A constrained transport method for the solution of Resistive Relativistic plasmas in the PLUTO code

Giancarlo Mattia[1], Andrea Mignone[2]

[1] *Max-Planck-Institut für Astronomie, Heidelberg, Germany*

[2] *Department of Physics, University of Turin, Turin, Italy*

Introduction

Resistive plasmas are of fundamental importance in the description of physical phenomena such as magnetic reconnection [1], which has been recently pointed out as an efficient site for particle acceleration. Here we present a new module for the PLUTO code [2] that solves the Relativistic Resistive MagnetoHydroDynamics (RRMHD) equations [3] using an IMplicit-EXplicit Runge-Kutta (IMEX-RK) method [4] for the evolution of the electric field. The divergence free condition and the conservation of the electric charge are treated using a constrained transport method [5], where both electric and magnetic fields have a staggered representation. The solution of the Riemann problem is obtained under the frozen limit condition on the direct combination of two Riemann solvers: one for the outermost electromagnetic waves across which only transverse components of electric and magnetic fields can change, and a second one across the sound waves where only hydrodynamical variables have non trivial jumps [5, 6].

Equations

The equations used to describe a relativistic resistive plasma are [3]:

$$\begin{cases} \partial_t D + \nabla\cdot(\rho\gamma\boldsymbol{v}) = 0 \\ \partial_t \boldsymbol{m} + \nabla\cdot(\rho h\gamma^2\boldsymbol{v}\boldsymbol{v} - \boldsymbol{E}\boldsymbol{E} - \boldsymbol{B}\boldsymbol{B} + p_{\rm tot}\mathsf{I}) = 0 \\ \partial_t \mathcal{E} + \nabla\cdot\boldsymbol{m} = 0 \\ \partial_t \boldsymbol{B} + \nabla\times\boldsymbol{E} = 0 \\ \partial_t \boldsymbol{E} - \nabla\times\boldsymbol{B} = -\boldsymbol{J} \end{cases}$$

where h is a function of p and ρ due to the equation of state. The explicit form of the current is:

$$\boldsymbol{J} = \frac{\gamma}{\eta}\left[\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} - (\boldsymbol{E}\cdot\boldsymbol{v})\boldsymbol{v}\right] + (\nabla\cdot\boldsymbol{E})\boldsymbol{v}$$

where η is the resistivity. Such system can be rewritten in a quasi-conservative form as

$$\partial_t \boldsymbol{U} + \boldsymbol{F}(\boldsymbol{U}) = \frac{1}{\eta}\boldsymbol{S}(\boldsymbol{U})$$

where $\boldsymbol{U} = (D, \boldsymbol{m}, \mathcal{E}, \boldsymbol{B}, \boldsymbol{E})$ is an array of conserved variables. Primary zone-centered zones include density, momentum and energy and are stored by their volume averages inside each zone. Conversely, electric and magnetic field are surfaces-averages located at cell interfaces.

IMEX Runge-Kutta

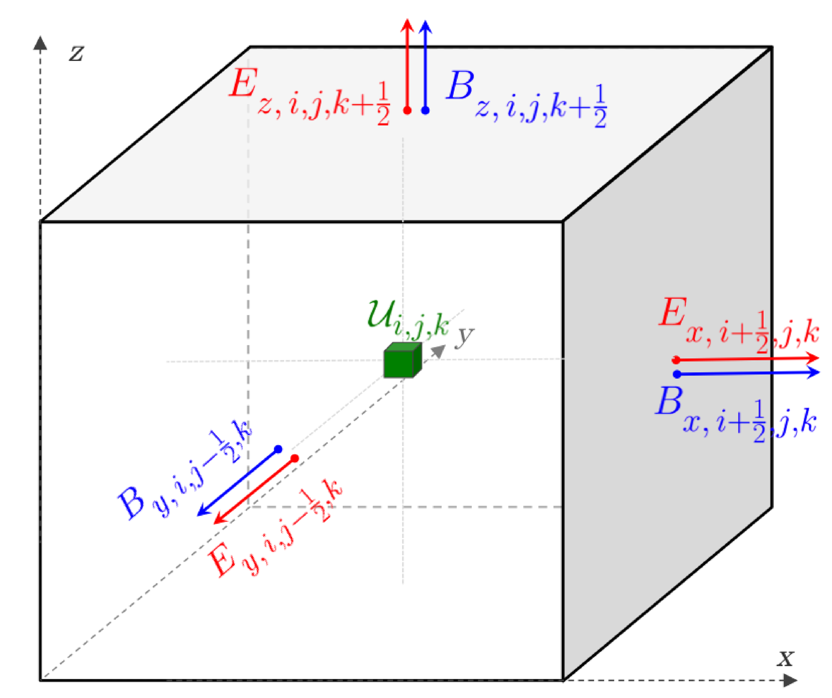
We employ a 2nd order IMEX-RK scheme [4] which consists of the following three stages:

$$\begin{aligned} \mathcal{U}^{(1)} &= \mathcal{U}^n + a\frac{\Delta t}{\eta}\mathcal{S}^{(1)} \\ \mathcal{U}^{(2)} &= \mathcal{U}^n + \Delta t\mathcal{R}^{(1)} \\ &\quad + \frac{\Delta t}{\eta}\left[(1-2a)\mathcal{S}^{(1)} + a\mathcal{S}^{(2)}\right] \\ \mathcal{U}^{n+1} &= \mathcal{U}^n + \frac{\Delta t}{2}\left(\mathcal{R}^{(1)} + \mathcal{R}^{(2)}\right) \\ &\quad + \frac{\Delta t}{2\eta}\left[\mathcal{S}^{(1)} + \mathcal{S}^{(2)}\right] \end{aligned}$$

where a is given from Butcher notation.
The implicite steps are solved through an iterative scheme, based on a multidimensional Newton-Broyden method.

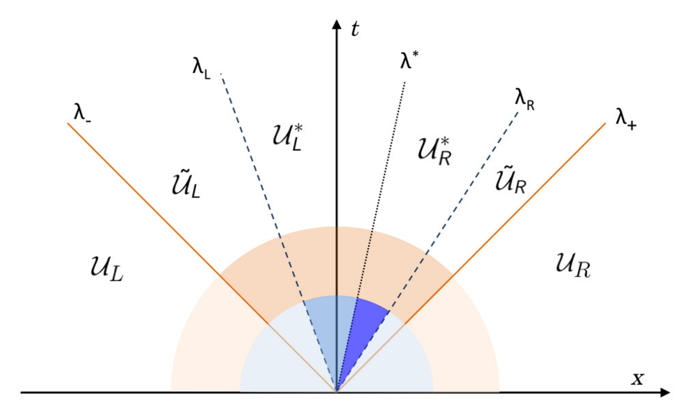
Constrained Transport Method

- start with zone-centered and face-centered quantities;
- average primary staggered fields and perform the implicit update at zone centers, then achieve the implicit step on the staggered electric field;
- compute interface Godunov fluxes and perform the predictor step. Also compute the explicit steps needed for the implicit step;
- repeat the implicit step;
- perform the corrector stage and obtain the solution at next time level.



Riemann Solver

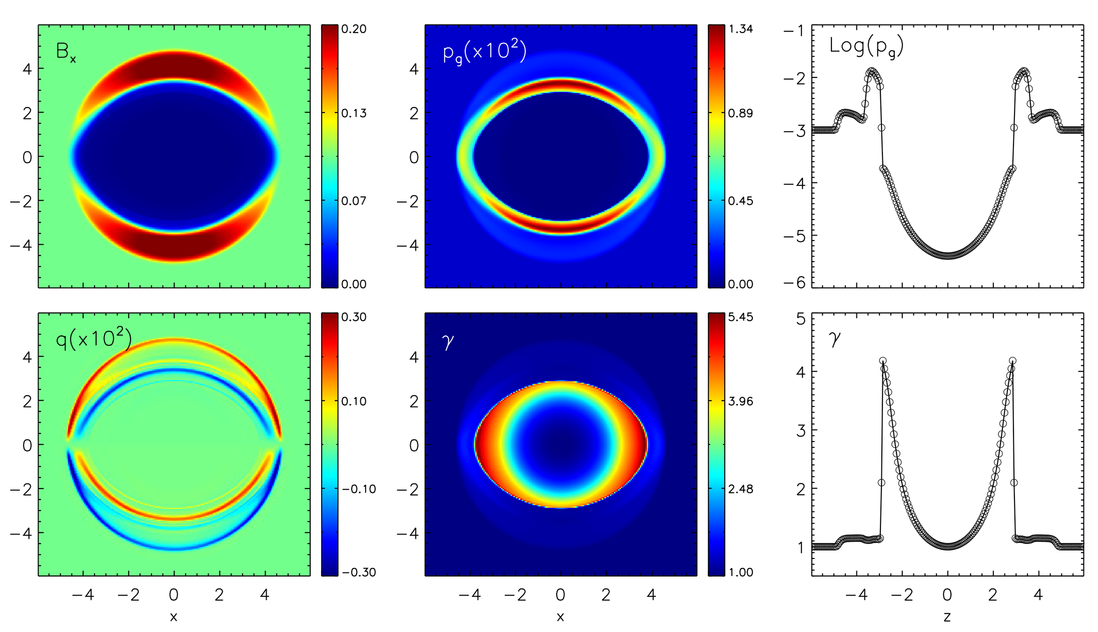
The solution to the Riemann problem is obtained under condition $\sigma = 0$ and is based on the combination of two solvers (M-HLLC):

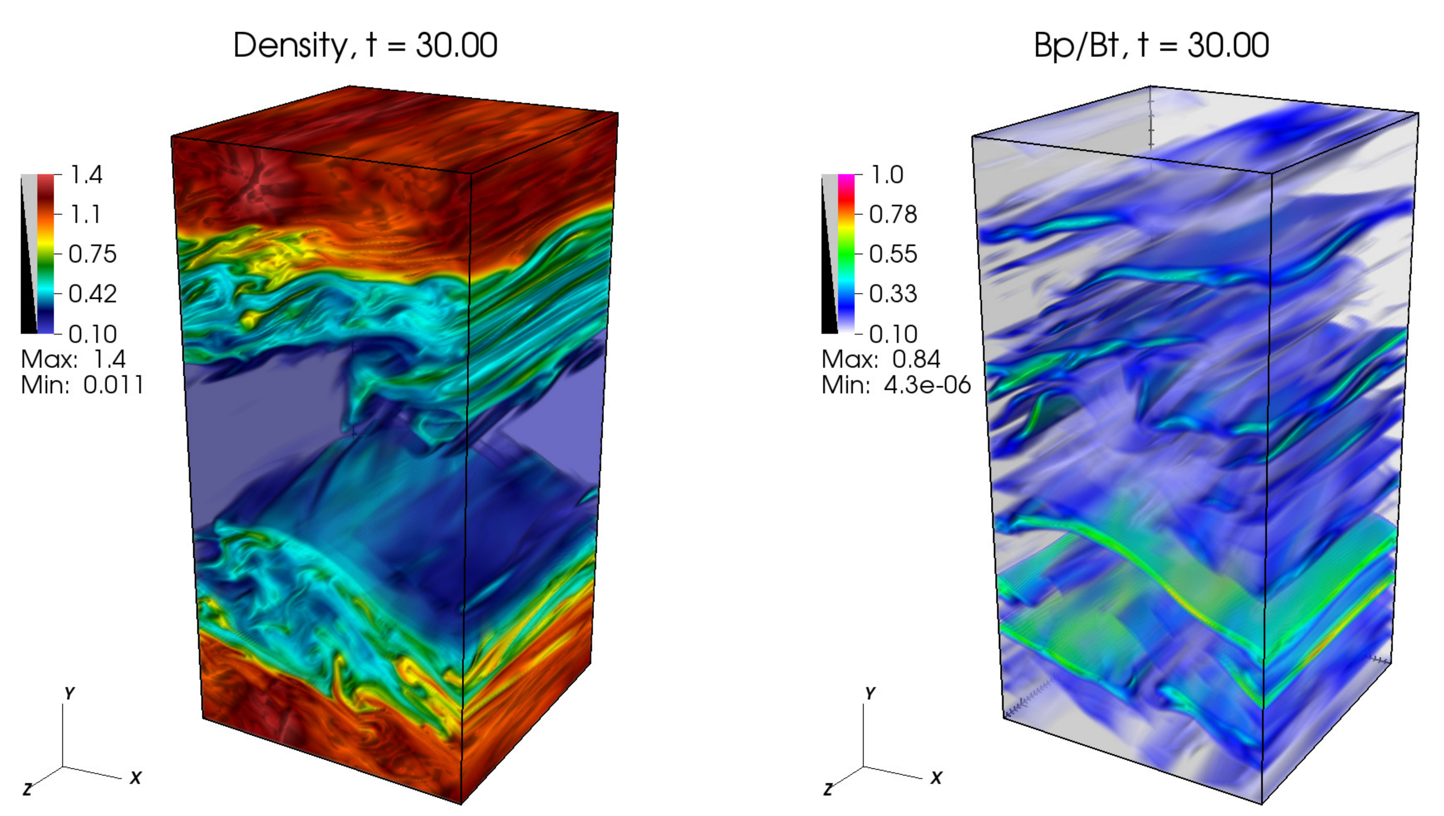


- a Maxwell solver for the outermost electromagnetic waves across which only transverse component of electric and magnetic field can change and for which jump conditions are derived directly from Maxwell's equation;
- a solver across the sound waves where only hydrodynamical variables have non trivial jumps, based on an approximate Harten-Lax-van Leer-Contact Riemann solver.

Results

Spherical explosion at $t = 4$ using $\eta = 10^{-6}$ and the CT scheme. The upper panels show, from left to right, 2D slices in the xy plane of B_x, gas pressure pg and its 1D profile along the z axis. In the bottom panels we show 2D slices of the charge, Lorentz factor together with its one-dimensional cuts of along the z axis. Unlike the 2D case, there is a local production of electric charge. Plasma is accelerated mostly in the $x-$direction to large Lorentz factor ($\gamma \approx 5.5$).





Volume rendering of the density (left panel) and poloidal to toroidal magnetic field (right panel) for the three-dimensional relativistic Kelvin-Helmholtz instability at $t = 30$. The conductivity is $\sigma = 10^5$ and the M-HLLC solver has been employed. The large-scale motion remains confined along the initial shear direction and sheet-like thin structures characterize the turbulent state.

Conclusion

The new module of PLUTO code [2] is able to solve RRMHD equations in 1D, 2D or 3D both with low and high resistivity. The solution tends to the ideal solution if the resistivity is low, while it becomes more diffusive (as it should be) as the resistivity increases. The module is available in the latest version of PLUTO (4.4) under conditions.

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