Analytical Solution of Magnetically Dominated Astrophysical Jets

— Jet Launching, Acceleration, and Collimation (Chen & Zhang, 2021, ApJ, 906, 105)

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Part I. Solve Equation

- Jet configuration ⇔ the radial balance equation

$$\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}-\frac{\cot\theta}{r^2}\frac{\partial\Psi}{\partial\theta}$$
$$+\Phi'\Phi-\left\{\frac{\Omega'}{\Omega}\left[\left(\frac{\partial\Psi}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial\Psi}{\partial\theta}\right)^2\right]+\frac{\partial^2\Psi}{\partial r^2}+\frac{2}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial\Psi}{\partial\theta}\right\}(\Omega r\sin\theta)^2=0$$

Simulations: jet configuration seems not depend on rotation (e.g., Tchekhovskoy et al. 2008)

$$\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}-\frac{\cot\theta}{r^2}\frac{\partial\Psi}{\partial\theta}=0$$

⇦ non-rotating

rotating ⇩

$$\frac{\Phi'\Phi}{\Omega^2 r^2\sin^2\theta}-\left\{\frac{\Omega'}{\Omega}\left[\left(\frac{\partial\Psi}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial\Psi}{\partial\theta}\right)^2\right]+\frac{\partial^2\Psi}{\partial r^2}+\frac{2}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial\Psi}{\partial\theta}\right\}=0$$

- The solutions of these two term equations are matching each other very well.

$$\Psi=Cr^\nu\sin^2\theta\;{}_2F_1\left(1-\frac{\nu}{2},\frac{1}{2}+\frac{\nu}{2},2,\sin^2\theta\right)\quad(0\le\nu\le2)$$

Part II. Jet Properties

1. Jet configuration: (quasi-parabolic at $\theta\ll1$)

$$\Psi=Cr^\nu\sin^2\theta\;{}_2F_1\left(1-\frac{\nu}{2},\frac{1}{2}+\frac{\nu}{2},2,\sin^2\theta\right)\;\Rightarrow\;R=C^{-1/2}\Psi^{1/2}z^{1-\nu/2}$$

2. Drift velocity well match cold plasma jet velocity

non-relativistic to relativistic

3. Jet acceleration: stages I, II III (non-relativistic to relativistic)

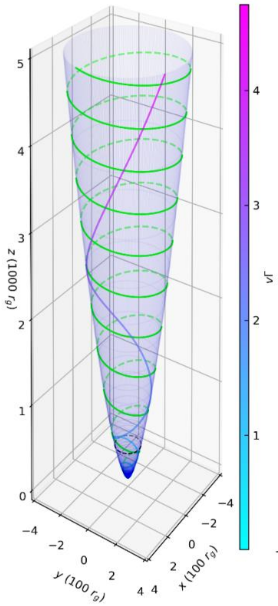
$$\frac{1}{(v\Gamma)^2}\simeq\frac{1}{(\Omega R)^2}+\frac{2-\nu}{4(c/\theta)^2}$$

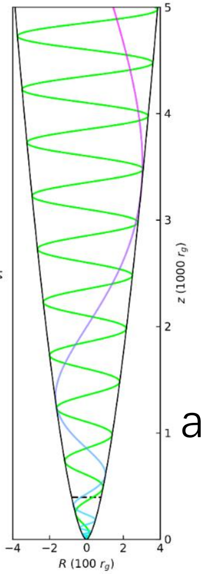
4. For a BZ jet (jet power $P_{\rm jet}=P_{44}\times10^{44}$ erg s^{-1}):

jet electric current → $J=\sqrt{cP_{\rm jet}}\approx5.8\times10^{17}\sqrt{P_{44}}$ A

electric potential difference ("gap") → $\Delta V=\sqrt{P_{\rm jet}/c}\approx1.7\times10^{19}\sqrt{P_{44}}$ Volts

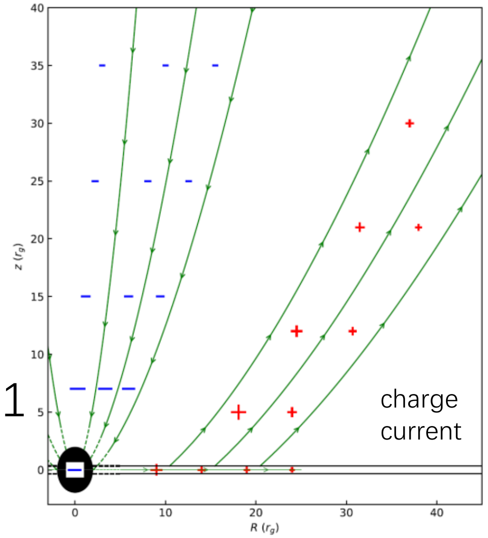
black hole charge → $r_{\rm Q}=\sqrt{G}Q/M\approx\sqrt{GP_{\rm jet}/c^5}\approx1.7\times10^{-8}\sqrt{P_{44}}$





BZ Jet

a=0.1



Consist with previous asymptotic results at ultra-relativistic regime (Blandford, Narayan, Tchekhovskoy, Beskin, Komissarov, Lyubarsky, ⋯)