot}/W$$

$\beta > 1/3$ corresponds to breakup speed of the cloud. So $0 < \beta < 1/3$

Basic rotational configurations III



For flattening: $T_{rot}/W > 0.1$

<u>Examples:</u>

$T_{rot} \approx I\Omega^2 = mr^2\Omega^2$

(I: moment of inertia, Ω: rotational velocity)

$W \approx Gm^2/r$

→ $T_{rot}/W \approx 1\mathrm{x}10^{-3}\ (\Omega/(1\mathrm{km\ s^{-1}pc^{-1}}))^2\ (r/(0.1\mathrm{pc}))^3\ (m/(10M_{sun}))^{-1}$

→ Cloud elongations do not arise from rotation, and centrifugal force NOT sufficient for cloud stability!

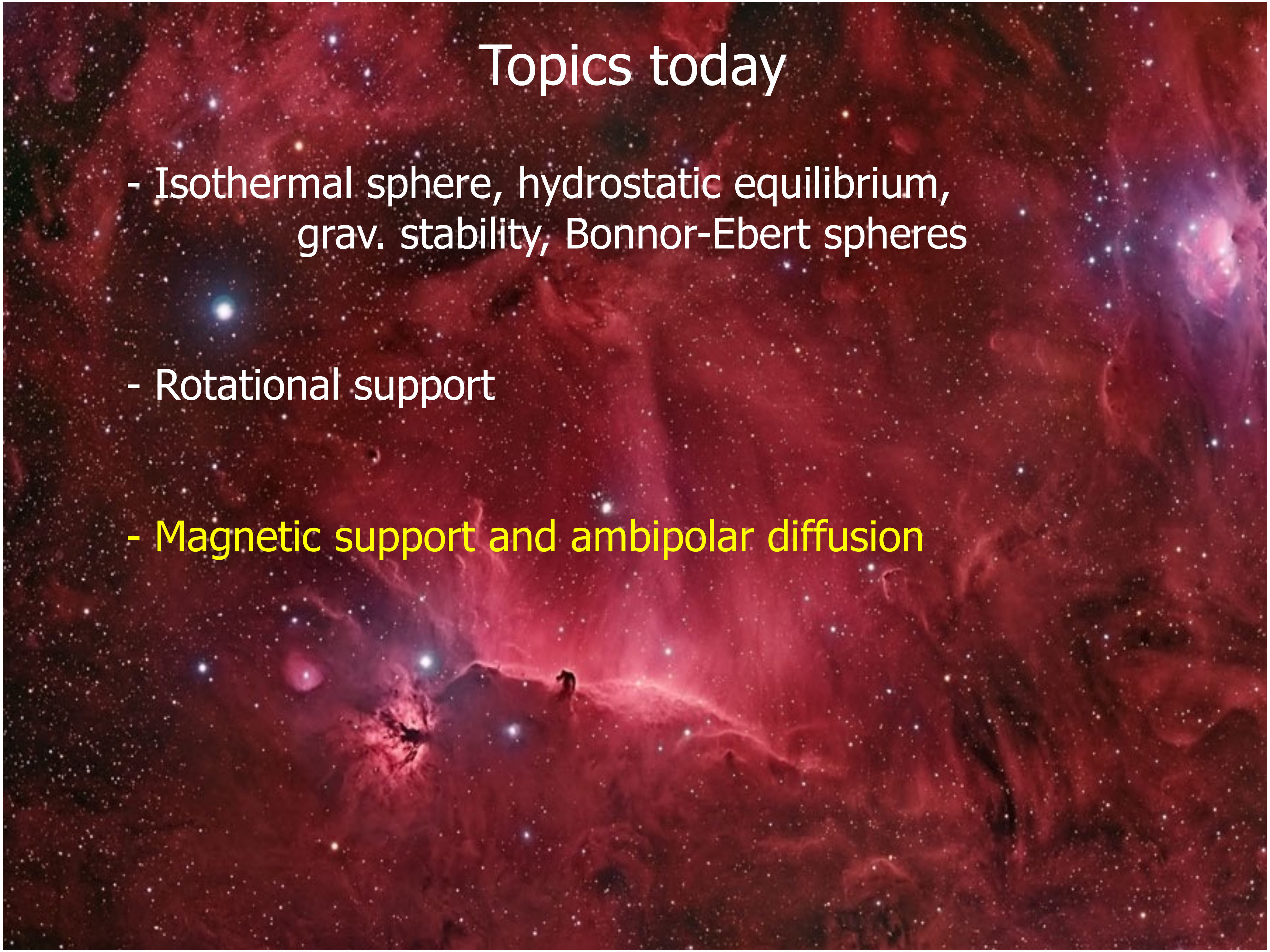
Other stability factors are necessary → Magnetic fields

Specific angular momentum

Specific angular momentum j=J/M is reduced from molecular cloud to star.

	J/M(cm^2/s)
Molecular clump	10^{23}
Binary (P~10^4yr)	$4x10^{20}$-10^{21}
Binary (P~10yr)	$4x10^{19}$-10^{20}
Binary (P~3d)	$4x10^{18}$-10^{19}
T Tauri star	10^{17}
Sun	10^{15}

→ Specific angular momentum reduced by 6 orders of magnitude
from molecular cloud to T Tauri star scale.



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Magnetic fields I

The equation for magneto-hydrodynamic equilibrium now:

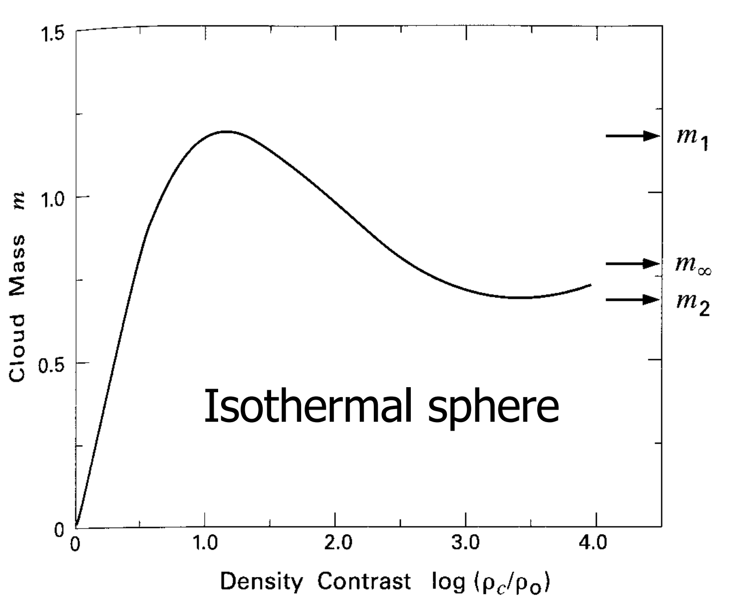
$$-1/\rho \;\mathrm{grad}(P) - \mathrm{grad}(\Phi_g) - 1/(\rho c)\, \mathbf{j} \times \mathbf{B} = 0$$

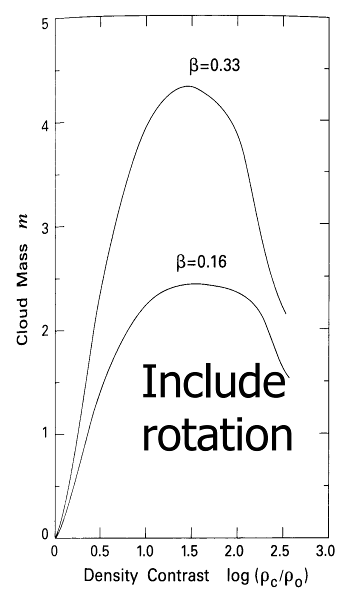
Numerical solving → solutions with 3 free parameters:
the density contrast ratio ρ_c/ρ_0,
the ratio α between magnetic to thermal pressure

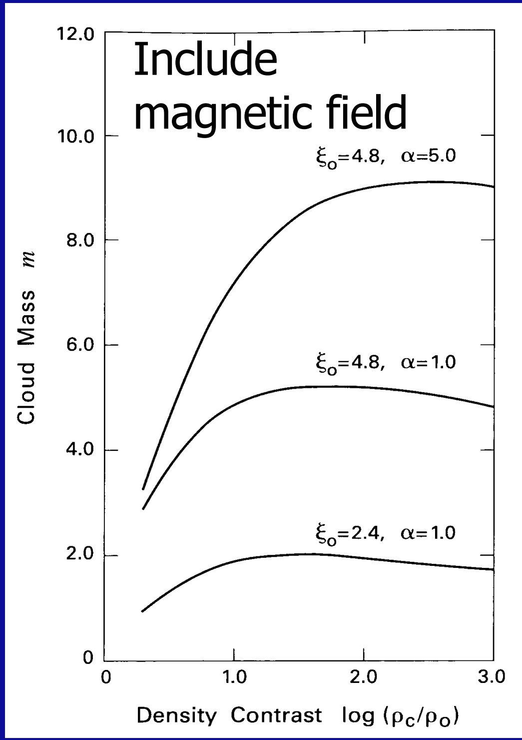
$$\alpha = B_0^2/(8\pi P_0)$$

and the dimensionless radius of the initial sphere

$$\xi_0 = (4\pi G \rho_0/a_t^2)^{1/2} * R_0$$







Fit to numerical results: $m_{crit} = 1.2 + 0.15\, \alpha^{1/2}\, \xi_0^2$

Magnetic fields II

Conversion to dimensional form (multiply by $a_t^4/(P_0^{1/2}G^{3/2})$):
→ first term equals the Bonnor-Ebert Mass: $M_{BE} = m_1 a_t^4/(P_0^{1/2}G^{3/2})$

$$M_{crit} = M_{BE} + M_{magn}$$

with $M_{magn} = 0.15\ \alpha^{1/2}\ \xi_0^2 a_t^4/(P_0^{1/2}G^{3/2})$
$= 0.15\ 2/\mathrm{sqrt}(2\pi)\ (B_0 \pi R_0^2/G^{1/2}) \propto B_0$

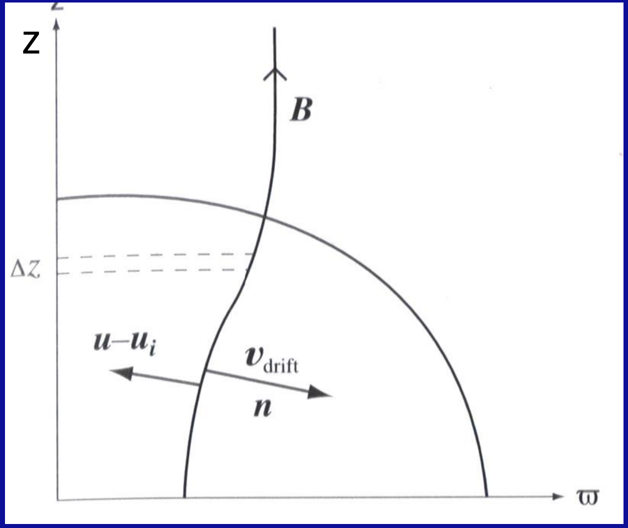
--> magnetic mass M_{magn} proportional to the B-field!

Qualitative difference between thermal and magnetized clouds.
If increase of outer pressure P_0 around core of mass M
→ Bonnor-Ebert mass decreases until $M_{BE} < M$ → then cloud collapse

However, in magnetic case: if $M < M_{magn}$ → cloud remains stable
because M_{magn} constant as long a flux-freezing applies.

Ambipolar diffusion I

- Lower density GMCs, large ionization degree → ions & neutrals strongly collisionally coupled.
- Dense cores: lower ionization degree → neutrals & ions easier decouple.



Neutrals stream through ions accelerated by gravity.

→ drag force between ions & neutrals from collisions.

- Furthermore, Lorentz force acts on ions.

- Drift velocity between ions and neutrals: $v_{drift} = v_i - v_n$
- Drag force between ions and neutrals is: $F_{drag} = n_n \langle \sigma_{in} v_{drift} \rangle m_n v_{drift}$

(average number of collision per unit time $n_n \langle \sigma_{in} v_{drift} \rangle$ times the transferred momentum $m_n v_{drift}$)

→ equation of motion with drag & Lorentz force:

$$n_i F_{drag} = \mathbf{j} \times \mathbf{B}/c = 1/(4\pi)\ (\text{rot}\ \mathbf{B}) \times \mathbf{B}$$

(with Ampere's law: rot $\mathbf{B} = 4\pi/c * \mathbf{j}$)

→ $v_{drift} = (\text{rot}\ \mathbf{B}) \times \mathbf{B} / (4\pi n_i n_n m_n \langle \sigma_{in} v_{drift} \rangle)$

n_n: neutral density
n_i: number of ions
σ_{in}: ion-neutral cross section
m_n: mass of neutral

Ambipolar diffusion II

Dense core with size L → time-scale for ambipolar diffusion:

$$t_{ad} = L/|v_{drift}| = (4\pi n_i n_n m_n <\sigma_{in} v_{drift}>)L \;/\; (|(\mathrm{rot}\,\mathbf{B}) \times \mathbf{B}|)$$

Approximating (rot **B** = B/L): $|(\mathrm{rot}\,\mathbf{B}) \times \mathbf{B}| = B^2/L$

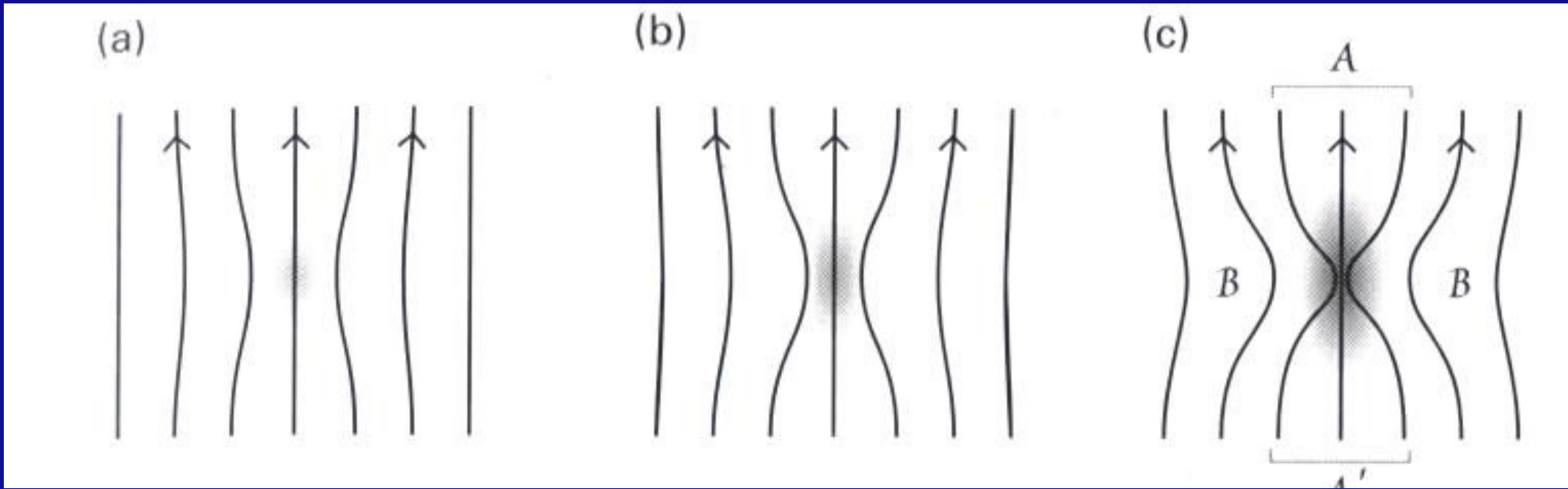
$$\rightarrow t_{ad} = (4\pi n_i n_n m_n <\sigma_{in} v_{drift}>)L^2 \;/\; B^2$$

→ ambipolar diffusion time-scale proportional to:
ionization degree, density & size of the cloud; inversely prop. to mag. field

$$\rightarrow t_{ad} \approx 3\mathrm{x}10^6\mathrm{yr}\,(n_{H2}/10^4\mathrm{cm}^{-3})^{3/2}\,(B/30\mu G)^{-2}\,(L/0.1\mathrm{pc})^2$$

Still under discussion whether this time-scale sets the star formation rate or whether it is too slow and other processes like turbulence are required.

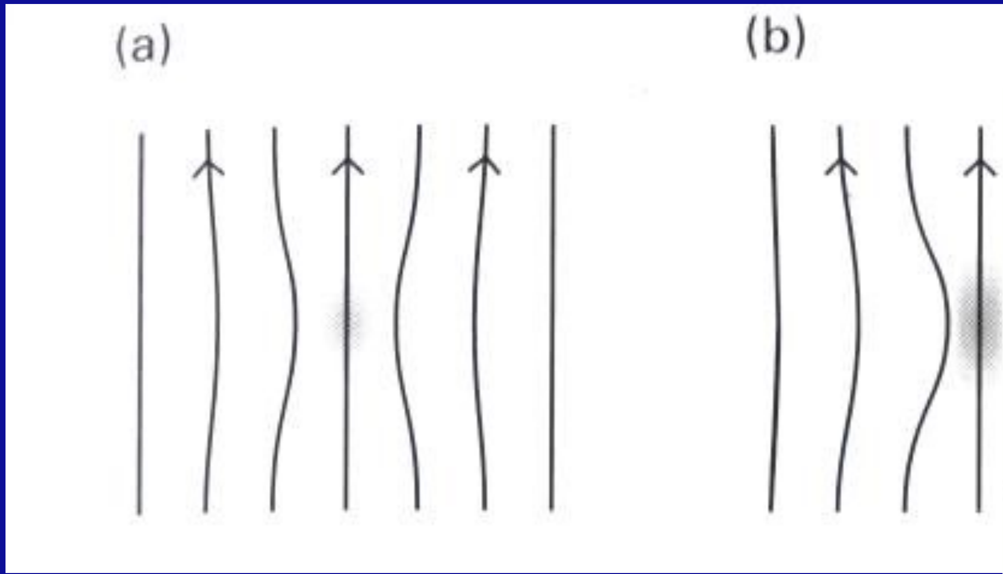
Ambipolar diffusion caveat

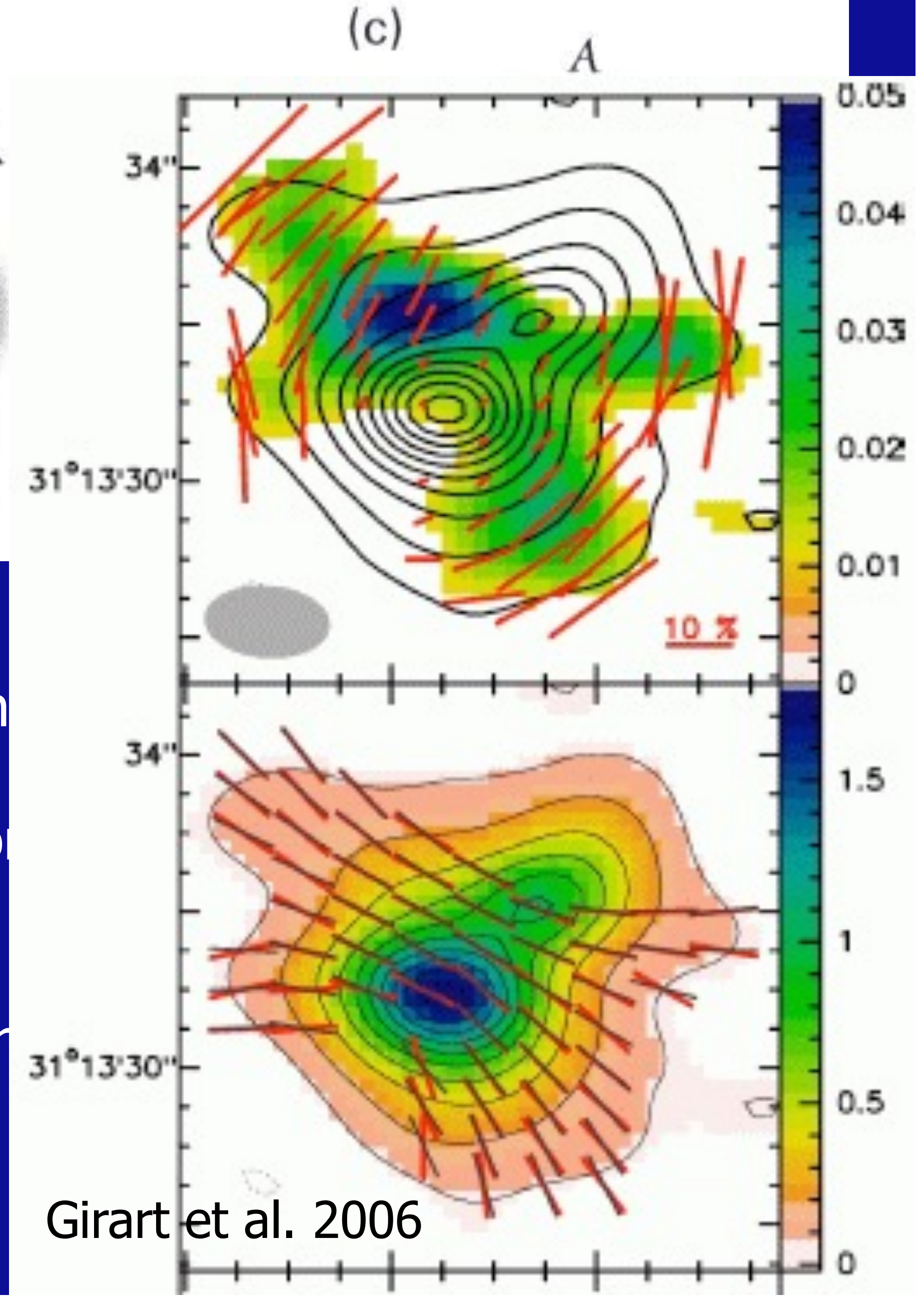


- Star Formation timescale: Observations indicate rapid star formation on the order 1-2 million years.
- Ambipolar diffusion usually requires longer cloud life-times.

→ Maybe gravo-turbulent fragmentation necessary …

Ambipolar diffusion caveat





Girart et al. 2006

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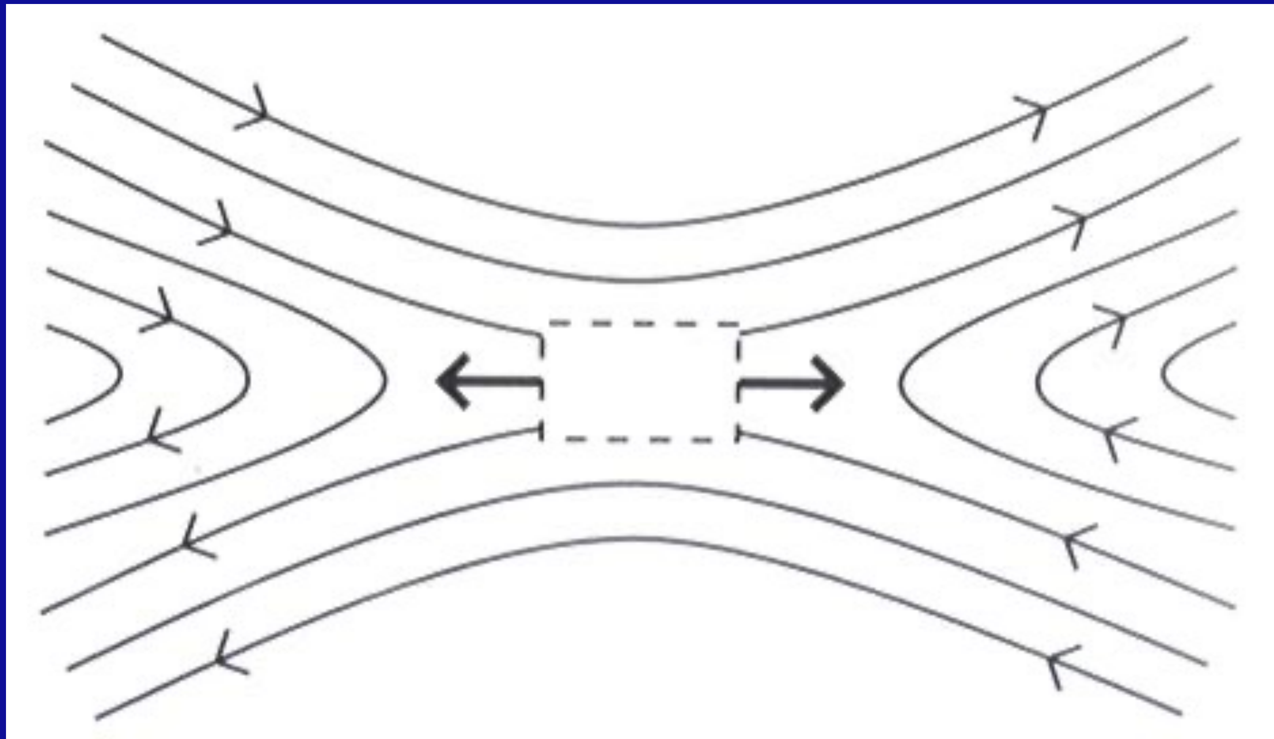
Magnetic reconnection



- Field lines of opposite direction are dragged together.
  → antiparallel B field lines annihilate and
  → magnetic energy dissipates as heat.
- This process was first invoked to explain large luminosities observed in solar flares.

Summary

- Hydrostatic equilibrium between thermal pressure and gravitational force.
  → Bonner Ebert mass for gravitationally stable cores.

- Can rotation support cloud stability?

- Magnetic cloud support and ambipolar diffusion

- Observational signatures of infall motions

Sternentstehung - Star Formation

Winter term 2024/2025

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