1. Übung zu: Astronomie & Astrophysik II

(Due: Thu., 22.4.2004)

Problem 1.1: Cooling of White Dwarfs

White Dwarfs (WD) still radiate after ejection of their envelope. The required energy is taken largely from the thermal energy of the remaining ions – since the electrons of the star are already in a degenerate state, not much energy can be generated from contraction. You are asked to first compute the thermal energy of a WD and then conclude a potential remaining radiation time from this.

a) The heat capacity per ion c_i shall be given (assumed to be only a function of temperature)

$$U_i = \int c_i dT.$$

Assume $c_i = 3/2k$ for a non-degenerate monoatomic gas and derive a relation for the total heat capacity (or thermal energy) U for a WD of mass M, temperature T and composed only of ions of effective mass $A \cdot m_u$ with $A = 5$.

b) How large is the heat capacity for $T = 10^7\,\mathrm{K}$ and $M = 1\,\mathrm{M}_\odot$?

C) How does this relate to the bolometric radiation of the sun?

d) From the ongoing radiation the thermal energy of the WD continuously decreases. Write down the relation between the change in thermal energy $-dU/dt$ and the luminosity L of the WD. Assume, that the luminosity is proportional to $T^{3.5}$ and the mass, with a constant of proportionality $C = 2 \cdot 10^6\ \mathrm{erg\,s^{-1}\,M_\odot^{-1}}$.

e) Determine the cooling time $\tau = (t - t_0)$ by integration, assume here that $T(t_0) \gg T(t)$ and express τ in terms of luminosity.

f) How large is τ for $L = 10^{-3} L_\odot$, with T and M as in part c)? What does this mean for the possibility to actually observe WDs?

Problem 1.2: Neutrinos from SN1987a

With the end of fusion inside a star also the outward forces stop that counteract the gravitational force and keep the star stable. Stars with masses from $\sim$8 to $\sim$25 $\mathrm{M}_\odot$ subsequently collapse from the influence of gravitation down to a much smaller radius at which the degeneration of neutrons creates a new counteracting force. They become Neutron Stars. In this collapse a large part of the stellar mass is ejected in a supernova explosion, while large amounts of energy is released.

a) Assume the core of a massive star, core mass $M = 1.4\,\mathrm{M}_\odot$, collapses from a radius of $r_i = 1000$ km to a radius of $r_f = 10$ km. Show that the emitted energy is $\sim 3.1 \cdot 10^{53}$ erg, under the assumption that both before and after the collapse the density in all parts of the core is constant.

b) $\sim$99% of the energy releasen in a supernova is emitted in the form of neutrinos that are created in process of converting normal elements to neutrons. The supernova SN 1987a is at a distance of $5.1 \cdot 10^4$ pc from earth. Compute, how many neutrinos from SN 1987a can be expected in experiments on earth. Assume as an energy the above $3.1 \cdot 10^{53}$ erg, as energy of the neutrinos $E_\nu = 20\,\mathrm{MeV}$, as detector surface $500\,\mathrm{m}^2$ and a detection probability of 10^{-16}.