Winds from clusters with non-uniform stellar distribution

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We present analytic and numerical models of the `cluster wind' resulting from the multiple interactions of the winds ejected by the stars of a dense cluster of massive stars. We consider the case in which the distribution of stars (i.e., the number of stars per unit volume) within the cluster is spherically symmetric, has a power-law radial dependence, and drops discontinuously to zero at the outer radius of the cluster. We carry out comparisons between an analytic model (in which the stars are considered in terms of a spatially continuous injection of mass and energy) and 3D gasdynamic simulations (in which we include 100 stars with identical winds, located in 3D space by statistically sampling the stellar distribution function). From the analytic model, we find that for stellar distributions with steep enough radial dependencies the cluster wind flow develops a very high central density and a non-zero central velocity, and for steeper dependencies it becomes fully supersonic throughout the volume of the cluster (these properties are partially reproduced by the 3D numerical simulations). Therefore, the wind solutions obtained for stratified clusters can differ dramatically from the case of a homogeneous stellar distribution (which produces a cluster wind with zero central velocity, and a fully subsonic flow within the cluster radius).

1. INTRODUCTION

Super star clusters (SSCs) are dense clusters of young massive stars first identified in NGC 1705 by Melnick et al. (1985) and in NGC 1569 by Arp & Sandage (1985). Recently, they have been observed in a wide range of star-forming galaxies, such as merging systems (NGC 4038/4039), dwarf galaxies, classical starbursts, as well as in our galaxy amongst many other systems (for a review see Whitmore 2001).

These star clusters can contain hundreds or thousands of very young, energetic stars, and have stellar densities far greater than those seen in normal OB associations. The ages of most of these star clusters are around 1- 10 Myr, their radii typically in the range of ~ 1 -10 pc, and their total cluster masses in the 10^3 - 10^6 M_{sun} range (Melo et al. 2005 reported a mean mass per star cluster, of ~ 2 X 10^5 M_{sun}, for M82). The central stellar densities of SSCs reach up to ~ 10^5 M_{sun} pc^{-3} . However, we can find SSCs with older ages and/or larger masses (Walcher et al. 2006 reports a cluster with 6 X 10^7 M_{sun}).

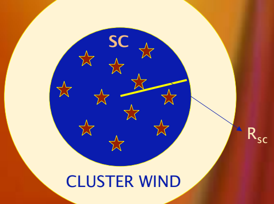
Both in the stationary solution for spherically symmetric winds (Chevalier & Clegg 1985, Canto et al. 2000) and in the numerical calculations of Raga et al. (2001) the stellar distribution (within the cluster) was assumed to be homogeneous. Also, these models are adiabatic (or, more precisely, non-radiative) solutions, which are appropriate for SSCs with low to intermediate mass and/or terminal velocity of SSCs (masses around 10^4 - 10^6 M_{sun} and terminal velocities of ~ 1000 km/s). For more massive stellar clusters, lower stellar wind terminal velocities or higher metallicities, radiative losses within the cluster wind may become important (see Silich et al. 2004).

2. THE ANALYTIC MODEL

We consider N identical stars in a spherical cluster with an outer radius R_c. The stars have a spatial distribution (number of star per unit volume) of the form:

$$n(R) = k_c R^\alpha = \frac{(3+\alpha)N}{4\pi R_c^{3+\alpha}} R^\alpha ,$$

Where R is the spherical radius and α and k_c are the constants.



For an adiabatic, spherically symmetric cluster wind (and neglecting the gravity due to the stellar distribution), the mass, momentum and energy equation are:

WITHIN CLUSTER VOLUME

R < Rsc

$$\frac{1}{R^2}\frac{d}{dR}(\rho V R^2) = n(R)\dot{M}_w , \quad \text{(i)}$$

$$\rho V \frac{dV}{dR} = -\frac{dP}{dR} - n(R)\dot{M}_w V , \quad \text{(ii)}$$

Solving the conservation equations we get,

$$\rho V = \frac{\dot{M}_c}{4\pi R_c^{3+\alpha}} R^{1+\alpha} , \quad \text{(iii)}$$

$$\frac{1}{R^2}\frac{d}{dR}\left[\rho V R^2 \left(\frac{V^2}{2} + h\right)\right] = n(R)\dot{E}_w ,$$

This indicates that the mass flux of the wind is inversely proportional to a positive power of *r*. On the other hand, solving the momentum equation (ii) obtained, (iv)

$$\frac{1-cv^2}{a+bv^2}\frac{dv^2}{v^2} = \frac{dr^2}{r^2} , \quad \text{(v)}$$

The equation (v) admit solutions,

$$v^p\left(a+bv^2\right)^q = Ar \qquad v = (-a/b)^{1/2} = const. \quad \text{(vi)}$$

where a, b, c, p, q are constants: $v = V/V_w \; r = R/R_c$

$a = 1+\alpha,\; b = (1+\alpha+[5+\alpha]\gamma)/(\gamma-1)$ $\quad c = (\gamma+1)/(\gamma-1)$

$p = 1/a,\; q = -(b+a\,c)/(2a\,b)$

CLUSTER WIND

R > Rsc

$$\frac{1}{R^2}\frac{d}{dR}(\rho V R^2) = 0 , \quad \text{(vii)}$$

$$\rho V \frac{dV}{dR} = -\frac{dP}{dR} \quad \text{(viii)}$$

Solving the conservation equations we get,

density ~ r^{-2}

temperature ~ $r^{-4/3}$

Combining the equation (vii) y (viii) we obtain

$$v\left(1-v^2\right)^{1/(\gamma-1)} = \frac{B}{r^2} , \quad \text{(ix)}$$

$$A = \left(\frac{\gamma-1}{\gamma+1}\right)^{p/2}\left(a + b\frac{\gamma-1}{\gamma+1}\right)^q$$

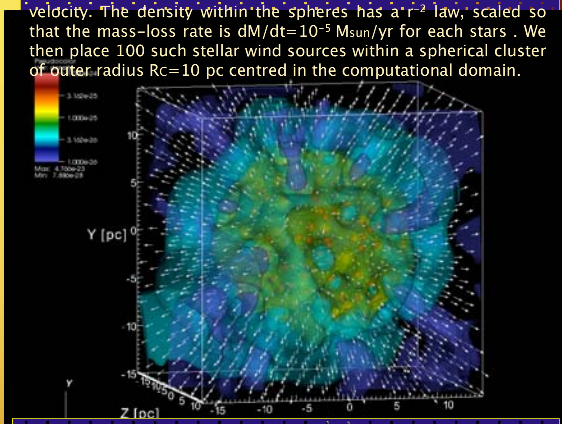
and

$$B = \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}\left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} .$$

3. THE ANALYTIC AND NUMERICAL MODELS

In order to illustrate the analytic star cluster wind solution, we have 3D numerical simulation with the full non-radiative gas dynamic equation. The simulation solve a multiple stellar wind interaction problem with 3D adaptive grid 'yguazu-a' code which is described in detail by Raga (2002). The simulation were computed with maximum reolution of 0.1172 pc (256^3) in a computational domain of 30 pc.

We assumed that the computational domain was initially filled by a homogeneous, stationary ambient medium with T=500 K and n=0.1 cm^{-3}. The stellar winds are imposed in sphere of radius R_w=2.2x10^{18} cm (6 pixels). Within these spheres we impose (at all time) a T_w=15000 K and outwardly directed V_w=1000 km/s velocity. The density within the spheres has a r^{-2} law, scaled so that the mass-loss rate is dM/dt=10^{-5} M_{sun}/yr for each stars . We then place 100 such stellar wind sources within a spherical cluster of outer radius Rc=10 pc centred in the computational domain.

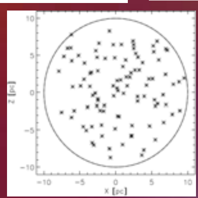


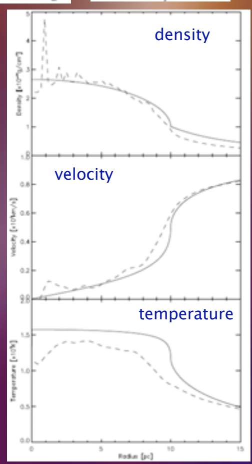
3D rendition of the model with 100 stars distributed homogeneously (α=0) inside of a sphere of 10 pc in radius. In logarithmic color-scale, we present five isosurfaces of density. Depicted by arrows we overlaid the velocity field, the largest arrows corresponds to a magnitude of 1000 km/s.

a. $\alpha \geqslant -1$ **:**

The wind velocity inside the cluster is subsonic, the velocity at the centre of the cluster is zero and the density is finite. The gas velocity increases towards the outer cluster boundary, and the flow at this boundary (r=1).The cluster wind at the centre of the cluster:

$$\rho_0 = \frac{\dot{M}_c}{4\pi A^a R_c^2 V_w} , \quad P_0 = \frac{\gamma-1}{2\gamma}\frac{\dot{M}_c V_w}{4\pi A^a R_c^2} , \quad T_0 = \frac{\gamma-1}{2\gamma}\frac{\mu}{k}V_w^2 .$$





Above figure shows the stellar distribution of x-y plane for the α=0 model. The solid line represent the outer boundary of the cluster. The right figures show the spherically average flow obtained from the α=0 model. The density (top), radial velocity (centre) and temperature (bottom) obtained from numerical simulation (dashed lines and the analytic model (solid lines) is shown as a function of spherical radius *R*.

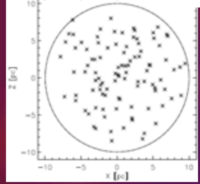
b. $\alpha_{min} \equiv -\frac{3\gamma+1}{\gamma+1} < \alpha \leqslant -1$ **:**

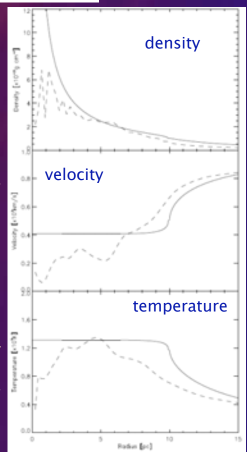
Thus the velocity inside the cluster is subsonic and has a non-zero value at the cluster centre (r=0):

$$v_0 = \sqrt{-\frac{a}{b}} . \quad \text{(x)}$$

The equation (iv) indicates that the mass flux of the wind is inversely proportional to a positive power of *r*. This implies that the central gas density tend to infinite. And the central temperature is,

$$T_0 = \frac{\gamma-1}{2\gamma}\frac{\mu}{k}\left(1+\frac{a}{b}\right)V_w^2 . \quad \text{(xi)}$$

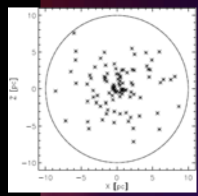


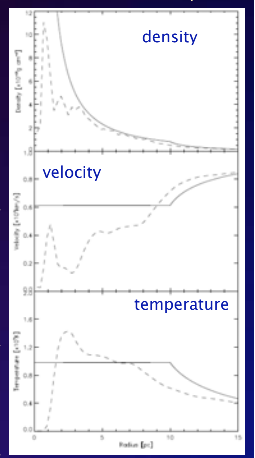


Above figure shows the stellar distribution of x-y plane for the α=-2 model. The right figures show the spherically average flow obtained from the α=-2 model. Substantial differences between the numerical and the analytical model are found for radii smaller than ~4 pc. This is direct result of the under sampling of the stellar distribution function which occurs as a result of the 'proximity criterion' applied for placing the stars in the computational grid. For larger radii, a reasonable agreement between the analytical and numerical result obtained.

c. $\alpha_{cr} \equiv -3 < \alpha \leqslant \alpha_{min}$**:**

The velocity of the wind remains constant (with a supersonic value) within the cluster. Its magnitude is given by equation (x). The temperature is also uniform inside the cluster and is given by equation (xi). The density also goes to infinity at the centre of the cluster and decreases outwards. But the wind escapes from the cluster surface supersonically, and accelerates outwards, until it reaches the terminal velocity.





Above figure shows the stellar distribution of x-y plane for the α=-2.5 model. The right figures show the spherically average flow obtained from the α=-2.5 model. For this model, the analytic solution has a supersonic, outwards velocity in the inner region of the cluster. Again, we obtain substantial deviation between the analytic and the numerical solution in the central region of the cluster and better agreement for larger radii.

We do not consider power-law stellar distribution with α=-3 because they have an infinit number of star (resulting from the strong divergence of the distribution function in the cluster centre).

4. CONCLUSIONS

We have extended the analytic cluster wind model of Paper I to the case of a non-uniform stellar distribution. In particular, we have studied the case of a radially dependent, n(R)=k_c $R^{-\alpha}$ power-law distribution.

Of particular interest is the α=-2 distribution, which corresponds to the stratification of a singular, isothermal, self-gravitating sphere. Such a structure is of interest for modelling the wind from a gravitationally bound stellar cluster. Power-law stellar distributions with other values of α do not have a clear physical justification, but can be considered as a parametrization of stellar distributions with different degrees of central condensation.

We find that for shallow distributions, with -1< α < 0, the cluster wind has zero velocity in the cluster centre. However, for more negative α values the cluster wind has a non-zero, outwards directed velocity (subsonic for α_{min} < α < -1 and supersonic for -3< α < α_{min}) in the centre of the cluster. The solutions with a non-zero central velocity have an infinite central density for the cluster wind.

We have then compared the analytic cluster wind solutions with 3D numerical simulations. For carrying out the simulations, we consider the winds from 100 stars, with a spatial distribution obtained by statistically sampling the appropriate stellar distribution function. We then carry out angular averages of the computed flow variables, and compare the radial dependence of these averages with the predictions obtained from the analytic model.

For different values of α, we obtain a good agreement between the analytic and numerical predictions in the outer regions of the cluster. However, the analytic and numerical solutions have large differences in the central region of the cluster. These differences are a direct result of the fact that only a small number of stars are present in this spatially reduced region, and therefore the continuous mass and energy source distribution assumed in the analytic model is inappropriate for describing the real cluster wind flow.

5. REFERENCES

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