3. Übung zu: Astronomie & Astrophysik II

(Abgabe: Do., 13.5.2004)

Problem 3.1: Free-fall time and accretion

The collapse of gas or molecular clouds in the ISM is occuring unter the influence of self gravitation. If one assumes that a spherical gas cloud starts to collapse at time t_0 then it is possible to compute the duration of such a collapse until the center, the so called free-fall time τ_{ff}.

a) Derive an expression for τ_{ff}, assuming a spherical region with initial density ρ_0 and initial radius r_0. Make the simplifying assumption that the acceleration during the whole collapse is constant. (Bonus question, 3 extra points (!): Show that the exact expression for τ_{ff} *without* this assumption is $\tau_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\rho_0}}$.) Interpret the formula for τ_{ff}.

Answer:

$$\begin{aligned}
r &= \frac{1}{2}at^2 \Leftrightarrow a = \frac{2r}{t^2} \qquad \text{(Motion with constant acceleration)} \\
a &= \frac{GM}{r_0^2} = \frac{4\pi G\rho}{3}\frac{r_0^3}{r_0^2} \\
\Rightarrow \frac{2r}{t^2} &= \frac{4\pi G\rho}{3} r_0 \\
\Leftrightarrow t &= \tau_{\mathrm{ff}} = \sqrt{\frac{3}{2\pi\rho G}}
\end{aligned}$$

b) Assuming that in the center of the region a star is located which accretes the infalling matter, then in a rough approximation the accretion rate is $\dot{M} \approx M_J/\tau_{\mathrm{ff}}$, thus Jeans mass of the collapsing region divided by the free-fall time. Express the accretion rate once as a function of temperature and once as a function of the sound speed in the medium. Interpret also this relation.

Answer:

$$\begin{aligned}
M_J &= \frac{\pi^{5/2}}{6}\sqrt[3]{\frac{kT}{G\mu m_H}}\rho^{-1/2} \qquad \text{(Jeans mass)} \\
\tau_{\mathrm{ff}} &= \sqrt{\frac{3\pi}{32G\rho_0}} \\
\dot{M} &\approx M_J/\tau_{\mathrm{ff}} \\
\Rightarrow \dot{M} &\approx \sqrt{8/27}\pi^2\left(\frac{kT}{m_H\mu}\right)^{3/2}\frac{1}{G} \approx 5.4\left(\frac{kT}{m_H\mu}\right)^{3/2}\frac{1}{G} = 5.4\frac{c_s^3}{G}
\end{aligned}$$

c) Compute the accretion rate for a sound speed of $c_s = 0.2$ km/s (in solar masses per year).

Answer:

$$\dot{M} \approx 5.4 \cdot \frac{c_s^3}{G} = 5.4 \cdot 200^3/(6.672 \cdot 10^{-11})\,\mathrm{kg/s} = 10^{-5}\,\mathrm{M_\odot/yr}$$

Problem 3.2: Proper motion of stars

Assume that the sun and a star move in the plane of Milky Way with the same velocity and the same circular orbit. Show that the observed proper motion of the star is independent of the distance sun–star. How is proper motion defined? How large is the proper motion of this star (arcseconds per year)?

Answer:

Star–sun–center of MW form an isosceles triangle. Motion of angle ϕ of the sun around the center lead to a rotation of ϕ of the line sun–star, independent of the angle between sun and star (and thus independent of distance) since the angles in the triangle are constant.

Proper motion is the (angular) motion with respect to an infinitely distant fix point.

230 million years for one orbit. Per year thus $1/2.3 \cdot 10^8$ of a full circle, thus $360 \cdot 60 \cdot 60''/2.3 \cdot 10^8\,\mathrm{yr} = 0.0056''\,\mathrm{yr}$.