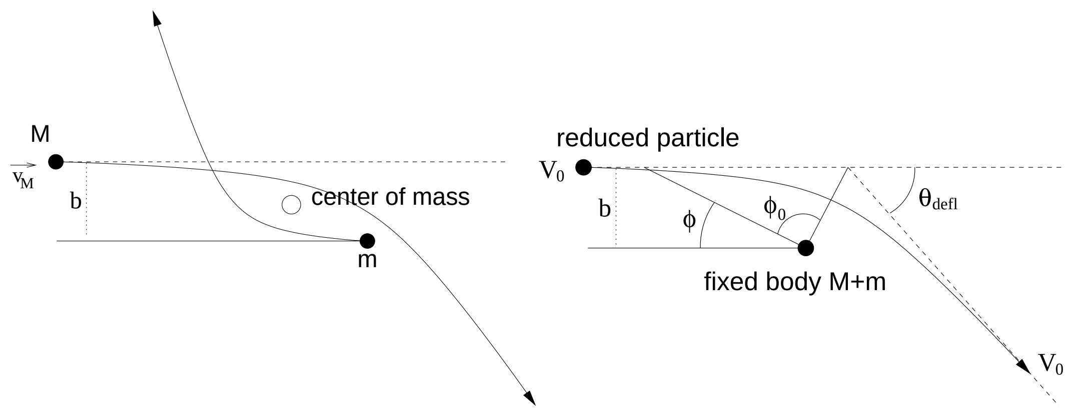
4. Übung zu: Astronomie & Astrophysik II

(Due: Thu., 10.6.2004)

Problem 4.1: Dynamic Friction

In a gravitational encounter of two object the momentum is conserved. If the masses are different then their velocities are not conserved. If a mass, e.g. a star moves through a field of other masses, e.g. in the Milky Way, its velocity will constantly be decreased by the stars around it. This effect is called *dynamic friction*. This problem is about the dynamic friction for a point like mass M passing through a population of stars of uniform individual mass m.



First derive an equation for the effect of the encounter with one star. Assume that $(\vec{x}_M, \vec{v}_M)$ and $(\vec{x}_m, \vec{v}_m)$ are the positions and velocities of M and m, respectively. Let $\vec{r} = \vec{x}_m - \vec{x}_M$, the separation vector between M and m, and $\vec{V} = \dot{\vec{r}}$. Then the separation vector $\vec{r}$ obeys the equation

$$\left(\frac{mM}{m+M}\right)\ddot{\vec{r}} = -\frac{GMm}{r^2}\vec{e}_{\vec{r}} \tag{1}$$

where $\vec{e}_{\vec{r}}$ is the unity vector in direction of $\vec{r}$. This is the equation of motion of a fictitious particle, the 'reduced particle', moving in the Keplerian potential of a fixed body of mass $m + M$. This is sketched in the figure.

a) Give a relation between the changes of the individual velocity vectors $\Delta\vec{v}_M$ and $\Delta\vec{v}_m$, and $\Delta\vec{V}$ (sign?).

b) Since the momentum is conserved, how do the changes in momenta of M and m relate?

c) Combine the equations from (a) and (b) to get a relation for $\Delta\vec{v}_M$ (the quantity that we are interested in), as a function of $\Delta\vec{V}$.

d) Now evaluating $\Delta\vec{V}$: Let the distance perpendicular to the initial direction of motion $\vec{V}_0 = \vec{V}(t = -\infty)$ have the length b (see figure). b is called the *impact parameter* of the encounter. The conserved angular momentum per unit mass associated with the motion of the reduced particle is $L = \vec{b}\vec{V}_0 = bV_0$. Then the radius r and azimuthal angle ϕ in a

Keplerian orbit are related by

$$\frac{1}{r} = C\cos(\phi - \phi_0) + \frac{G(M+m)}{b^2 V_0^2}, \tag{2}$$

where ϕ is shown again in the figure. Differentiate this equation to derive an expression for dr/dt as a function od bV_0. (Hints: Re-use equation 2. And what can $r^2\dot{\phi}$ be replaced by?)
e) Evaluate equation 2 and the expression for dr/dt derived in in (d) for $t = -\infty$. Assume that $\phi = 0$ for $t = -\infty$. (Hint: In which direction does $\vec{V}$ point at $t = -\infty$? What is the relation between dr/dt and V_0?) Combine those two to eliminate the constant C and receive an expression for the angle ϕ_0.
f) From the figure one can see that the point of closest approach is when $\phi = \phi_0$. Then the orbit is symmetrical about this point and the angle through which the reduced particle's velocity is deflected is $\theta_{\mathrm{defl}} = 2\phi_0 - \pi$. From conservation of energy the speed before and after the encounter are equal (this is all the reduced particle!) thus equal to V_0. Compute the components $\left|\Delta\vec{V}_{\parallel}\right|$ and $\left|\Delta\vec{V}_{\perp}\right|$ of $\Delta\vec{V}$, parallel and perpendicular to the original velocity vector V_0.
g) Use the result from (c) to derive an equation also for the components of $\Delta\vec{v}_M$ from (f).
h) What happens when the test mass encounters many stars of mass m in a field? What is the quantitative result for $\Delta\vec{v}_{M\parallel}$ and $\Delta\vec{v}_{M\perp}$? How is $\Delta\vec{v}_{M\parallel}$ oriented with respect to $\vec{v}_M$? Does this result in an acceleration or deceleration?

Problem 4.2: Angular momentum of Galaxy

The large and small Magellanic Clouds (LMC, SMC) have distances of 51 and 63 kpc to the Milky Way (MW), respectively. If the two neighbors are gravitationally bound to the MW, is there more angular momentum in their orbits around the MW or in the rotation of the disk of the MW itself?
Assume masses of 2×10^{10} and 2×10^{9} $\mathrm{M}_\odot$ for LMC and SMC. The MW disk can be simplyfied to be infinitely thin, and to have an exponential density profile $\Sigma = \Sigma(r_0)\exp(-r/r_0)$ with $\Sigma(r_0) = 193$ $\mathrm{M}_\odot$ pc^{-2} and $r_0 = 2$ kpc. Further assume that the disk is rotating rigidly and one full rotation of the MW takes 150 Myr, and that the total mass of the MW inside the orbit of the Magellanic Clouds is 7×10^{11} $\mathrm{M}_\odot$.