6. Übung zu: Astronomie & Astrophysik II

(Due: Thu., 8.7.2004)

Problem 6.1: The age of the universe

The expansion, size and general structure of the universe can be described with the Friedman equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2(\Omega_m \cdot a^{-3} + \Omega_k \cdot a^{-2} + \Omega_\Lambda).$$

Different value combinations of the three "energies" Ω_m (matter), Ω_k (curvature) and Ω_Λ (cosmological constant, dark energy) as measured today lead to solutions that show vastly different properties in geometry (finite/infinite), expansion history and future, or existence of a big bang. Their combination in the Friedman equation connects these properties directly to the value for the scale parameter a, its change with time $\dot{a}$ and thus the Hubble constant H, normalized to the Hubble constant today, H_0.
In this exercise you are supposed to give general expressions for the current scale parameter $a = a(t)$, and velocity $\dot{a} = \dot{a}(t)$ and expression and value for the present age of the universe t_0 and for different cosmologies. Use the definition of a that $a(t_0) = a_0 \equiv 1$ that together with the above equation gives

$$\Omega_m + \Omega_k + \Omega_\Lambda = 1.$$

At a measured Hubble constant $H_0 = 70$ km s^{-1} Mpc^{-1} how do the derived ages relate to the ages of the oldest known stars (13 Gyr)? Discuss how each solution compares to the observed reality.
The cosmological models are:

a) $\Omega_m = 1$, $\Omega_\Lambda = 0$

b) $\Omega_k = 1$, $\Omega_m = 0$

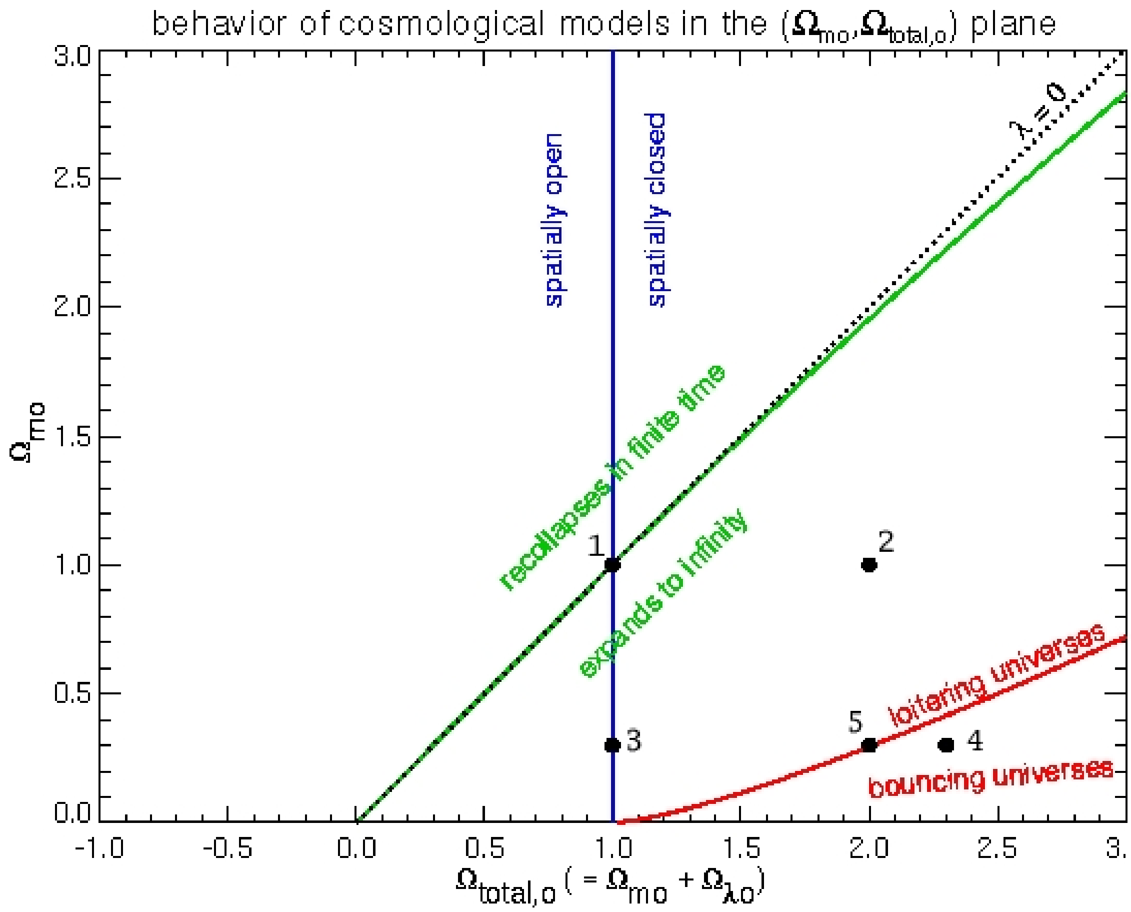
c) $\Omega_\Lambda = 1$, $\Omega_m = 0$ (don't compute the age here)

d) $\Omega_k = 0$ (give expressions for a and for t, and compute t_0 for the special case $\Omega_m = 0.3$). Here you will need to take into account that $(\Omega_m - 1) < 0$, which, when inside a square-root, will give an imaginary number. Use the complex expression and the identity $-i \arcsin(iz) = \operatorname{arcsinh}(z)$ to solve the integral.

Problem 6.2: Λ cosmologies

In case of a non-zero cosmological constant the expansion history (and future) of the universe can be very diverse, depending on the absolute values and the relation between Ω_Λ and Ω_m.

a) If there was a big bang and the universe is expanding now with a positive cosmological constant, how does its motion in the t-a–plane schematically look like? Draw curves for the different Ω_Λ–Ω_m combinations 1–5 shown in the figure.



b) One class of these models is called Loitering Cosmologies (auf deutsch in etwa: "Bummelkosmologien", number 5 in the figure). For these $\Omega_\Lambda > 1$, and the universe is spatially closed ($\Omega_k > 0$) but the universe expands to infinity. This loitering combination is at the limit of the bounce models and has the effect that there is a redshift at which many objects with the same redshift are observed, and few beyond it. This loitering redshift is depending on the value of Ω_m:

$$1 + z_{\text{many}} = 2 \cosh \left(\frac{1}{3} \text{arccosh} \left(\Omega_m^{-1} - 1 \right) \right)$$

Plot the dependence of z_{many} on Ω_m. Discuss what the observation of the (very conservative) estimate of $\Omega_m = 0.04$ – which only takes the luminous matter into account and disregards e.g. dark matter – has on the possibility of a loitering cosmology being realised for our universe, given the observational evidence on distant galaxies.