Analytical Model of Magnetically Dominated Jet:

— jet launching, acceleration and collimation

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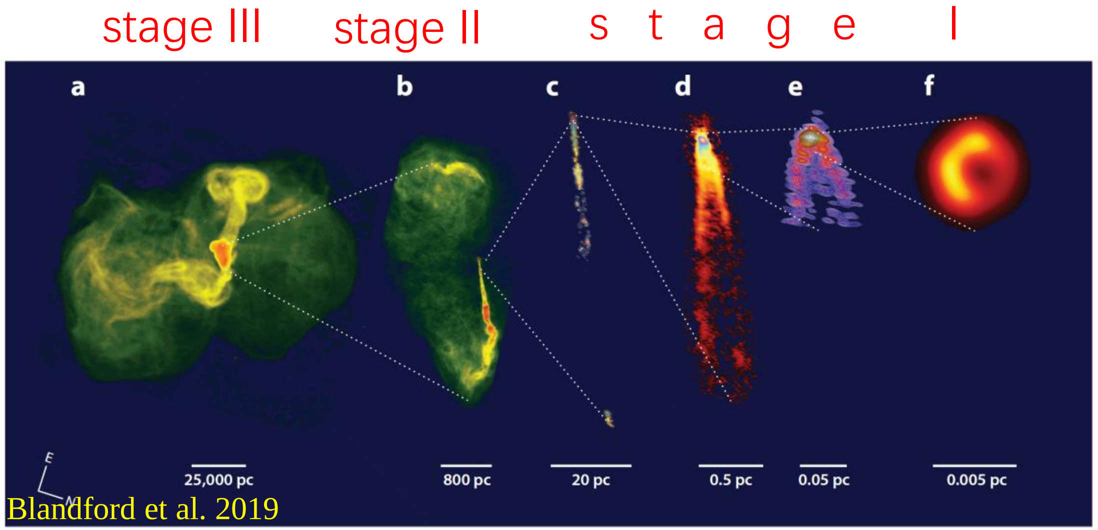
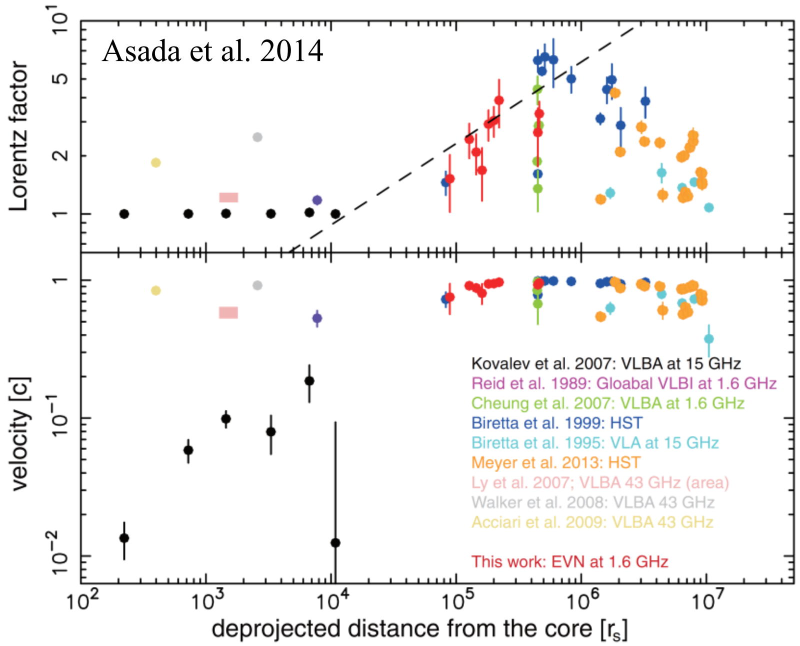
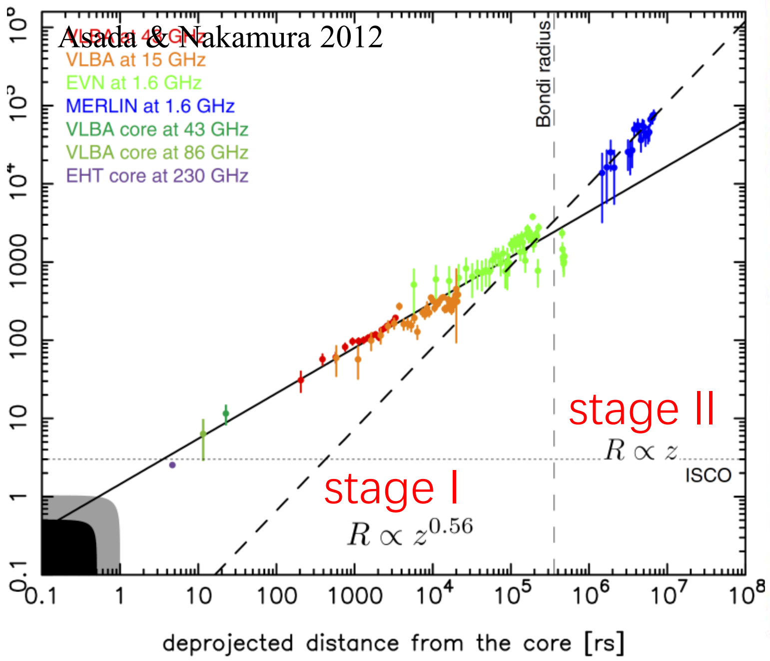
Chen & Zhang 2021, ApJ, 906, 105

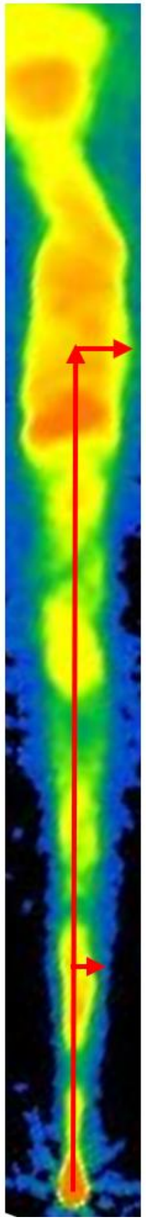
Jets: evolution

- stage I:
  - collimating
  - accelerating
  - “parabolic”
- stage II:
  - “collimating”
  - “conical”
- “stage III”
  - terminal, lobe









Jets: stage I: magnetically dominated ⇨ "force-free"

$$\mathbf{B} = \frac{1}{r^2 \sin\theta}\frac{\partial \Psi}{\partial \theta}\hat{\mathbf{r}} - \frac{1}{r\sin\theta}\frac{\partial \Psi}{\partial r}\hat{\theta} + \frac{\Phi}{r\sin\theta}\hat{\phi}$$

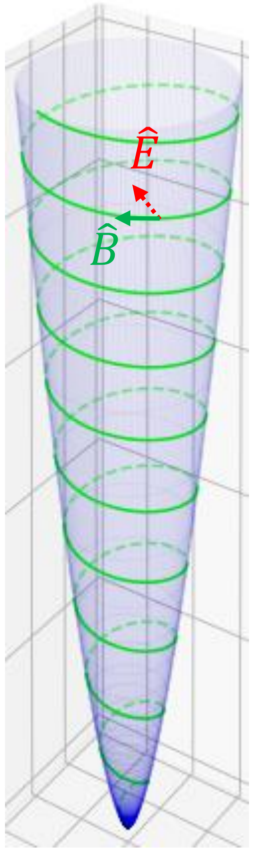
$$\boxed{\Psi} = r\sin\theta A_\phi \qquad \boxed{\Phi} = r\sin\theta B_\phi$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\boxed{\Omega}\nabla\Psi = -\Omega r \sin\theta \hat{\phi} \times \mathbf{B}$$

$$\frac{\mathbf{j}}{\sigma_c} = \mathbf{E} + \mathbf{v} \times \mathbf{B} \overset{\text{ideal MHD}}{=\!=\!=\!=} 0$$

$$\rho\,(\mathbf{u}\cdot\triangledown)\,\mathbf{u} = \rho_e \mathbf{E} + \mathbf{j}\times\mathbf{B} \overset{\text{force free}}{=\!=\!=\!=} 0.$$



$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \theta^2} - \frac{\cot\theta}{r^2}\frac{\partial \Psi}{\partial \theta}$$
$$+\Phi'\Phi - \left\{\frac{\Omega'}{\Omega}\left[\left(\frac{\partial \Psi}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial \Psi}{\partial \theta}\right)^2\right] + \frac{\partial^2 \Psi}{\partial r^2} + \frac{2}{r}\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial \Psi}{\partial \theta}\right\}(\Omega r \sin\theta)^2 = 0$$

the "pulsar" equation (established 1960s)

Jets: solve equation

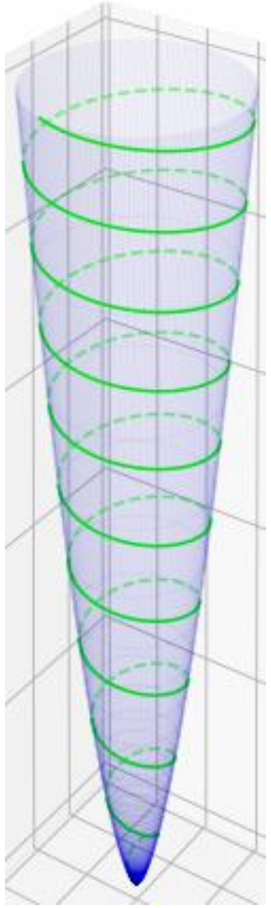
- simulation: BH rotating slow or fast produce similar jet configuration
  (e.g., Tchekhovskoy, McKinney & Narayan 2008)
- math expect: two terms (equations): non-rotating and rotating

$$\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}-\frac{\cot\theta}{r^2}\frac{\partial\Psi}{\partial\theta}$$
$$+\Phi'\Phi-\left\{\frac{\Omega'}{\Omega}\left[\left(\frac{\partial\Psi}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial\Psi}{\partial\theta}\right)^2\right]+\frac{\partial^2\Psi}{\partial r^2}+\frac{2}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial\Psi}{\partial\theta}\right\}(\Omega r\sin\theta)^2=0$$

if real

non-rotating	$\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}-\frac{\cot\theta}{r^2}\frac{\partial\Psi}{\partial\theta}=0$
rotating	$\frac{\Phi'\Phi}{\Omega^2r^2\sin^2\theta}-\left\{\frac{\Omega'}{\Omega}\left[\left(\frac{\partial\Psi}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial\Psi}{\partial\theta}\right)^2\right]+\frac{\partial^2\Psi}{\partial r^2}+\frac{2}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial\Psi}{\partial\theta}\right\}=0$

- The two solutions match each other!



Jets: solve equation

$$\Psi = r^{\nu} T_{\mathrm{nr}}(\theta) \qquad 0 \leq \nu \leq 2$$

$$T_{\mathrm{nr}}(\theta) = C_2 y \; {}_2F_1\left(1 - \frac{\nu}{2}, \frac{1}{2} + \frac{\nu}{2}, 2, y\right)$$

$$T_{\mathrm{r}}(y) = A_2 e^{\frac{\nu}{s+\nu} \int_1^y \frac{G_1(t) + A_1 G_2(t)}{A_1 G_3(t) + G_4(t)} dt}$$

non-relativistic to relativistic regimes

apply: $\theta \ll 1$ or $\theta \to \pi/2$

$\Omega r \sin\theta \gg 1$ or $\Omega r \sin\theta \ll 1$

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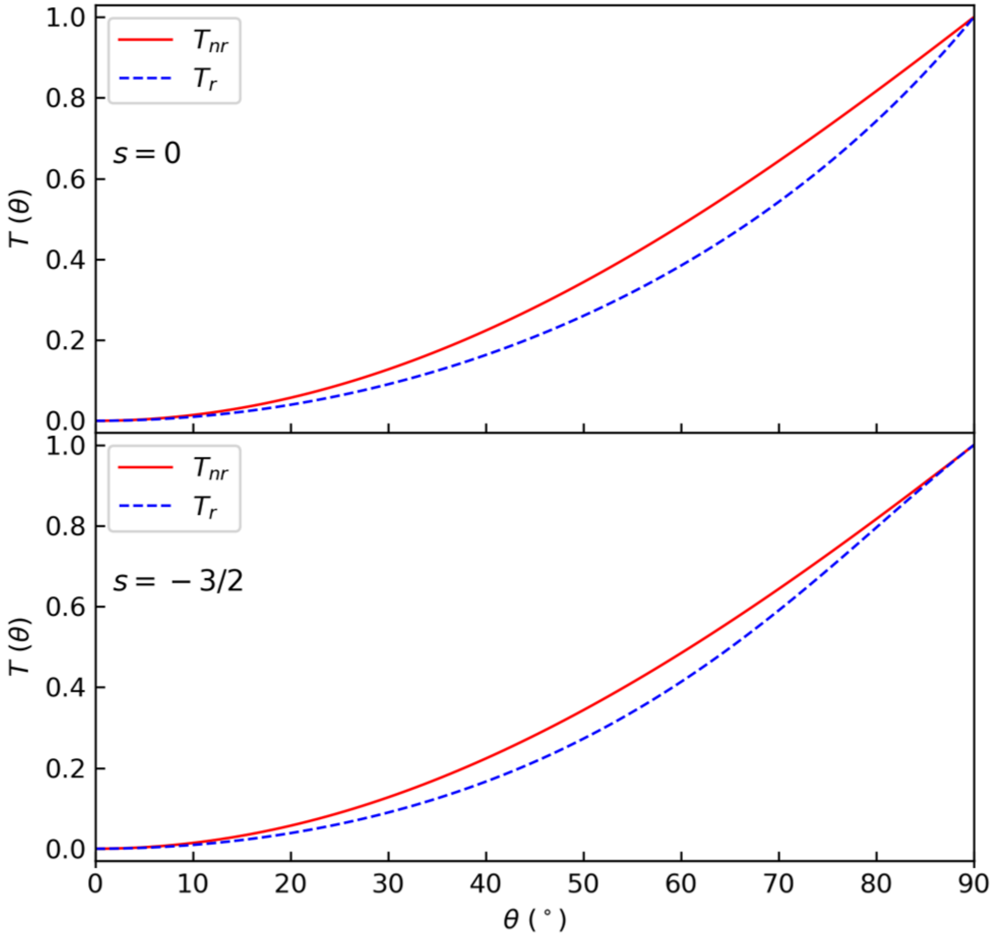


Jets: magnetic field and velocity

- magnetic field

$$B_p = \frac{2\Psi}{R^2}$$
$$B_\phi = -\frac{2\Omega\Psi}{R}$$

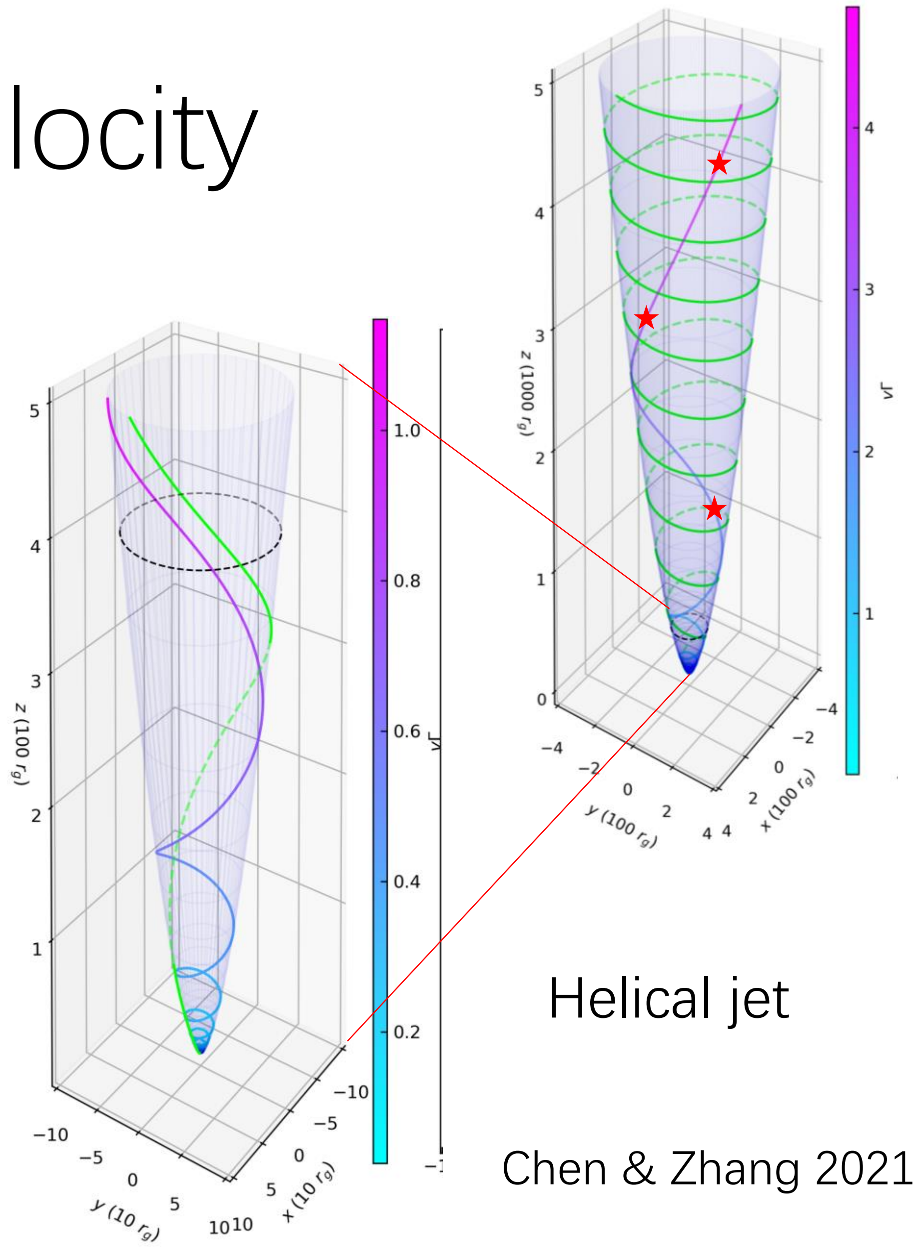
- drift velocity ⇔ cold plasma velocity

$$D_{\rm fd} \equiv \frac{(v\Gamma)^2 - (v_{\rm d}\Gamma_{\rm d})^2}{(v_{\rm d}\Gamma_{\rm d})^2} \ll 1 \qquad (\Omega R \gg 1 \text{ or } \Omega R \ll 1)$$

- velocity

$$\begin{aligned} v_\phi &= \Omega r \sin\theta \frac{B_p^2}{B^2} \approx \frac{\Omega R}{1+(\Omega R)^2}, \\ v_p &= -\Omega r \sin\theta \frac{B_\phi B_p}{B^2} \approx \frac{(\Omega R)^2}{1+(\Omega R)^2}, \\ v &= \Omega r \sin\theta \frac{B_p}{B} \approx \frac{\Omega R}{\sqrt{1+(\Omega R)^2}}, \\ v\Gamma &\approx \Omega R. \end{aligned}$$

From non-relativistic to relativistic



Helical jet

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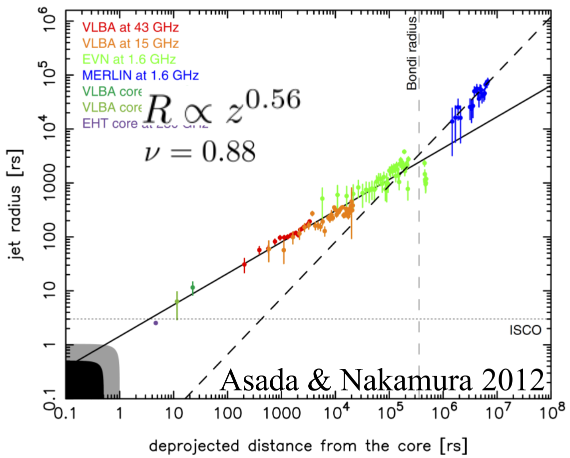
Consist with previous asymptotic results at ultra-relativistic regime (Blandford, Narayan, Tchekhovskoy, Beskin, Komissarov, Lyubarsky, …)

Jets: acceleration and collimation

- configuration : (quasi-parabolic at $\theta \ll 1$)

$$\Psi = C r^{\nu} \sin^2\theta \; {}_2F_1\left(1-\frac{\nu}{2}, \frac{1}{2}+\frac{\nu}{2}, 2, \sin^2\theta\right) \quad \Rightarrow \quad R = C_2^{-1/2}\Psi^{1/2} z^{1-\nu/2}$$

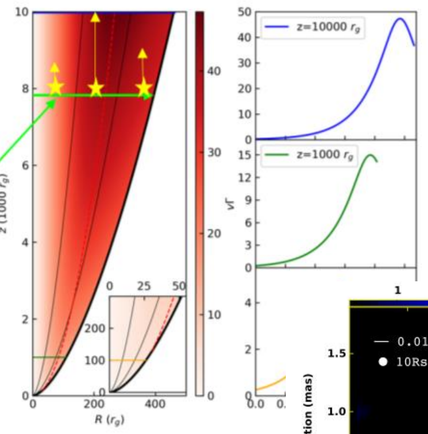


- acceleration: stages I, II, III (non-relativistic to relativistic)

$$\frac{1}{(v\Gamma)^2} \simeq \frac{1}{(\Omega R)^2} + \frac{2-\nu}{4\,(c/\theta)^2}$$

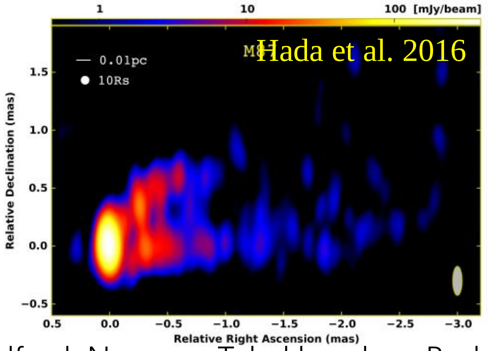
- spine/layer jet

$$v\Gamma = \Omega R$$



- hollow jet (for a BZ jet)

$$S_z = \frac{B_\phi^2}{4\pi} = \frac{C_2}{\pi}\Omega^2\Psi r^{\nu-2} = \frac{C_2^{2/\nu}}{\pi}\Omega^2\Psi^{2-2/\nu}\theta^{-2+4/\nu} = \frac{\alpha^2}{\pi}C_2^{2\lambda+2} z^{2\lambda\nu+2\nu-4\lambda-4} R^{4\lambda+2}$$



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Jets: current and charge

- electric current

$$J = \sqrt{cP_{\rm jet}} \approx 5.8 \times 10^{17} \sqrt{P_{44}}\ {\rm A}$$

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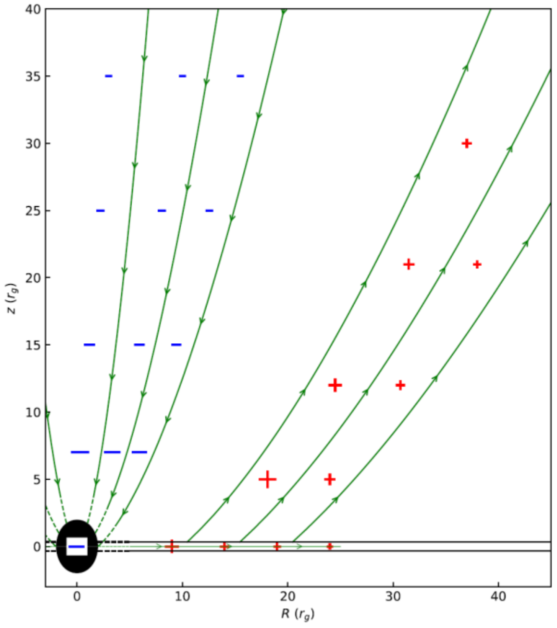
$\sim 3.9 \times 10^{18}\ {\rm A}$ Kronberg et al. 2011

$\sim 1.0 \times 10^{46}\ {\rm erg\ s^{-1}}$ Zhang et al. 2018



- electric potential difference
  ("gap" near BH horizon)

$$\Delta V = \sqrt{P_{\rm jet}/c} \approx 1.7 \times 10^{19} \sqrt{P_{44}}\ {\rm Volts}$$

- black hole charge

$$r_{\rm Q} = \sqrt{G}Q/M \approx \sqrt{GP_{\rm jet}/c^5} \approx 1.7 \times 10^{-8} \sqrt{P_{44}}$$

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Consist with previous asymptotic results at ultra-relativistic regime (Blandford, Narayan, Tchekhovskoy, Beskin, Komissarov, Lyubarsky, ···)

Thanks!