Sternentstehung - Star Formation

Winter term 2017/2018

Henrik Beuther & Thomas Henning

17.10 Today: Introduction & Overview (H.B.)
24.10 Physical processes I (H.B.)
31.10 no lecture – Reformationstag
07.11 Physcial processes II (H.B.)
14.11 Molecular clouds as birth places of stars (H.L.)
21.11 Molecular clouds cont., virial & Jeans Analysis (H.B.)
28.11 Collapse models I (H.B.)
05.12 Collapse models II (T.H.)
12.12 Protostellar evolution (T.H.)
19.12 Pre-main sequence evolution & outflows/jets (T.H.)
09.01 Accretion disks I (T.H.)
16.01 Accretion disks II (T.H.)
23.01 High-mass star formation, clusters and the IMF (H.B.)
30.01 Planet formation (T.H.)
06.02 Examination week, no star formation lecture

Book: Stahler & Palla: The Formation of Stars, Wileys

More Information and the current lecture files: http://www.mpia.de/homes/beuther/lecture_ws1718.html

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Last week

- Virial theorem and applications to cloud (in)stabilty

- Jeans analysis and applications to fragmentation

- Magnetic fields on clouds scales

- Turbulence

Topics today

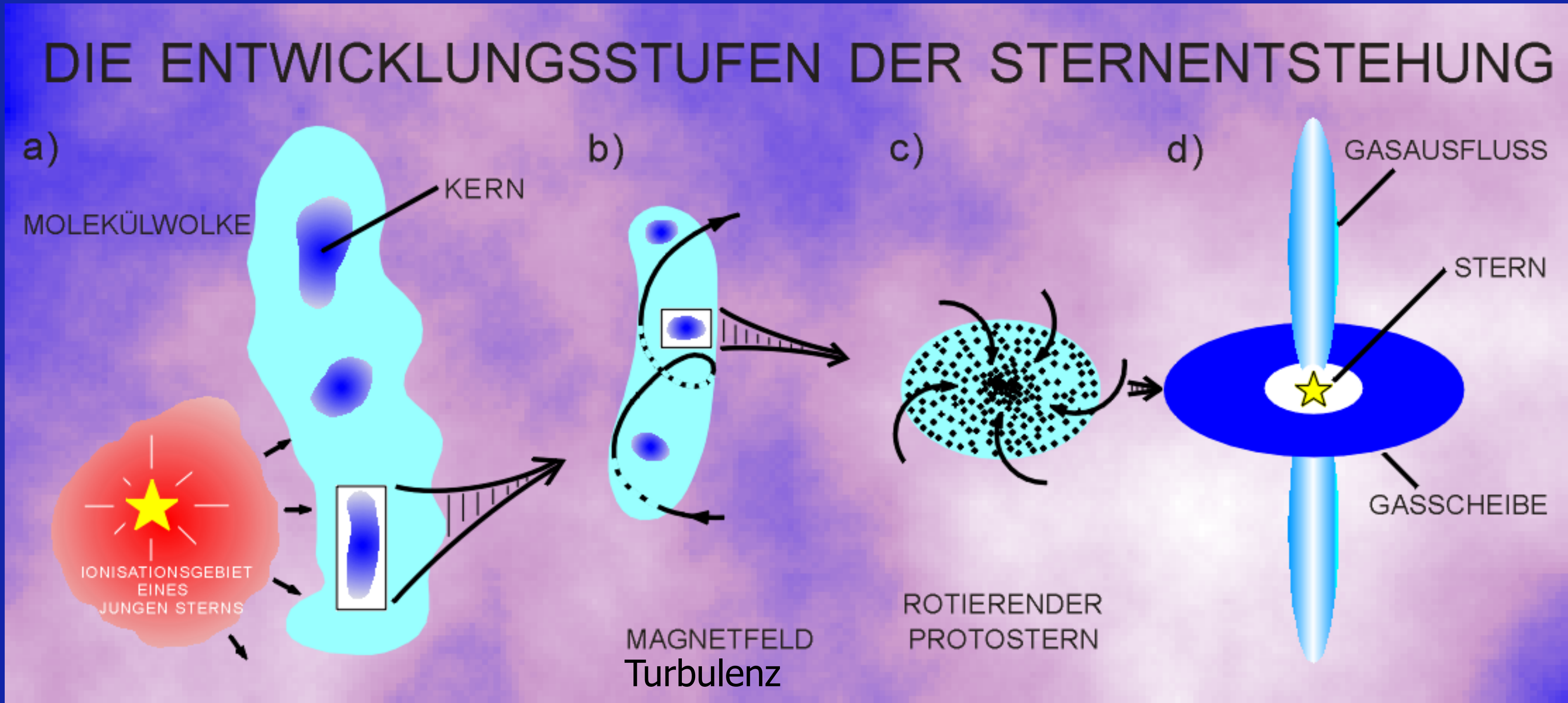
- Isothermal sphere, hydrostatic equilibrium, grav. stability, Bonnor-Ebert spheres

- Rotational support

- Magnetic support and ambipolar diffusion

- Infall signatures

Star formation paradigm



Isothermal Sphere I

Three equations governing the equilibrium are:

Hydrostatic equilibrium

$$-\frac{1}{\rho}\nabla P - \nabla\Phi_g = 0 \qquad (1)$$

Ideal isothermal gas

$$P = \rho a_t^2 \qquad (2)$$

where the Φ_g obeys Poisson equation

$$\nabla^2\Phi_g = 4\pi G\rho \qquad (3)$$

Substituting equation 2 in 1 and after integration

$$ln\rho + \Phi_g/a^2 = const. \qquad (4)$$

In the spherical case, this is

$$\rho(r) = \rho_c exp(-\Phi_g/a^2) \qquad (5)$$

P: Pressure
ρ: density
Φ_g: grav. Potential
a_t: sound speed

Isothermal Sphere II

With ρ_c the density at the center and $\Phi_g(r=0)=0$, the Poisson eq. becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi_g}{dr}\right) = 4\pi G\rho \qquad (1)$$

$$= 4\pi G\rho_c exp(-\Phi_g/a^2) \qquad (2)$$

Often, this equations is used in dimensionless form with the dimensionless potential:

$$\phi = \Phi_g/a^2$$

and the dimensionless length ξ

$$\xi = \sqrt{\frac{4\pi G\rho_c}{a^2}}r$$

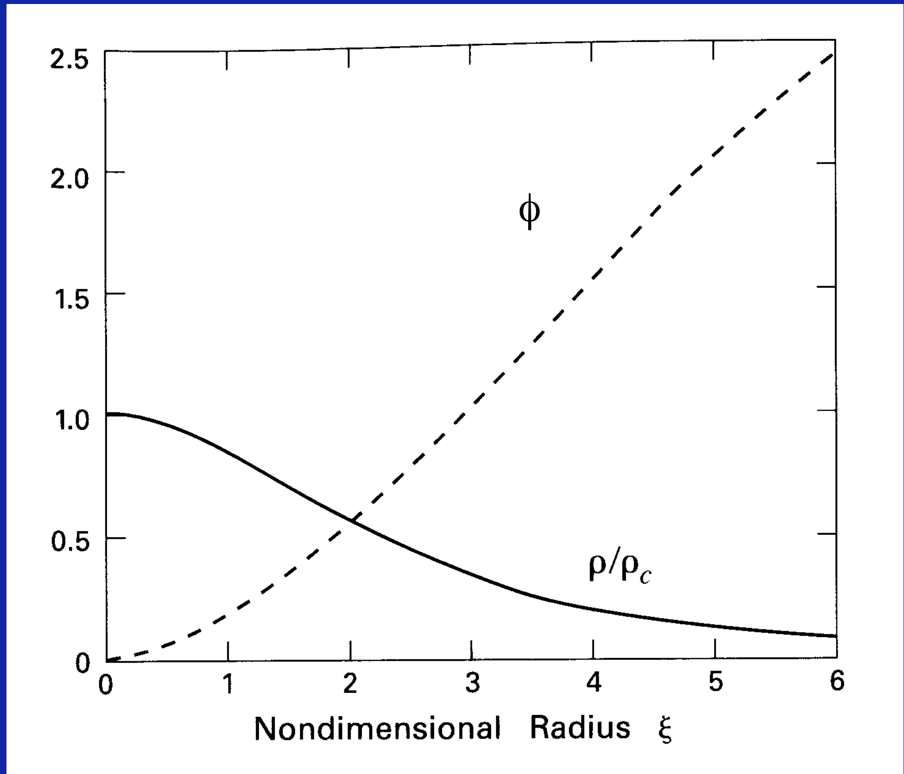
Then the Poisson eq. turns into the Lane-Emden eq.

$$\frac{1}{\xi^2}\frac{1}{d\xi}\left(\xi^2\frac{d\phi}{d\xi}\right) = exp(-\phi) \qquad (3)$$

Boundary conditions:
$\phi(0) = 0$
$\phi'(0) = 0$
Gravitational potential and force are 0 at the center.

→ Numerical integration: gravitational potential versus radius … then density

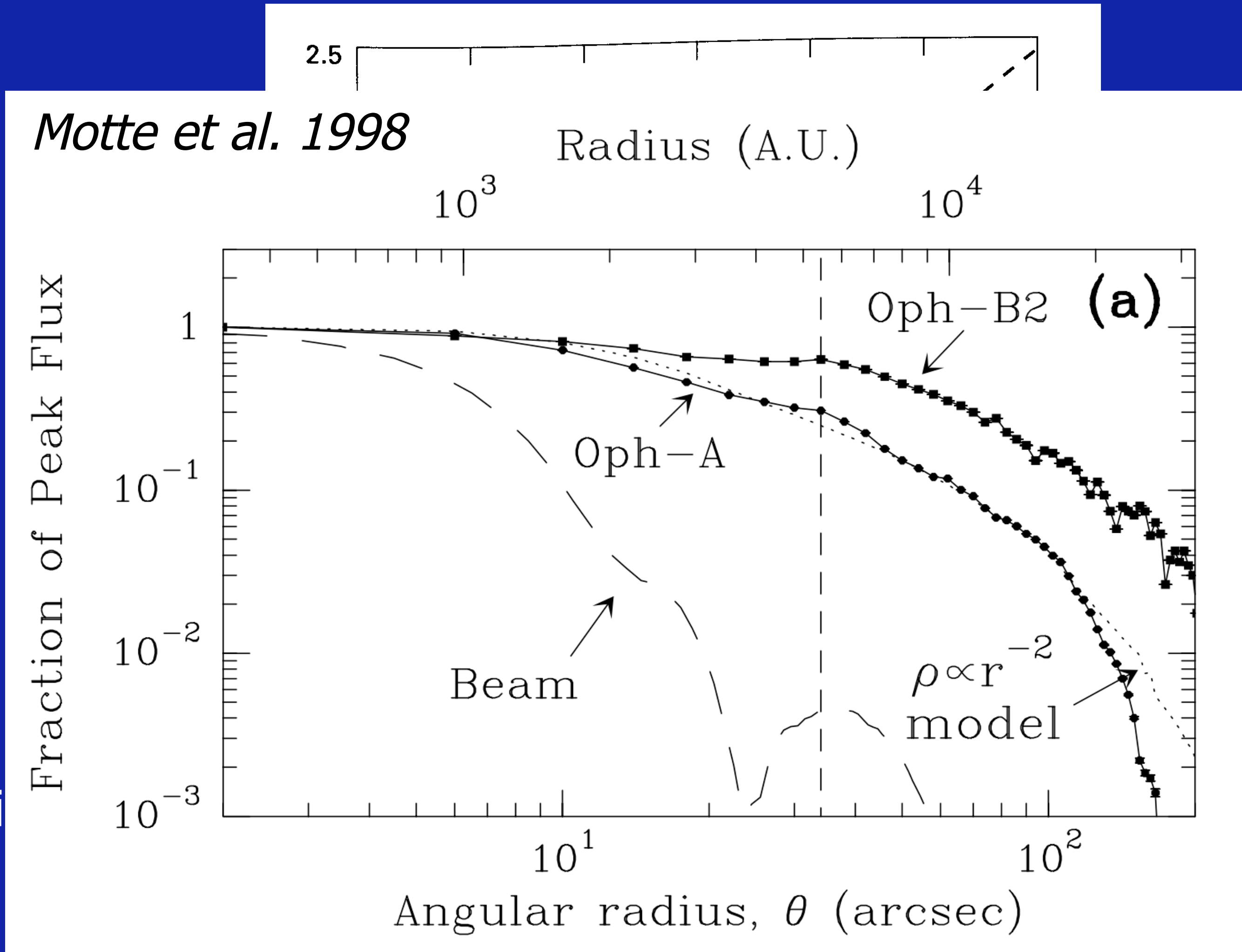
Isothermal Sphere III



- Density and pressure ($P=\rho a^2$) drop monotonically away from the center.
  → important to offset inward pull from gravity for grav. collapse.
- After numerical integration of the Lane-Emden equation
  → density ρ/ρ_c approaches asymtotically $2/\xi^2$.
- Hence the dimensional density profile of the isothermal sphere is:

$$\rho(r) = a^2/(2\pi G r^2) \sim 1/r^2.$$

Isothermal Sphere III



- Densi … the center.
… collapse.

- After …

→ density ρ/ρ_c approaches asymtotically $2/\xi^2$.

- Hence the dimensional density profile of the isothermal sphere is:

$$\rho(r) = a^2/(2\pi G r^2) \sim 1/r^2.$$

Isothermal Sphere IV

The dimensional mass is

$$M = 4\pi \int_0^{r_0} \rho r^2 dr \quad (1)$$

$$= 4\pi\rho_c \left(\frac{a_t^2}{4\pi G\rho_c}\right)^{3/2} \int_0^{\xi_0} e^{-\phi}\xi^2 d\xi \quad (2)$$

Using the Lane-Emden eq. and the boundary condition $\phi'(0) = 0$

$$\rightarrow M = 4\pi\rho_c \left(\frac{a_t^2}{4\pi G\rho_c}\right)^{3/2} \left(\xi^2 \frac{d\phi}{d\xi}\right)_{\xi_0} \quad (3)$$

Defining furthermore a dimensionless mass m

$$m = \frac{P_0^{1/2} G^{3/2} M}{a_t^4}, \text{ with } \mathrm{P}_0 = \rho_0 \mathrm{a}_\mathrm{t}^2 \quad (4)$$

the dimensionless mass equals

$$m = \left(4\pi\frac{\rho_c}{\rho_0}\right)^{-1/2} \left(\xi^2 \frac{d\phi}{d\xi}\right)_{\xi_0} \quad (5)$$

Since ξ_0 is known for each ρ_c/ρ_0, and $\left(\xi^2\frac{d\phi}{d\xi}\right)_{\xi_0}$ can be read from the previous figure, one can evaluate m.

With:
$r = \sqrt{(a_t^2/(4\pi G\rho_c))}*\xi$
$\rho = \rho_c \exp(-\phi)$
Subscript 0 at cloud edge

Isothermal Sphere IV

The dimensional mass is

$$M = 4\pi \int_0^{r_0} \rho r^2 dr \qquad (1)$$

$$= 4\pi\rho_c \left(\frac{a_t^2}{4\pi G\rho_c}\right)^{3/2} \int_0^{\xi_0} e^{-\phi}\xi^2 d\xi \qquad (2)$$

Using the Lane-Emden eq. and the boundary condition $\phi'(0) = 0$

$$\rightarrow M = 4\pi\rho_c \left(\frac{a_t^2}{4\pi G\rho_c}\right)^{3/2} \left(\xi^2 \frac{d\phi}{d\xi}\right)_{\xi_0}$$

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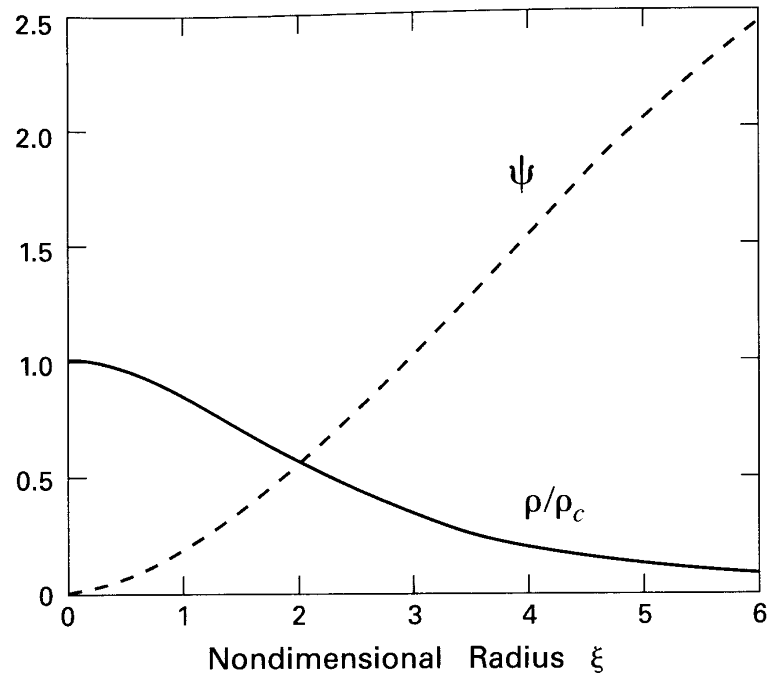
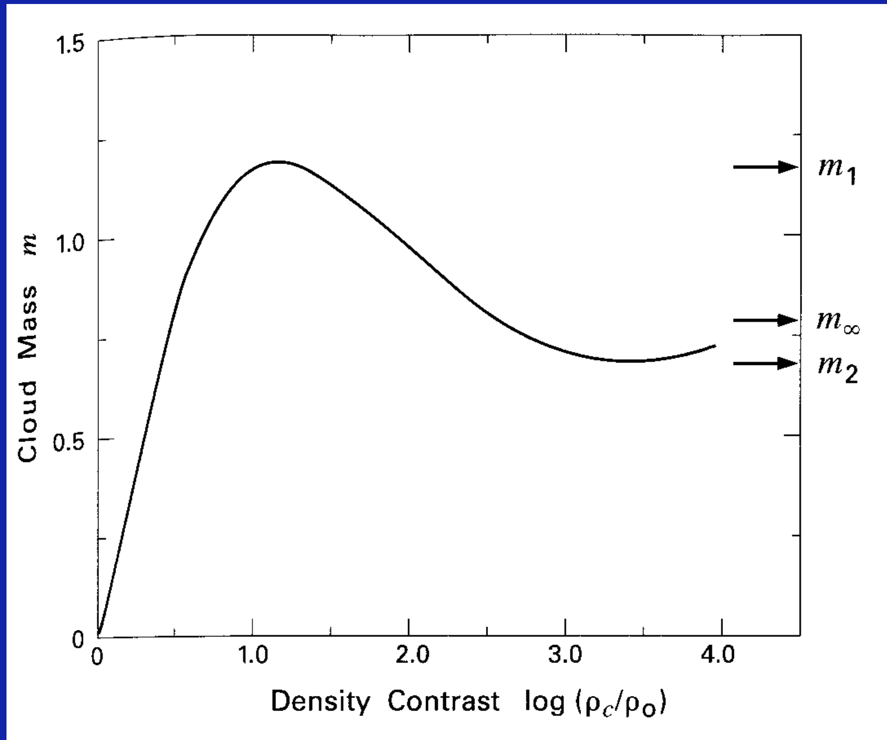
Since ξ_0 is known for each ρ_c/ρ_0, and $\left(\xi^2\frac{d\phi}{d\xi}\right)_{\xi_0}$ can be read from the previous figure, one can evaluate m.

With:
$r = \sqrt{(a_t^2/(4\pi G\rho_c))} * \xi$
$\rho = \rho_c \exp(-\phi)$
Subscript 0 at cloud edge



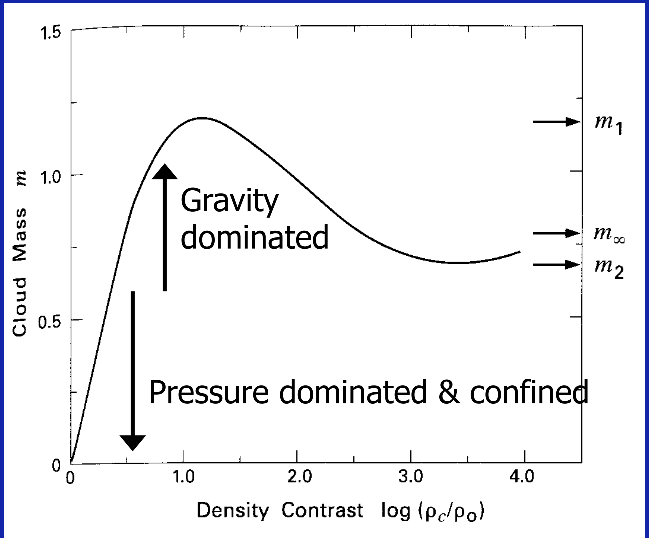
Isothermal Sphere V



The beginning is for a radius ξ_0=0, hence ρ_c/ρ_0=1 and m=0.

For increasing ρ_c/ρ_0, m (and Φ) then increases until ρ_c/ρ_0=14.1, corresponding to the dimensionless radius ξ_0=6.5.

Gravitational stability



- Low density-contrast cloud: Increasing outer pressure P_0 → rise of m & ρ_c/ρ_0.
- With internal pressure $P=\rho a_t^2$ and $\rho\sim 1/r^2$ decreasing outward → inner P rises more strongly than P_0 and the cloud remains stable.

- Following the Boyle-Mariotte law for an ideal gas:

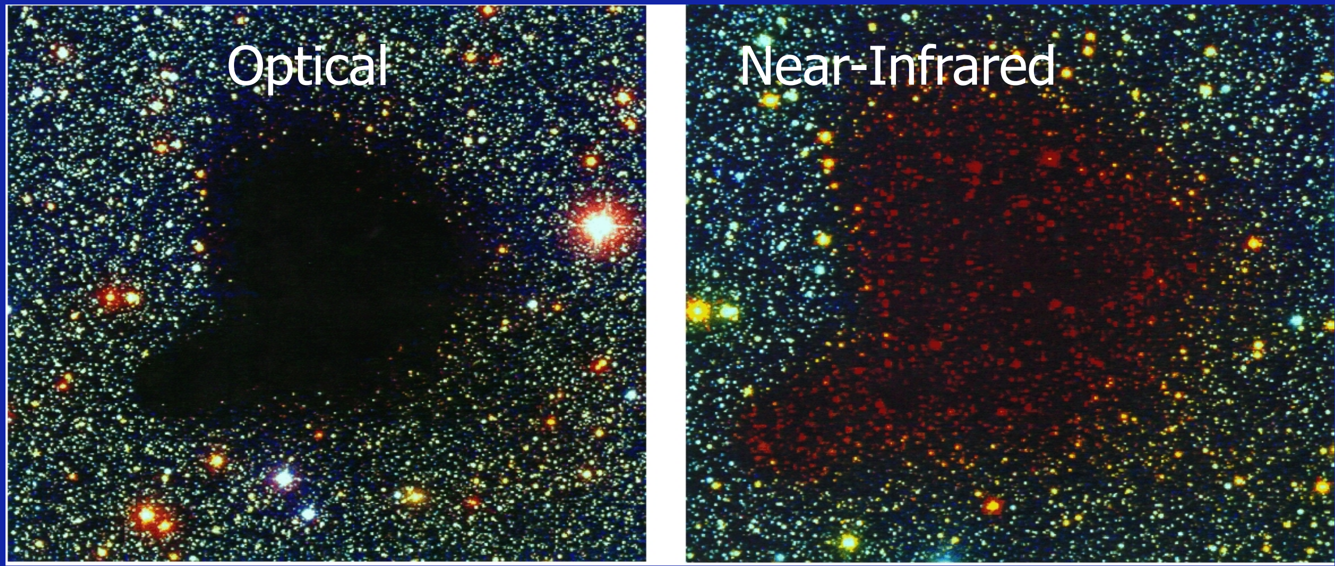
$$PV = \text{const} \rightarrow P*4/3\pi r^3 = \text{const}$$

the core actually shrinks with increasing outer pressure P_0.

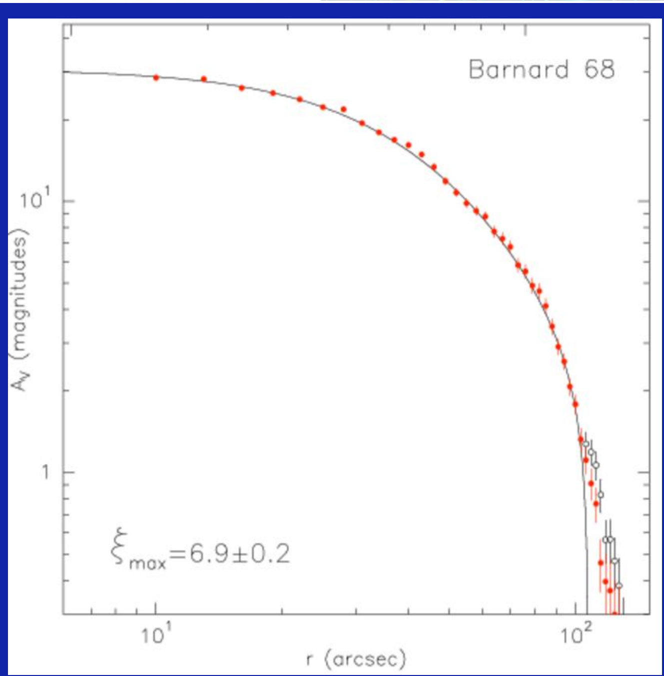
- All clouds with $\rho_c/\rho_0 > 14.1$ ($\xi_0=6.5$) are gravitationally unstable, the critical mass is the Bonnor-Ebert mass (eq. 4, 2 slides ago, Ebert 1955, Bonnor 1956)

$$M_{BE} = (m_1 a_t^4)/(P_0^{1/2} G^{3/2})$$

Gravitational stability: The case of B68



ξ_0=6.9 is only marginally about the critical value 6.5
→ gravitational stable or at the verge of collapse



Topics today

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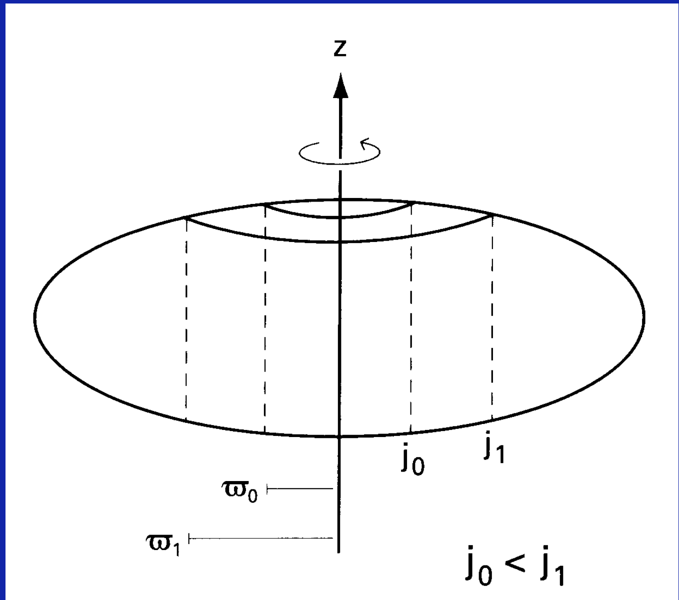
- Rotational support

- Magnetic support and ambipolar diffusion

- Infall signatures

Basic rotational configurations I



Adding a centrifugal potential Φ_{cen}, the hydrodynamic equation reads

$$-1/\rho \ \mathrm{grad}(P) - \mathrm{grad}(\Phi_g) - \mathrm{grad}(\Phi_{cen}) = 0$$

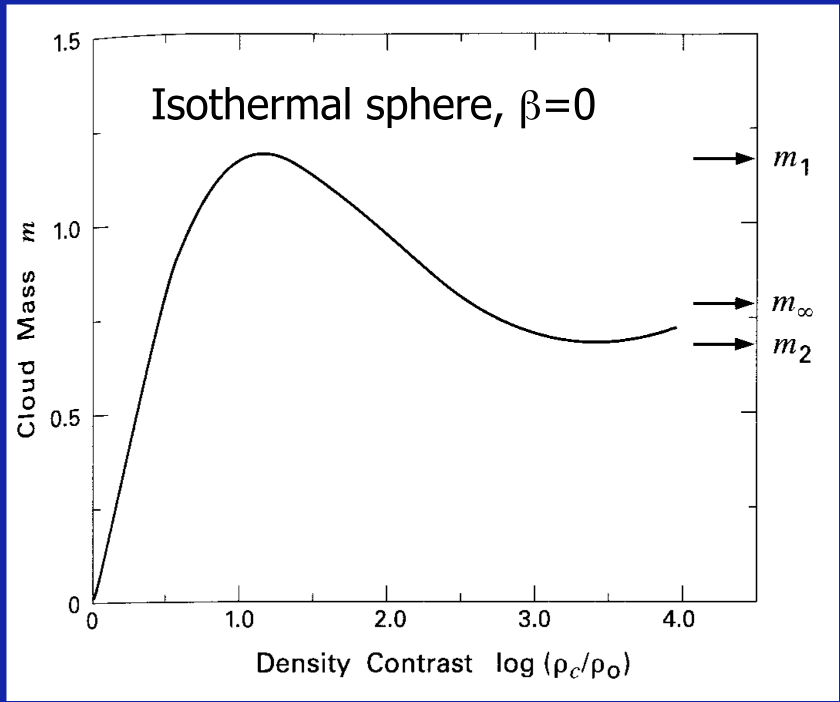
With Φ_{cen} defined as

$$\Phi_{cen} = - \int (j^2/\omega^3)\, d\omega$$

j: specific angular momentum
ω: cylindrical radius
and $j=\omega u$ with u the velocity around the rotation axis

Rotation flattens cores and may (?) be additional support against collapse.

Basic rotational configurations II

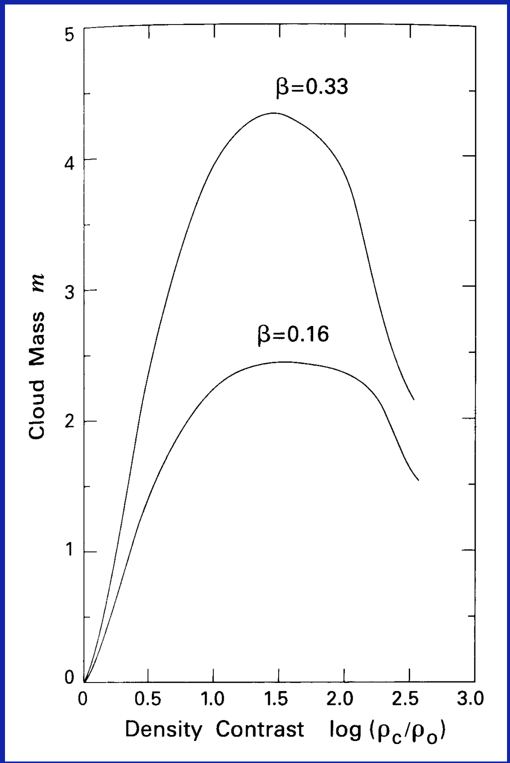


Compared to previous Bonnor-Ebert models, these rotational models have (in addition to the density contrast ρ_c/ρ_0) the parameter β quantifying the degree of rotation. β defined as ratio of rotational to gravitational energy:

$$\beta = T_{rot}/W$$

$\beta > 1/3$ corresponds to breakup speed of the cloud. So $0 < \beta < 1/3$

Basic rotational configurations III



In realistic clouds, for flattening to appear, the rotational energy has to be at least 10% of the gravitational energy. T_{rot}/W equals approximately β.

<u>Examples:</u>

$T_{rot} \approx I\Omega^2 = mr^2\Omega^2$

(I: moment of inertia, Ω: rotational velocity)

$W \approx Gm^2/r$

→ $T_{rot}/W \approx 1\times10^{-3}\,(\Omega/(1\mathrm{km\ s^{-1}pc^{-1}}))^2\,(r/(0.1\mathrm{pc}))^3\,(m/(10M_{sun}))^{-1}$

Dense cores: → $T_{rot}/W \sim 10^{-3}$

GMCs: Velocity gradient of 0.05km/s representing solid body rotation, $200M_{sun}$ and 2pc size imply also $T_{rot}/W \sim 10^{-3}$

→ Cloud elongations do not arise from rotation, and centrifugal force NOT sufficient for cloud stability!

Other stability factors are necessary --> Magnetic fields

Specific angular momentum

Specific angular momentum J/M ($=I\omega/M=Mr^2\omega/M=r^2\omega$) must be reduced from molecular cloud to star.

	J/M(cm^2/s)
Molecular clump	10^{23}
Binary (P~10^4yr)	$4\times10^{20}-10^{21}$
Binary (P~10yr)	$4\times10^{19}-10^{20}$
Binary (P~3d)	$4\times10^{18}-10^{19}$
T Tauri star	10^{17}
Sun	10^{15}

→ Specific angular momentum needs to be reduced by 6 orders of magnitude from molecular cloud to T Tauri star scale.

Topics today

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- Rotational support

- Magnetic support and ambipolar diffusion

- Infall signatures

Magnetic fields I

The equation for magneto-hydrodynamic equilibrium now is:

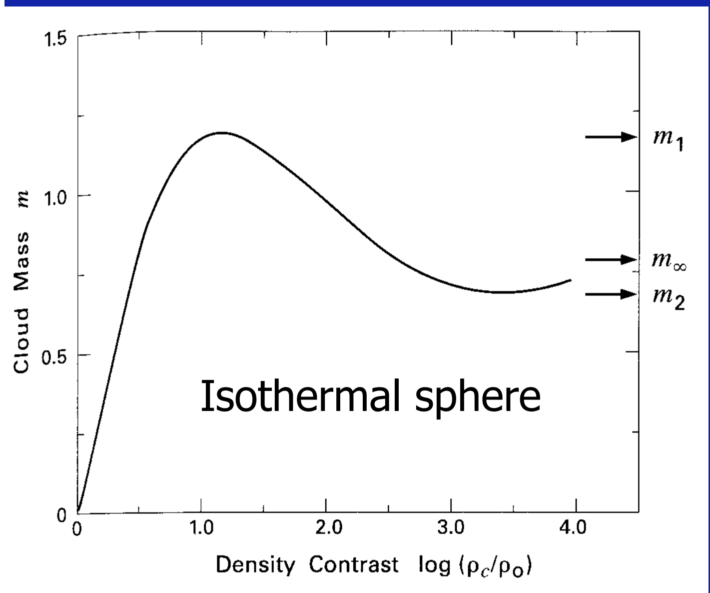
$$-1/\rho \ \text{grad}(P) - \text{grad}(\Phi_g) - 1/(\rho c)\ \mathbf{j} \times \mathbf{B} = 0$$

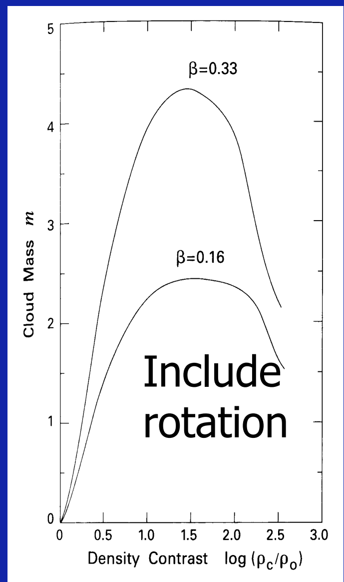
Solving the equations again numerically, one gets solutions with 3 free parameters: the density contrast ratio ρ_c/ρ_0,
the ratio α between magnetic to thermal pressure

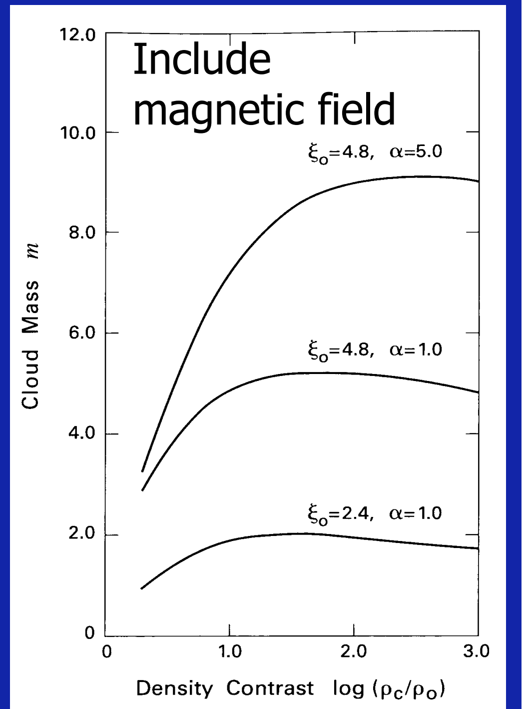
$$\alpha = B_0^2/(8\pi P_0)$$

and the dimensionless radius of the initial sphere

$$\xi_0 = (4\pi G\rho_0/a_t^2)^{1/2} * R_0$$







A good fit to the numerical results is given by: $m_{crit} = 1.2 + 0.15\ \alpha^{1/2}\ \xi_0^2$

Magnetic fields II

Converting this to dimensional form (multiply by $a_t^4/(P_0^{1/2}G^{3/2})$), the first term equals the Bonnor-Ebert Mass ($M_{BE} = m_1 a_t^4/(P_0^{1/2}G^{3/2})$)

$$M_{crit} = M_{BE} + M_{magn}$$

with $M_{magn} = 0.15\ \alpha^{1/2}\ \xi_0^2 a_t^4/(P_0^{1/2}G^{3/2})$
$= 0.15\ 2/\mathrm{sqrt}(2\pi)\ (B_0 \pi R_0^2/G^{1/2}) \propto B_0$

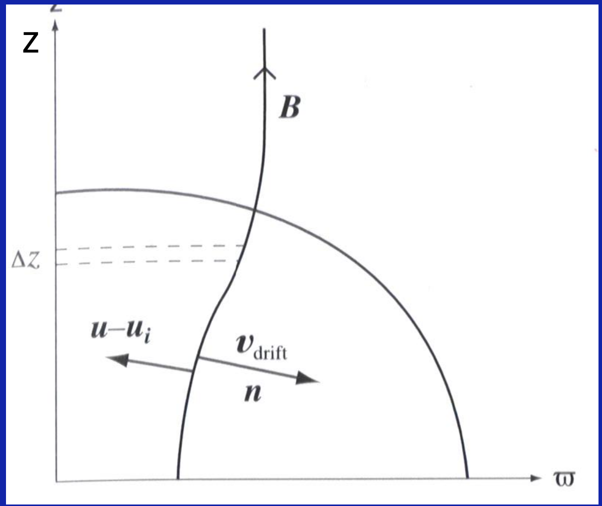
--> the magnetic mass M_{magn} is proportional to the B-field!

Qualitative difference between purely thermal clouds and magnetized clouds. If one increases the outer pressure P_0 around a low-mass core of mass M, the Bonnor-Ebert mass will decrease until $M_{BE} < M$, and then the cloud collapses.

However, in the magnetic case, if $M < M_{magn}$ the cloud will always remain stable because M_{magn} is constant as long a flux-freezing applies.

Ambipolar diffusion I

In less dense GMCs, the ionization degree is relatively large and ions and neutrals are strongly collisionly coupled. Going to denser molecular cores, the ionization degree decreases, and neutrals and ions can easier decouple.



Neutrals stream through the ions accelerated by gravity.

- There is a drag force between ions and neutrals from collisions.
- Furthermore, Lorentz force acts on ions.

The drift velocity between ions and neutrals is $v_{drift} = v_i - v_n$

And the drag force between ions and neutrals is: $F_{drag} = n_n \langle\sigma_{in} v_{drift}\rangle m_n v_{drift}$

(average number of collision per unit time $n_n\langle\sigma_{in} v_{drift}\rangle$ times the transferred momentum $m_n v_{drift}$)

The equation of motion with the Lorentz force is then:

n_n: neutral density
n_i: number of ions
σ_{in}: ion-neutral cross section
m_n: mass of neutral

$$n_i F_{drag} = \mathbf{j} \times \mathbf{B}/c = 1/(4\pi)\,(\mathrm{rot}\,\mathbf{B}) \times \mathbf{B}$$

(with Ampere's law: $\mathrm{rot}\,\mathbf{B} = 4\pi/c * \mathbf{j}$)

$$\rightarrow v_{drift} = (\mathrm{rot}\,\mathbf{B}) \times \mathbf{B} \,/\, (4\pi n_i n_n m_n \langle\sigma_{in} v_{drift}\rangle)$$

Ambipolar diffusion II

For a dense core with a size L, the time-scale for ambipolar diffusion is:

$$t_{ad} = L/|v_{drift}| = (4\pi n_i n_n m_n <\sigma_{in} v_{drift}>)L \,/\, (|(\text{rot}\,\mathbf{B}) \times \mathbf{B}|)$$

Approximating (rot **B** = B/L): $|(\text{rot}\,\mathbf{B}) \times \mathbf{B}| = B^2/L$ we get

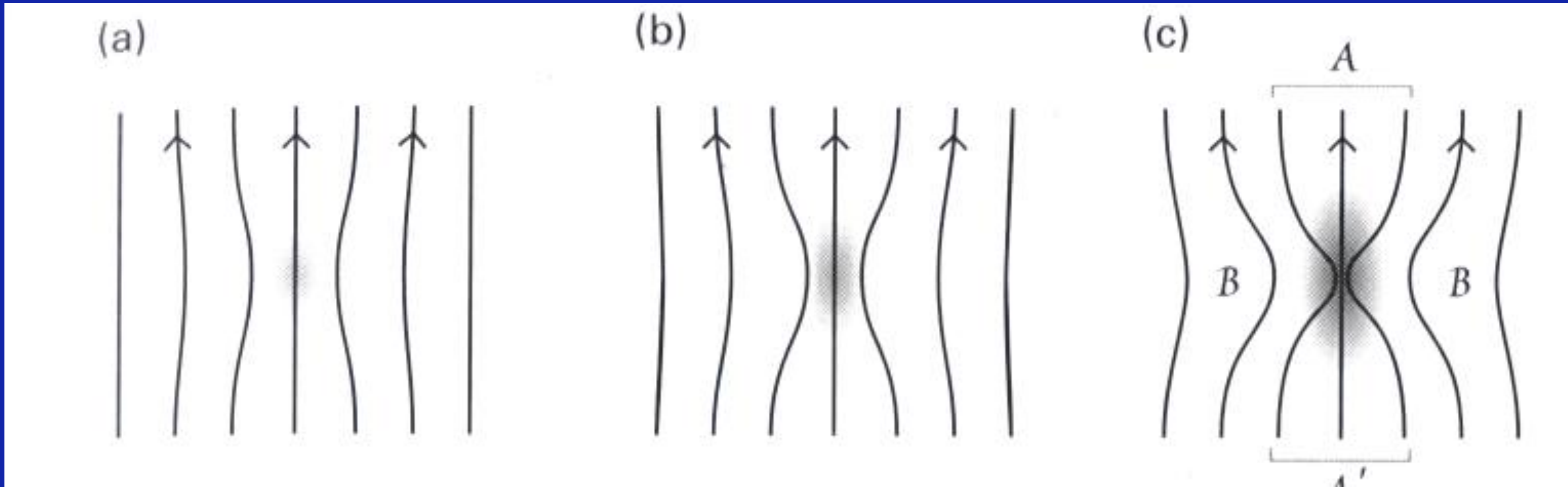
$$t_{ad} = (4\pi n_i n_n m_n <\sigma_{in} v_{drift}>)L^2 \,/\, B^2$$

Hence ambipolar diffusion time-scale is proportional to ionization degree, density and size of the cloud, and inversely proportional to magnetic field.

$$\rightarrow t_{ad} \approx 3\text{x}10^6\text{yr}\,(n_{H2}/10^4\text{cm}^{-3})^{3/2}\,(B/30\mu\text{G})^{-2}\,(L/0.1\text{pc})^2$$

It is still much under discussion whether this time-scale sets the rate where star formation takes place or whether it is too slow and other processes like turbulence are required.

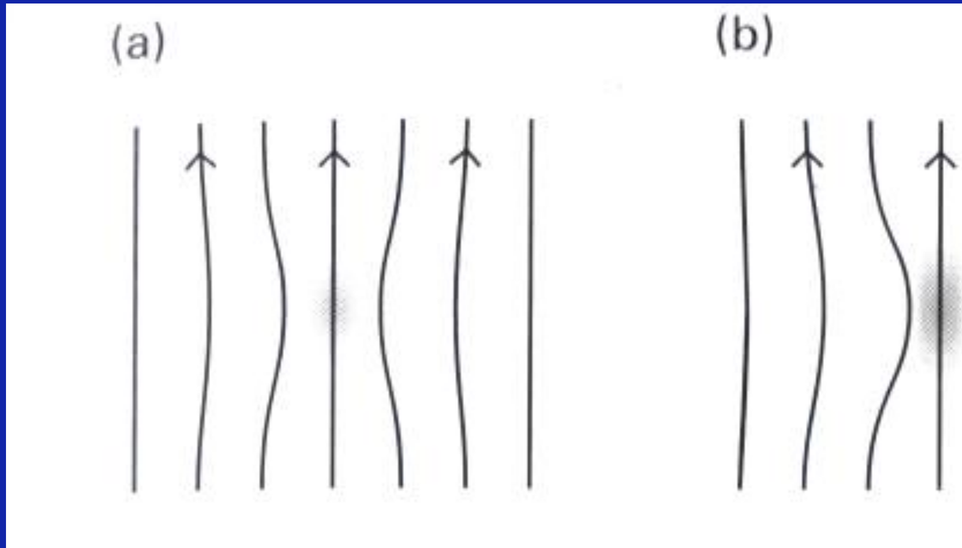
Ambipolar diffusion caveat

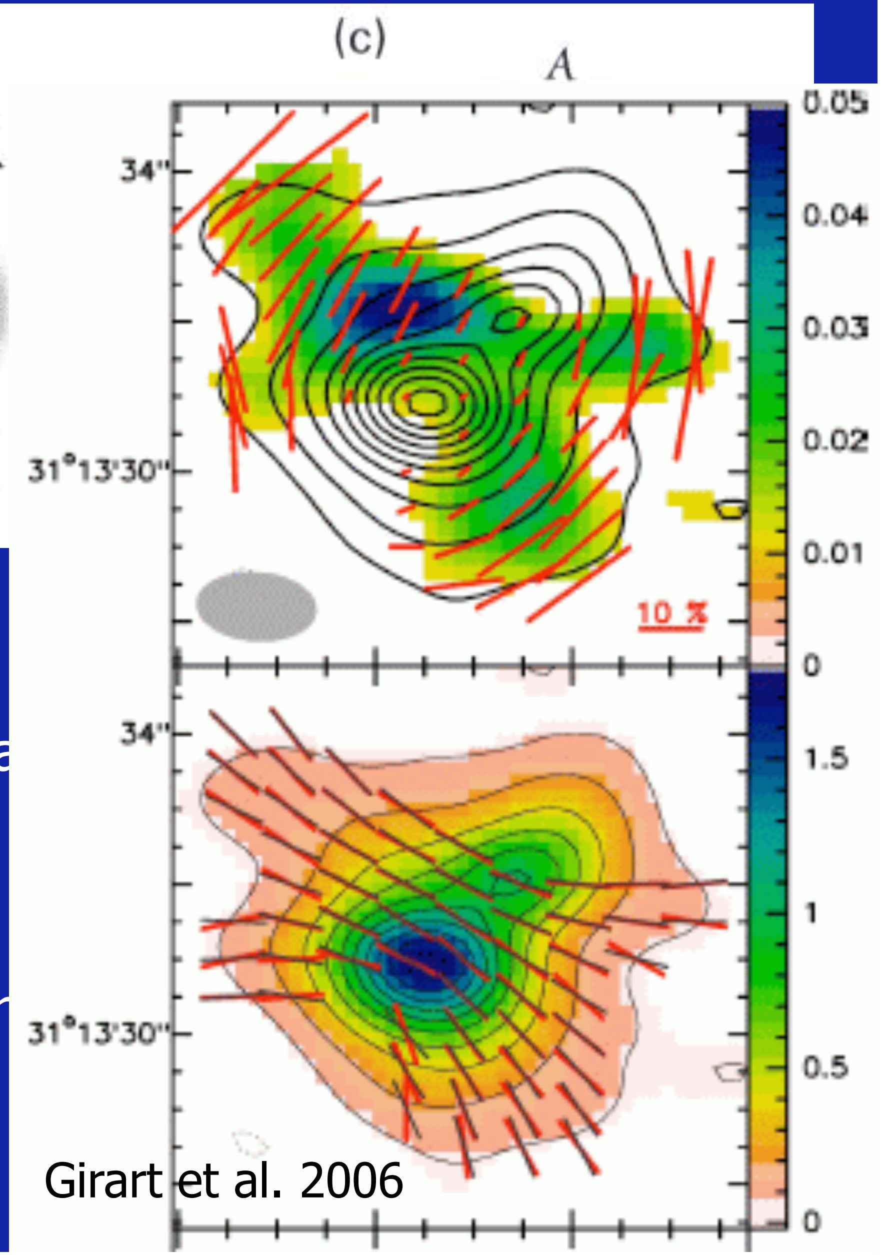


Star Formation timescale: Observations indicate rapid star formation on the order 1-2 million years. Ambipolar diffusion usually requires longer cloud life-times.

→ Maybe gravo-turbulent fragmentation necessary …

Ambipolar diffusion caveat



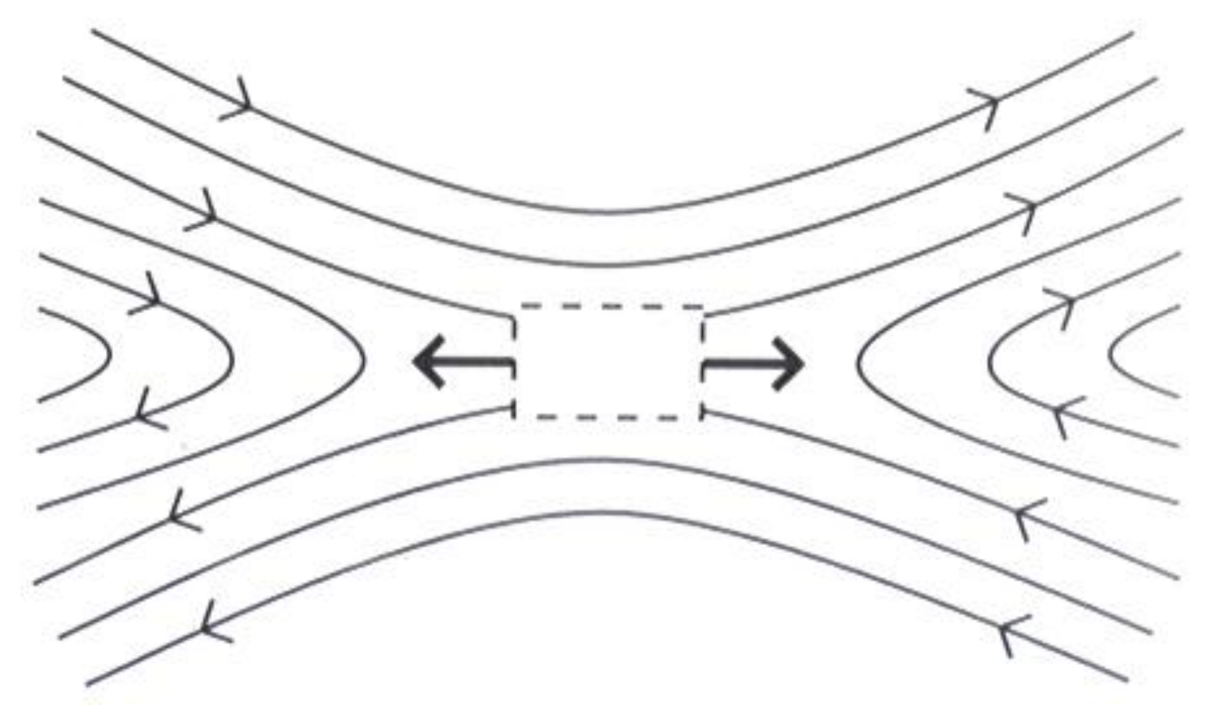


Girart et al. 2006

Star Formation timescale: Observations
on the order 1-2 million years. Ambipola
longer cloud life-times.

→ Maybe gravo-turbulent fragmentation

Magnetic reconnection



- Field lines of opposite direction are dragged together
  → antiparallel B field lines annihilate and magnetic energy is dissipated as heat.
- This process was first invoked to explain large luminosities observed in solar flares.

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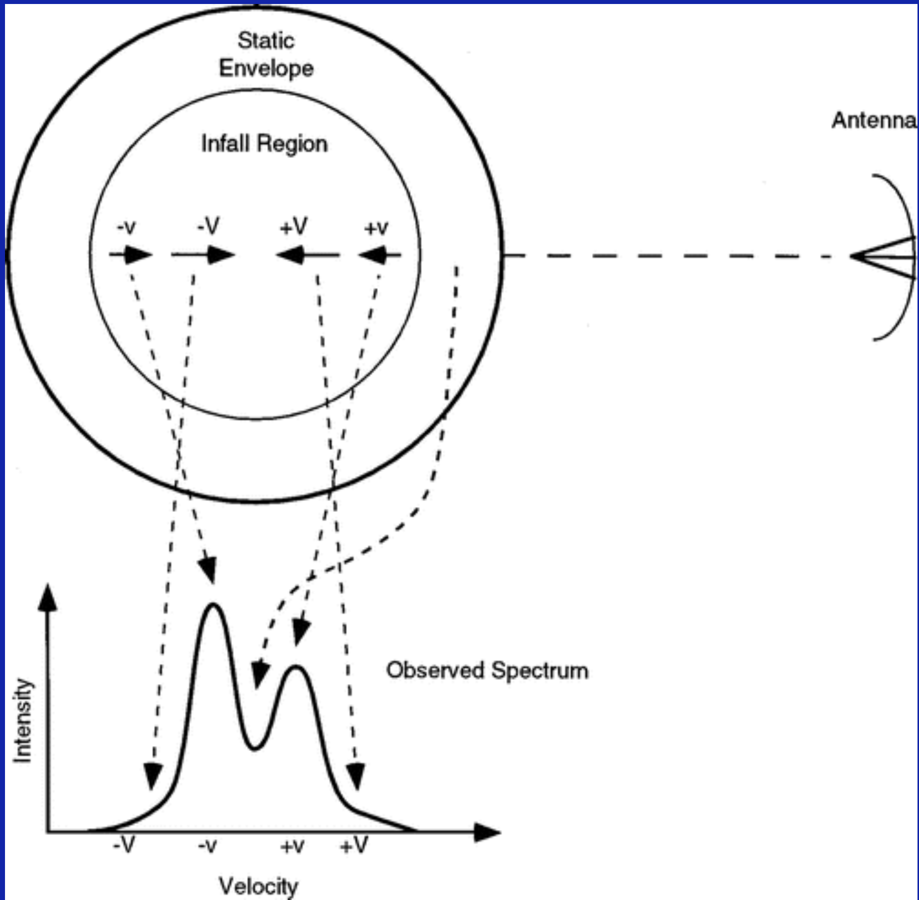
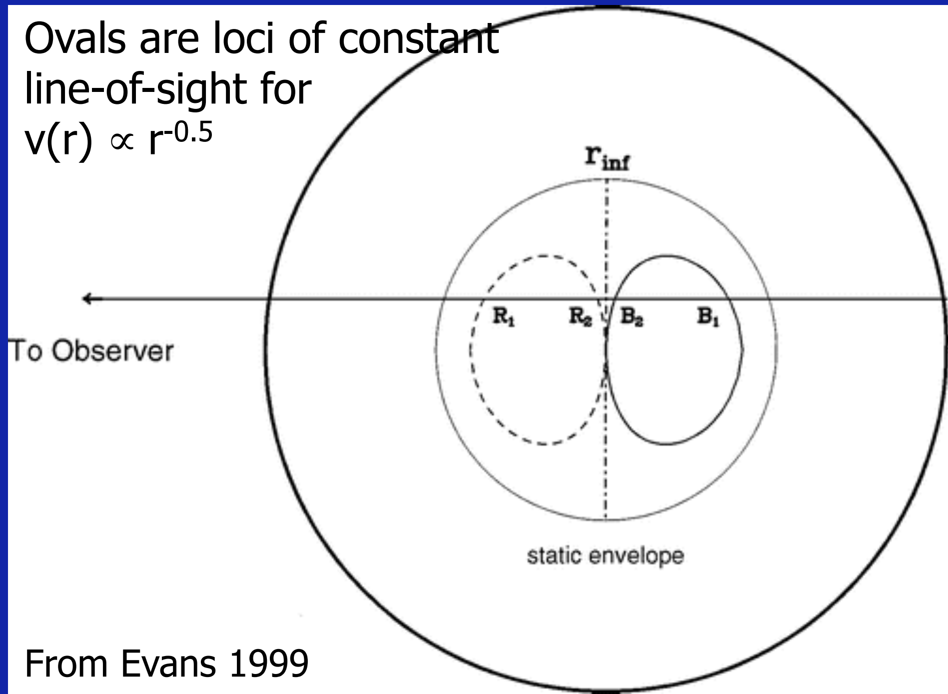
Infall signatures I

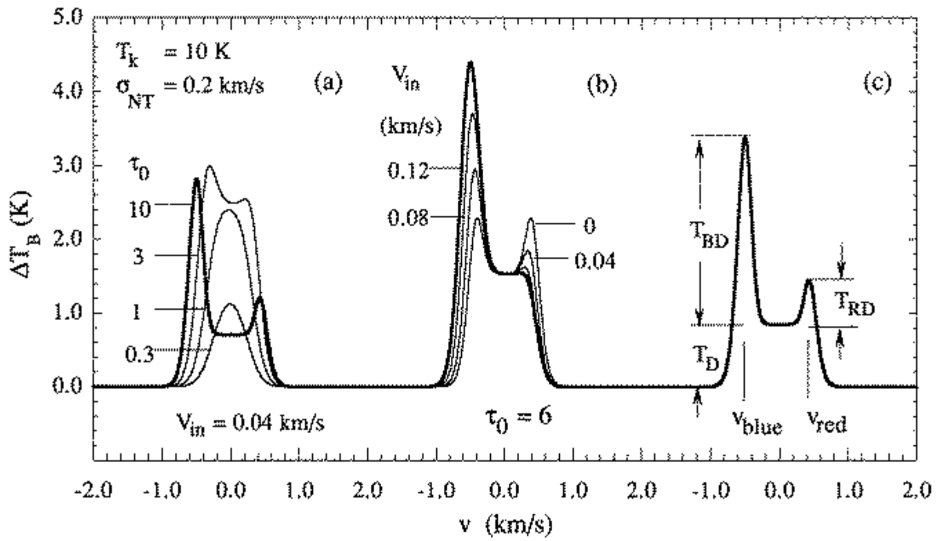




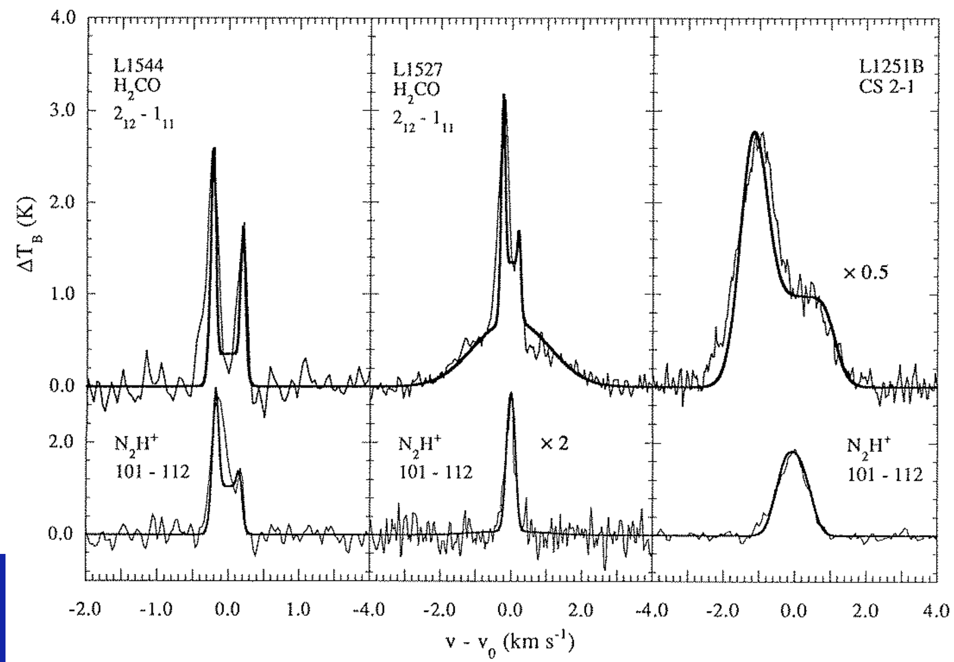
1. Rising T_{ex} along line of sight
2. Velocity gradient
3. Line optically thick
4. An additional optically thin line peaks at center

Infall signatures II

Models



Spectra and fits (Myers et al. 1996)



Model with two uniform regions along the line of sight with velocity dispersion σ and peak optical depth τ_0 → infall velocity v_{in}:

$$v_{in} \approx \sigma^2/(v_{red}-v_{blue}) * \ln((1+\exp(T_{BD}/T_D))/(1+\exp(T_{RD}/T_D)))$$

In low-mass regions v_{in} is usually of the order 0.1 km/s. In high-mass regions V_{in} can exceed 1km/s and hence be supersonic.

Summary

- Hydrostatic equilibrium between thermal pressure and gravitational force.
  → Bonner Ebert mass for gravitationally stable cores.

- Can rotation support cloud stability?

- Magnetic cloud support and ambipolar diffusion

- Observational signatures of infall motions

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Henrik Beuther & Thomas Henning

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