2. Übung zu: Astronomie & Astrophysik II

(Due: Thu., 29.4.2004)

Problem 2.1:

The Jeans instability criterion for the collapse of ISM gas can be exactly derived from pertubation theory or estimated in a simple way by means of the virial theorem. Show that the virialansatz yields

$$M_J \simeq 5.46 \left(\frac{k_B T}{Gm}\right)^{3/2} \rho^{-1/2}$$

for the Jeans mass. Here m is the molecular weight. In that process give an expression also for the Jeans length. Assume as in Problem 1.1 a nondegenerate monoatomic gas and as in Problem 1.2 that the density of the region (sphere) is constant.

Answer:

Virial theorem: $2E_{\text{kin}}/-E_{\text{pot}} = 1$ **in equilibrium**

$$\begin{aligned}
E_{\text{kin}} &= 3/2 k_B T \cdot N_{\text{molecules}} = 3/2 k_B T M/m \\
E_{\text{pot}} &= -3/5 G M^2/R \\
\Rightarrow 2E_{\text{kin}}/-E_{\text{pot}} &= \frac{3k_B T M/m}{3/5 G M^2/R} \\
\Rightarrow R_{\text{J}} &= \frac{1}{5}\frac{GMm}{k_B T} \\
\text{with} \quad M(R) &= \frac{4\pi}{3}\rho R^3 \\
\Rightarrow R_{\text{J}} &= 1.09\sqrt{\frac{k_B T}{\rho G m}} \\
\text{now mass :} & \\
\Leftrightarrow R &= \sqrt[3]{\frac{3M}{4\pi\rho}} \\
\Rightarrow \sqrt[3]{\frac{3M}{4\pi\rho}} &= \frac{1}{5}\frac{GMm}{k_B T} \\
\Longrightarrow M_{\text{J}} &= \left(\frac{3}{4\pi}\right)^{1/2} \rho^{-1/2} 5^{3/2} \left(\frac{k_B T}{Gm}\right)^{3/2} \\
&= 5.46 \cdot \rho^{-1/2} \left(\frac{k_B T}{Gm}\right)^{3/2}
\end{aligned}$$

Problem 2.2:

Compute the Jeans length and mass for a) an ISM with a density $\rho = 30\,\mathrm{H/cm^3}$, a temperature $T = 200\,\mathrm{K}$ and only composed of hydrogen as gas, and b) for a gas of density, temperature, and compostion as the earth's atmosphere at sea level ($\sim$100% nitrogen, 1018 hPa, 15° C; assume the above formula is also valid for N_2). c) Interpret the mass- and lengthscales in both cases in the corresponding context (interstellar space and earth's surface, respectively).

Answer:

a)

$$\begin{aligned}
m_{HI} &= 1\mathrm{u} = 1.66 \cdot 10^{-27}\,\mathrm{kg} \\
R_J(\mathrm{ISM}) &= 1.09\sqrt{\frac{k_B \cdot 200\,\mathrm{K}}{4.98 \cdot 10^{-20}\,\mathrm{kg/m^3} \cdot Gu}}\,\mathrm{m} = 7.7 \cdot 10^{17} = 25\,\mathrm{pc} \\
M_J(\mathrm{ISM}) &= 5.46 \cdot 3.9 \cdot 10^{24} \cdot 4.48 \cdot 10^{9}\,\mathrm{kg} \\
&= 9.55 \cdot 10^{34}\,\mathrm{kg} = 4.8 \cdot 10^{4}\,\mathrm{M_\odot}
\end{aligned}$$

b)

$$\begin{aligned}
m_{N_2} &= 2 \times 14\mathrm{u} = 4.65 \cdot 10^{-26}\,\mathrm{kg} \\
pV &= NkT \qquad \text{(ideale gas law)} \\
\Rightarrow N &= pV/kT = 1013\,\mathrm{hPa} \cdot 1\,\mathrm{m^3}/(1.38 \cdot 10^{-23} \cdot 289\,\mathrm{K}) \\
\Rightarrow N &= 2.5 \cdot 10^{25}\,\mathrm{N_2/m^3} \\
\Rightarrow \rho &= 1.18\,\mathrm{kg/m^3} \\
R_J(\mathrm{Erde}) &= 1.09\sqrt{\frac{k_B \cdot 289\,\mathrm{K}}{1.18\,\mathrm{kg/m^3} \cdot Gu}} = 1.9 \cdot 10^{7}\,\mathrm{m} = 19000\,\mathrm{km} \\
M_J(\mathrm{Erde}) &= 5.46 \cdot 4.6 \cdot 10^{22} \cdot 0.91\,\mathrm{kg} \\
&= 2.3 \cdot 10^{23}\,\mathrm{kg}
\end{aligned}$$

c)

Star clusters can be formed, but not yet individual stars. 25 pc is a large region, but corresponds to size of observed star formation regions. The earth's atmosphere can not collapse, because the Jeans mass can not be reached. Turbulences in the atmosphere (=wind) continuously mix the air and dominate the energy budget (=internal energy).